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ABSTRACT

This research is part of the London-based Concepts in Secondary Mathematics and Science Project. It is derived from developmental psychology, with particular emphasis on the implications of the results to the teaching of mathematics in secondary schools. A basic premise is that if one first identifies the child's present level of understanding of a particular mathematical topic, it will then be possible to select appropriate experiences to facilitate the movement of the child from this level to a higher one. In this portion of the research, tests were developed to enable teachers to classify children on the continuum of understanding, and data from large-scale testing were used to generate detailed descriptions of these levels for a number of mathematical topics. The eight chapters of the report focus on needs, a review of related literature, and an outline of the methodology used; the development of items, including a summary of the content of each test; the development of a marking scheme; selection of the sample and administration of tests; statistical methods of forming a hierarchy; the hierarchies for each topic area; strategies and errors; and a summary. Eight appendices, most related to statistical concerns, are also included. (MNS)

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FOREWORD

The research reported here is based on developmental psychology with particular emphasis on the implications of the results to the teaching of mathematics in secondary schools. A basic premise is that if one first identifies the child's present level of understanding of a particular mathematical topic it will then be possible to select appropriate experiences which will facilitate the movement of the child from this level to a higher level. One limitation of the present research is that only the first part of the premise has been investigated. The research team members are to be commended for their work in developing tests which will enable teachers or researchers to classify children as being at some point or level on a continuum of understanding and their use of data obtained from large scale testing to generate detailed descriptions of these levels for each of a number of mathematical topics as well as more general descriptions of stages across topics. There is now the need for further research to demonstrate that there is some 'appropriate experience' which, for a given group of children at a particular level of understanding for a given topic, facilitates their movement to a higher level.

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PREFACE

The research programme 'Concepts in Secondary Mathematics and Science' (CSMS) based at Chelsea College, University of London was funded by the Social Science Research Council for the years 1974-1979. The research was firmly based in schools and on the school curriculum so that the topics investigated would be recognisable to teachers as the type of mathematics they taught.

The original directors of the programme were Professor Kevin Keohane and Professor Geoffrey Matthews; later when they were no longer at Chelsea Colleg Professor Paul Black became director. Professors Eric Lunzer and Geoffrey Matthews acted as consultants to the mathematics team and we would like to acknowledge the help they gave us.

At various times mathematics educators have joined the team to help write test items, interview children etc. Their help has always been welcome. We would particularly wish to express our appreciation to Tony Malpas (the first leader of the Mathematics team), Brian Joyce, Ian Stewart, Lesley Booth and Gabrielle Cornwell.

Finally our work would have been impossible without the active involvement of many teachers and children. We thank all those schools which volunteered their help and allowed us to test their children.

This research monograph is intended for fellow researchers in the field of mathematics education. Other source documents are the following Ph.D., M.Phil. (London) theses which resulted from the work on individual topics carried out by members of the CSMS Mathematics research workers:

The Concept of Graphs in Secondary School Pupils Aged 12-14 Years
Daphne Kerslake (1977)

Secondary School Children's Understanding of Ratio and Proportion
K.M. Hart (1980)

Children's Understanding of Vectors and Matrices
G. Ruddock (1980)

Two other research reports are currently in progress. These represent detailed analyses of the results of the following topic areas:

Number Operations, Place Value and Decimals
(M. Brown)

Algebra, Generalised Arithmetic (D. Küchemann)

A number of articles have been written for teachers, a list of these appears in Appendix 8. In addition a book Children's Understanding of Mathematics, 11-16, written for teachers is to be published by John Murray, London in 1980. The book provides a general description of the results of the research with particular emphasis on implications for teachers.

Eleven of the CSMS Mathematics test papers together with teachers' guides will be available from the National Foundation for Educational Research, Windsor, Bucks (scheduled for publication in 1980).

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CHAPTER ONE

INTRODUCTION

This monograph reports on the mathematics component of the research programme 'Concepts in Secondary Mathematics and Science', CSMS. The programme was financed by the Social Science Research Council, SSRC, for five years, September 1974-September 1979, and was based at the Centre for Science Education, Chelsea College, University of London. There were two teams of researchers, one for mathematics and one for science; they investigated areas of common concern, but with different methodologies and emphasis. This monograph is concerned with the mathematics work only and CSMS(M) will be used to denote the mathematics component of the CSMS project.

The Aims of the Research

In the proposal three of the primary aims of the CSMS research were as follows:

(i) to identify order of difficulty throughout the treatment of individual topics in currently developed courses in science and mathematics; and to formulate and test hypotheses concerning the difficulties;

(ii) to develop a concept 'map' of secondary level mathematics and science and to indicate probable outcomes of different partially-ordered teaching sequences within its framework;

(iii) to provide evaluatory procedures designed to help teachers to identify the stages reached in their pupils' thinking and which would also stand up to external scrutiny.

This monograph describes the research which was designed to fulfil these aims for secondary school mathematics.

It had been proposed to adopt Piaget's stage theory for the research, with the intention of producing a clearer and more detailed description of the distinction between concrete and formal operational thinking, through the identification of the 'concept map'. The major work of Piaget which investigates formal thinking (The Growth of Logical Thinking, 1958) contains few examples which are strictly mathematical and the opportunities for using his work directly in this research were severely limited. For this reason, and others noted later in this report, it was decided that it was not feasible to interpret the work strictly in terms of a Piagetian framework or theory. However the underlying assumption throughout the research was that it is possible to describe children's understanding in terms of levels or stages each of which is more comprehensive and complex than the preceding stages. In this regard, many examples from the work of the Geneva school were incorporated in the set of items which were used to investigate the nature of mathematics learning and a separate class test composed entirely of items taken from Piaget (referred to as The Piaget Test) was written.

Two aspects of Piaget's type of research which formed an important basis of the research were, to quote Lovell (1979), that the system "puts the focus on pupils' thinking as shown by their explanations and justifications" and that "it gives recognition to the active construction of knowledge by each pupil". These aspects imply that a hierarchy of understanding in mathematics cannot be identified simply by examining the logical structure of mathematics or by analysing a given concept as a network of prerequisite concepts or skills.

Thus, for example, though it may be obvious that an understanding of long multiplication is dependent upon, and substantially more difficult, than understanding the multiplication of numbers less than ten, say, without testing children there is no way of knowing the degree of extra difficulty, nor whether children's modes of understanding match the formulated network.

Reference will be made throughout this report to items taken and adapted from the work of other researchers including Piaget.

Need for the Research

The traditional form of assessment of mathematics understanding is that of giving a test which is essentially used to rank children. The test might be to assess the effectiveness of what had been taught in a single topic or what had been taught in an entire course e.g. 'O' Level papers. At one time the test would have been entirely algorithmic in content and require the child to repeat rules used in questions of a recognisable type. This form is still common in many classrooms when the teacher writes a test paper, it is not so common in national examinations such as C.S.E. or 'O' Level. The knowledge gained from such testing enables one to put children in rank order and to state which questions were correctly solved by certain children and by how many children. In general, it does not include ideas of how certain children failed to solve questions nor how they might use their mathematical knowledge sometime after the teaching of a topic. Little or no attention is given to the notion of a sequence of attainment or a hierarchy of understanding.

Researchers in Mathematics learning have often on the other hand deliberately chosen to avoid testing rule acquisition and instead substituted the testing of 'concepts' (although few define concept). Piaget is foremost in this field. However the resultant ideas of how children interpret the problem, the methods they use for solution and the relative difficulty of the concepts is necessarily tied to problems which have little to do with what happens in the normal mathematics lesson. One aim of CSMS(M) therefore was to use problems which were recognisably connected to the mathematics curriculum but which would require the child to use methods which were not obviously "rules". In addition it was not intended to rank children on performance but to give information on the type of question at which the child succeeded and the understanding necessary for this success as well as pinpointing the type of understanding which was missing and which accounted for the failure on different items.

Related Literature

The literature available on learning hierarchies in Mathematics tends to fall into three main categories:

a) a learning sequence or sequence of understanding, which is essentially in the learner

b) a teaching sequence which is in the teacher, textbook or teaching machine program

c) a logic sequence which is in the topic.

Suppes (1966) has noted that there is some evidence to suggest that although connected these three aspects are not the same:

For anyone interested in the psychological foundations of mathematical concept formation it is natural to ask what is the sort of connection that holds between the logical structure of mathematical concepts and the psychological processes of acquisition of the concept. As far as I know, not very much has been written in the psychological literature about this kind of question. My present view, based partly on our experiments and partly on conjecture, is that the psychological stratification of mathematical concepts will seldom, if ever, do violence to the logical structure of these concepts; but it will markedly deviate from the mathematical analysis of the same concepts with respect to the amount of detail that must be considered.

The claims made by the Piagetian school of research would seem to embody aspect (a) in that the research is based on clinical interviews where the child is asked to explain attempts at certain tasks. The responses are categorised and are judged to be indicative of certain modes of thought (from pre-concrete to late formal). Piaget's stage theory also embodies what one might consider to be all the requirements of a hierarchy e.g. integration of lower levels into higher levels, gradual consolidation, unifying characteristics shown by inter-related behaviours, concepts and skills and equilibration. It is not a model of an additive nature, stage A need not be obvious within stage B and the integration of the two would constitute a transformation and entail re-structuring and coordination. Consolidation always involves an aspect of the recently acquired behaviour and an aspect of preparation for the behaviour of the following level. The actions or operations of a given level are not simply juxtaposed in an additive fashion but are organically interconnected and form a total structure "structure d'ensemble". There should also be a succession of levels of equilibrium at the heart of the evolutionary process. The theory is vast in its implications and researchers tend to take small sections of it and attempt to validate or provide information for the teacher on that specific part.

The research of the Geneva school tended to investigate one task with a group of children and then another task with a different group of children so that often the 'unifying characteristics' of the stage theory were not tested. Considerable argument on the feasibility of an overall stage which was not task specific has recently appeared (Laurendau and Pinard 1969, Hughes 1979) although Lovell (1961) and Shayer (1978) provide evidence which supports the stage theory. The integration of lower levels into higher levels and the suggested order of acquisition of concepts have been tested by a number of researchers who have assumed the Piagetian order as an hypothesis and then attempted to validate it. Some of this research has used statistical techniques to assess the child's performance in terms of integration of levels. Elim Kofsky (1966) is one such researcher. She took a sequence of classifactory skills with children aged four to nine years. She ranked the tasks by frequency of success and then split them into six groups the order of which correlated significantly with the hypothesised Piagetian order at $p < .01$ level. Then she looked at the requirement that the mastery of a particular task showed that all previous tasks had been mastered and used a Loevinger coefficient to quantify the connection (and concluded that this hypothesis was not supported). In addition she looked at the success pattern for each child; there should have been 12 patterns if success at harder meant success at each easier (in order) task. Thus arranging the tasks in order of difficulty we should have:

Score	Easy					Hard						
1	1	0	0	0	0	0	0	0	0	-	-	-
2	1	1	0	0	0	0	0	-	-	-	-	-
3	1	1	1	0	0	0	-	-	-	-	-	-

but not score 4 of say 000110011--. Only 27 per cent of the sample showed perfect patterns, 63 patterns in all occurred. Thus she concludes that the scalability of the stages was open to doubt.

The same type of experiment was carried out by Wohlwill (1960) who investigated the development of number concepts with young children aged 4 to 7 years. The child had to match numbers in seven tasks which had an hypothesised order of acquisition.

A pass on a set of trials was taken as 5/6 or 10/12; subjects who attained this mark were given three additional tasks but since they almost invariably succeeded on these items the tougher pass mark of 8/9 was abandoned in favour of 5/6. Only 45 of the 72 subjects had perfect scale type responses and when those with everything correct or everything incorrect were deleted, half of the remaining group had perfect responses. Wohlwill concluded that the data did not warrant the acceptance of each test representing a discrete, well determined point on the scale of conceptual development, and he was particularly concerned that there was no significant separation between pairs of adjacent tasks. He looked too at the performance within each task to see whether the children tended to get all or nothing correct.

Some recent research by Noelting (1978) is also in the Piagetian mode. He has taken the same task and increased its complexity by varying the numbers involved. The task involved mixtures of water and orange juice; the child being required to state the relative taste of the mixture. The subjects were aged 6-16 and came from mathematically advanced classes in the upper middle class socio-economic group in Quebec City. The items were ranked according to difficulty, and a Guttman scalogram analysis used to test for scalability. Noelting then assigned Piagetian levels to sub-groups on the basis of his knowledge of Piagetian theory and the ages of the children tested. This leads one to question whether one needs a cross check that the children who are labelled 'early formal' on this task should also be given a further task or tasks which have been described by Piaget. Is a task at the formal operational level because a) it is successfully completed by children capable of/consistently using formal operational thought, b) it demands abstract thought for completion (the abstraction judged by the experimenter), c) it is successfully completed mainly by children in late adolescence, d) it is hard, so completed by few children?

To assign a child to an operational level is difficult if the child is presented with different tasks; on one task one may be able to distinguish between the quality of responses, but on a number of tasks the child performance is often task related. If one gives a task which requires formal thought for successful completion then the lower level responses are of necessity incorrect in that they contain erroneous reasoning. To try to assign a child to a cognitive level based entirely on precise errors on particular tasks is bound to be difficult. Very often the error has not appeared in the descriptions given by Piaget and one must interpret its significance in the light of his general theory but besides this the level of response (or type of error) might be very task specific.

Gagné (1968) has proposed the idea of a learning hierarchy and has further allied this with ideas of sequence of instruction. To Gagné learning hierarchies are characterised as "an ordered set of intellectual skills such that each entity generates a substantial amount of positive transfer to the learning of a not previously acquired higher-order capability". The assumptions made by Gagné include:

1) Any human task may be analysed into a set of component tasks which are quite distinct from each other in terms of the operations needed.

2) The presence of these tasks ensures positive transfer to a final performance, their absence reduces such transfer to near zero.

3) The design of instruction requires a) the identification of the component tasks, b) ensuring each task is fully achieved, c) arranging the total in a sequence which will produce optimal transfer.

The problems one faces with this type of hierarchy are numerous, to start with how does one construct the hierarchy and how does one know that it *is* a hierarchy? Gagné suggests that to form the tasks one simply asks "what would the individual already have to know how to do in order to learn this new capability simply by being given verbal instructions". He also suggests that the hierarchy can be checked empirically for positive transfer by taking as a hypothesis that an individual will not be able to learn a particular topic if he has failed to achieve any of the subordinate topics that support it. The study by Gagné and Paradise (1961) tested this hypothesis by administering an examination immediately after the instructional programme and considering pass/fail patterns. The index used was

PPT - Proportion Positive Transfer

$$PPT = \frac{1 + 2}{1 + 2 + 3} \quad \text{where}$$

1: Number of children who correctly answered questions for higher skills and all relevant lower skills.

2: Number of children who were incorrect on a higher skill and at least one lower skill.

3: Number of children who were correct on a higher skill and incorrect on at least one lower skill.

White (1974) points out that although PPT values of greater than .91 were found for all the ratios, the experiment should be tried on different samples, the criterion for success on a skill is open to debate and that high values of PPT can be obtained even if the subordinate skills are indepedent.

This raises a number of points about learning hierarchies viewed as a sequence of tasks:

1) one can show that a skill is a prerequisite if it is to a far greater degree less complex than the next in the chain, see Figure 1.1.

Figure 1.1

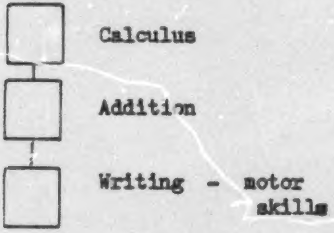
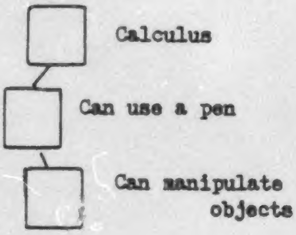


Figure 1.2



2) One can obtain a high PPT value even if the skills are not immediately relevant to each other i.e. the hierarchy is not valid in a content sense, for example a high PPT value could arise from the sequence given in Figure 1.2.

3) The gaps between composite skills seem to be significant; although one needs each of the lower skills, what else is required?

Merrill's (1965) work illustrates some of these points. He tested the hypothesis that learning and retention of a hierarchical task are facilitated by mastering each successive component of the hierarchy before continuing in the instructional programme. Merrill channelled students who failed on a component into a review/correction procedure but his results tend to infer that it is not necessary to master one level before procedding to the next. Briggs (1968) suggested that Merrill's task analysis might be faulty - he questions the validity of the hierarchy. Note too that we must also distinguish between what the student knows and what the student can do although we tend to test the former by the latter.

The aim of CSMS(M) in trying to form hierarchies of understanding in a number of mathematical topics resulted in an amalgam of a topic sequence and a sequence of understanding for each topic. In order to write items which varied in complexity, the topics were analysed for key concepts but no attempt to describe a teaching sequence was made nor were the degrees of complexity stated in terms other than 'this is harder'. Items were then written to embody the key concepts and the performance of children on these items provided the basis for the determination of a hierarchy of understanding. It is recognised that the research was assessing understanding in the sense of 'what can a child do?', rather than 'what does the child know?'.

Methodology

In order to present a picture of secondary school children's mathematical understanding which was representative of the English child population the decision was taken that the final assessment instrument should be in a form easily administered to large numbers of children. Hence an important component of the research was the development of written tests. These were written by the CSMS(M) team and were designed mainly in a problem-solving format in order to probe understanding rather than to test whether certain skills and algorithms had been acquired. The content to be included was identified through a careful analysis of the curriculum materials being used in schools and through a review of relevant research carried out by other mathematics educators. The general topic areas selected for investigation and age groups to be assessed are given in Table 1.1.

Items were written which were free of technical words and these were tried on interview with children and then replaced or revised. The interviews were used for a second purpose; that of finding the methods used and errors made by children when confronted with a mathematical problem. About 30 children of the appropriate age range, from different schools, were interviewed on each topic and their replies tape-recorded and later transcribed and analysed. The methods used by the children were often other than those taught by teachers - and such methods were not necessarily idiosyncratic as some were frequently repeated by children from different schools. The errors made could similarly be generalised and the answers obtained could later be observed in the written tests and interpreted with respect to the strategies which were likely to have led to them. The interviews were thus an integral part of the methodology, being used to inform and interpret the results of the written tests besides being used as a method of assessing the suitability of the test items.

Table 1.1

CSMS(M) Topics and Age Groups

Topics	Ages
Measurement (Length, Area Volume)	12-14+
Number Operations (the meaning of the operations on whole numbers)	11-12+
Place Value and Decimals	12-15+
Fractions (Fractional notation, addition and subtraction. Extended to include multiplication and division for ages 14-15+).	12-13 14-15+
Positive and Negative Numbers (notation, addition, subtraction and multiplication)	13-15+
Ratio and Proportion	13-15+
Algebra (Generalised Arithmetic)	13-15+
Graphs	13-15+
Vectors	14-15+
Matrices	14-15+

The final forms of the tests were administered to over 10,000 children (with the intention that every child in the sample was to be tested on at least two topics). The data were then analysed and the results used to postulate a hierarchy (both within and across topics), identify common (frequently occurring) errors or inappropriate strategies, and indicate implications for teaching. The general implications are given in a book written for teachers, called Children's Understanding of Mathematics, 11-16. The book contains chapters on each individual topic tested as well as chapters on the overall hierarchies and implications for teaching.

Copies of all tests with the marking schemes are in Appendix 1. While all tests are referenced throughout this monograph, it is not feasible within the space available to include a complete description of each stage of the research - methodology, design, and analysis - for each topic. Hence, when it is necessary to illustrate a particular point or stage, this has been done in the context of the topic of Ratio (to give some continuity to the overall report). Results, conclusions, and implications (Chapters 6-8) are, of course, discussed for all topics.

Chapters Two and Three of this monograph describe the development of the tests. Chapter Four deals with the sample and administration of the written papers for each phase of the project with particular reference to the large scale survey and the three longitudinal studies. A discussion of the statistical and procedural techniques employed for obtaining the hierarchies for each topic area is given in Chapter Five along with alternative techniques which were considered and either modified for inclusion or rejected.

Chapter Six reports on the major findings of the research. The hierarchies for each of the topic areas in terms of levels of understanding are described and the chapter concludes with a consideration of stages of development across topics (a crossmatching of levels leading to a very general description of stages).

Chapters Seven and Eight include some further discussion of errors and strategies and aspects of needed further research.

CHAPTER TWO

THE DEVELOPMENT OF THE ITEMS

The two main phases of the research, the interviews and the wide scale testing using written tests, were clearly dependent on careful selection or development of items/exercises (hereafter referred to as problems) which were consistent with the school mathematics curriculum and were potential (valid) indications of children's level of understanding of mathematical concepts. These problems had to be suitable for discussion with children in the interviews and also sufficiently self-contained to be intelligible when given in a written test.

In the writing of items and development of the test for each topic the writer(s) of the test followed the following procedure:

1) Analyse the topic as it appears in commonly used mathematics textbooks and where relevant as it appears in the science curriculum (e.g. ratio).

2) Investigate the results and examples used by other researchers with a view to using selected items to support or extend some of their findings.

3) Write (or select and modify) a series of problems in the topic so that they span a wide range of difficulty, are free of technical terms and thought to be testing understanding rather than the rote recall of a skill or technique.

4) Discuss the items with other members of the team and other mathematics educators who were acting in a consultative capacity, in order to ascertain as far as possible that the items were in fact testing the aspect of the topic stated by the writer and that the background information required for the understanding of the question could reasonably be assumed.

5) Interview some 30 secondary school children covering the relevant ages and abilities, using the items as a basis for the interviews. The interviews were to serve two purposes: i) to test the items for readability and general understanding, ii) to find the methods used by children for the solution of the problems and the strategies which led to errors.

6) Modify or replace the test items, in the light of the interviews.

7) Prepare the preliminary version of the class test and try it out in selected London secondary schools. The results being used to further modify items prior to the large scale testing.

8) Discuss possible levels of demand of the items in the light of the facilities obtained from the preliminary testing with classes. In certain cases try to match the items with possible Piagetian levels, particularly when items taken from the works of Piaget had been incorporated in the test. (In some cases e.g. Graphs and Ratio, the items were given to Piagetian 'experts' who were asked to hypothesise the cognitive level required for the successful solution of each of the items.)

9) Carry out large sample testing using the written class tests, marking codes for which had already been formulated to include the classifying or coding of errors which had appeared on the interviews.

Each of these steps is discussed in the following sections with particular reference to the topic of Ratio.

The Analysis of Textbooks

Schools in Great Britain choose which ever textbook they feel suits their pupils; there are some such as S.M.P. which are used in many schools. In order that the researcher's view of the topic should not be at variance with the teacher's view of the same topic, commonly used textbooks were analysed for key concepts and the stage at which they were introduced. The following is the analysis of the topic of Ratio as it appears in popular texts used in British schools.

The mathematics textbook series chosen for analysis were:

School Mathematics Project, SMP, Number Series 1, 2, 3, 4
School Mathematics Project, SMP, Letter Series A-H
The Scottish Mathematics Group 1-5
Pattern and Power of Mathematics (Moakes) Books 1-7
Midland Mathematics Experiment, MME, GCE and CSE
New Mathematics (Knight) Books 1-4

In addition various Science books, particularly Nuffield (Biology, Chemistry and Physics) and the project "Science Uses Mathematics", were investigated in order to ascertain where ratio and proportion appeared in the science curriculum. An analysis of the topic of ratio and proportion as it occurs in both mathematics and science books appears in Table 2.1.

SMP Book A introduces the idea of rate from a situation where something occurs three times per minute, the child is asked to record this as $x \rightarrow 3x$. Book D provides examples of the ratio 2:1 and also the writing in simplest form of 2 to 10 etc. Book F gives the statements on the properties of similar figures. Scale drawing appears in Book E. If one assumes that two books are used each year then the introduction to ratio occurs in the second year of the secondary school, no algorithm appears to be introduced. Similarly in the numbered series of SMP, book 2 discusses similarity and enlargement, the comparison of fractions representing proportions is used. Similar figures also occur in Book 2. Book 4 produces a method for solving problems involving ratio and proportion, not unlike the algorithm $\frac{x}{a} = \frac{y}{b}$.

The Scottish Mathematics Group series tends to introduce ratio earlier i.e. in Book 1. It also uses formal notation. Similar triangles are not dealt with until Book 4 however. A thorough discussion of direct and inverse variation takes place in Book 5, the finding of an unknown x in the form of $\frac{x}{a} = \frac{y}{b}$ is work set in Book 5.

Moakes spends some time in Book 2 on reducing fractions obtained from ratios, to their lowest terms, scale factors do not appear until Book 3.

Knight, and SMP, use set notations at some stage to describe ratio. There is no strict adherence in any series to an algorithm such as "Unitary Method" or $\frac{x}{a} = \frac{y}{b}$ (three amounts known, one to be found). The examples are varied and involve rate in scientific settings. The requirements of the Science subjects very often demand a concept prior to its appearance in any of the mathematics texts. The Science subjects themselves do not require the concepts at the same time. Physics always seems to require a particular aspect of ratio earlier than the other science subjects.

Table 2.1

An Analysis of the Topic of Ratio and Proportion as it Appears in Secondary Mathematics and Science Textbooks

	Books: SMP A-H	SMP 1-4	MME CSE	MME	Knight	Moakes	Scottish	S.U.M.	Nuffield Biol.	Nuffield Chem.	Nuffield Phys.
Ratio Words and Conventions	D(90) H(55)		1A(311) 2A(81)	1A(311)						Bk.1	Bk.1
Equivalent fractions (lowest form)	D(90) B(51)	2(206)	1A(110)	1A(110)	1(32)	3(108)	1(165)				
Fraction multiplier	H(96)	2(208)				6	3(171)	1			
Inverse multiplier		2(208)				6	3(171)				
Same units	D(90) H(96)	2(206)				3(108)	1(175)				
Written a:b or $^a/_b$	D(90)	2(208)					1(175)				
x:y, y:z find x:z		2(212)									
Scale											
model → real	E(26)	2(46)	2B(48)		1(149)	2(156)	1(175)	Bio.1-3	1		
real → model	F(82				1(149)	2(156)	3(179)				
representative fraction					1(149)	5(145)	3(179)				
Gradient	H(6,14) Z		2B(146)	2A(158)	2(44) 4(66)	7	4(115)	Bio.1-3		3/4	
Percentage	D(130)	2(220)	2A(155)	2A(73)		4(99)	1(177)				

Table 2.1 Continued

	Books: SMP A-H	SMP 1-4	MME CSE	MME GCE	Knight	Moakes	Scottish	S.U.M.	Nuffield Biol.	Chem.	Phys.
Rate Cost	H(10) D(140)						3(171)				2
Speed	B C(187)			2A(158)	4(59)		3(182)				
Interest					4(195)	4(99)	4(203)				
Conversion			1A(311)	1A(311)	1(149)	6	2(145) 3(171)				
"Rate"								3	2	3/4	1
Similar Figures											
Enlargement	D(24)	2(43) 2(51)	1A(183)	1A(183) 2A(36)	1(137) 2(137)	3(101) 3(124) 6	4(101)	4 Bio.1-3			2
Scale factor	D(24) C(85)	2(206)				5(143)		4			2
Area sim. figures	F(82)	2(56)		2A(36)	2(77)	5(143)	4(101)				
Volume sim. figures	F(82)	2(56)			2(77)	5(143)	4(101)				
Recognition sim. figures - 2D						3(126)					
3D						3(126)					
Triangles	D(24) H(196)			2B(119)	4(75)	3(126)	4(101)				
Properties of sim. triangles	H(96)				4(75)	3(126)	4(101)				
$\frac{a}{b} = \frac{p}{q} = \frac{a}{p} = \frac{b}{q}$		2(212)				6(158)					
Sharing in ratio	D(90) F(96)	2(127)									

Table 2.1 Continued

	Books: SMP A-H	SMP 1-4	MME CSE	MME GCE	Knight	Moakes	Scottish	S.U.M.	Nuffield		
									Biol.	Chem.	Phys.
Similar Figures											
Total number of parts	H(96)					3					
Pie chart	D(100)										
Notation											
$\propto$					3(101)		5(19)				
Variation					3(101)		5(9)				
Inverse					3(111)		4(4) 5(19)				
'Proportion'		2(212)					3(171)			3/4	1
Defined $y = kx$	H(55)	2(206)			3(101)		5(19)				
Proportion relation between 2 ratios		2(206)					5A(1)			3	
Unitary Method							3(171-9)				
Direct Method					3(101)		5A(1)				
$\frac{x}{y} = \frac{a}{b}$		4				3	5				

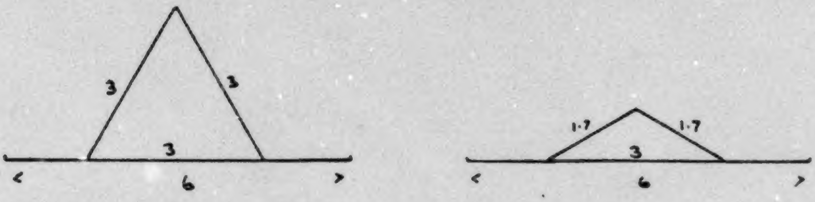
Other Researchers and Related Literature

In Chapter One some literature related to hierarchies was reported. Piaget's stage theory satisfied many of the criteria one might require for the existence of a hierarchy, therefore whenever possible items from the works of the Geneva school were incorporated in the tests. This was particularly true of Ratio and Measurement since in the former Piaget has stated that the use of a correct reasoning on proportion problems was indicative of formal operational thought and in the latter he has suggested an order of concept formation in various aspects of measurement. The formation of the Algebra test relied heavily on the work of Collis (1975) who has expanded the work of Piaget to more specifically describe understanding in mathematics. The use of research in the writing of test items is illustrated below within the context of Ratio and Proportion.

Piaget's Research on Proportion

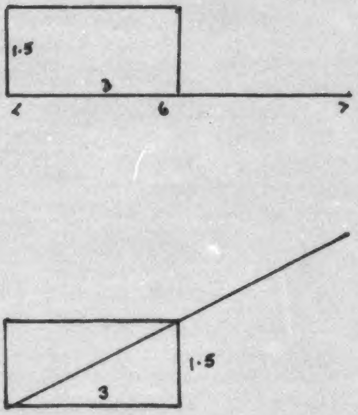
Piaget and Inhelder (1967) investigated the problem of metric proportion with respect to the triangle and rectangle. The task which presented similar triangles was to have two outcomes, firstly to explore the parallelism of the sides as a criterion of similarity between triangles and secondly to facilitate the study of the relationship between this criterion and that of the equality of the angles compared with length of sides. The method used was to provide the child with a triangle, a new base and ask the child to draw the correct triangle to circumscribe the one given originally. The midpoint of the new base was the same as the midpoint of the original base. The first part used only isosceles triangles, later irregular triangles were introduced. The emphasis was on the discovery of the parallelism of the sides. Figure 2.1 shows the type of example used.

Figure 2.1



Secondly the child was presented with a triangle on which two sides had been extended and the problem was to provide just one side. The investigation was extended to examples where the base was covered, depriving the child of a line to aid in drawing another parallel, then the child was presented with triangles already drawn and asked to draw similar ones which were not circumscribed. Lastly the subject was asked to sort sets of triangles into similar and non similar pairs. For comparison the child was asked to circumscribe a given rectangle with a similar one, compare similar and non similar rectangles and draw a rectangle circumscribing the given one which had its diagonal extended (in order to see whether the child made use of the extension). See Figure 2.2.

Figure 2.2



A further group of questions dealt with similarity of triangles. In this context the subject was allowed to handle them freely - superimpose and choose similar ones. The two types of presentation allowed the authors to categorise the children's responses in substages.

Piaget and his co-workers have generally postulated that the four stages of cognitive development occur at roughly the following ages:

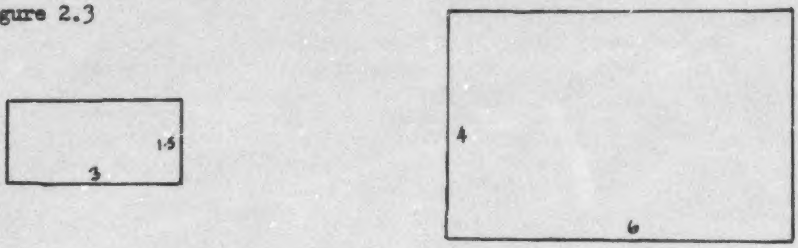
early concrete	age 7-8
late concrete	age 9-10
early formal	age 11-12
late formal	age 13+

The notation they ascribe to each of these stages is not consistent, i.e. in some books stages I, II, IIIA, IV are used and in others 2A, 2B etc. In the Child's Conception of Space (presently being discussed), the roman numerals correspond roughly with ages thus: IIA, IIB (age 4-7), IIIA (age 7-8), IIIB (age 9-10), IV (age 11-12). Some idea of the level of response and its position on a concrete operational-formal operational continuum can be obtained by comparing ages. Table 2.3 gives the notation and corresponding ages for a number of Piaget's works.

Children performing at substage IIA (on the triangle tasks) took no account of parallelism or equality of angles, at substage IIB they showed an intuitive idea of parallelism in a few cases. "Stage III marks the appearance of operations facilitating general comparison of parallels, angles and simple dimensional relations" (p.326). At stage IIIB the children made comparisons which took account of both parallelism of elementary dimensional relations and when superimposing triangles they related parallel sides and equal angles. Stage IV showed the attainment of true proportionality for all dimensional relations. Judging the similarity of rectangles was more difficult than judging the similarity of triangles.

The study of similar rectangles subsequently reported in the same book was undertaken because although rectangles are less complex than triangles in that all the angles are right angles, they provide less clue to the lack of similarity (which is apparent in non similar triangles). Using phrases such as "is the same shape but bigger" Piaget presented rectangles, the smallest of which was 1.5cm x 3.0cm; the child was asked to choose those which were similar to the small one (see Figure 2.3).

Figure 2.3



Next the child was asked to draw a rectangle similar to the 1.5cm x 3.0cm one given.

It was impossible to define a stage for the very youngest children. Stage II children chose rectangles which were too long, this occurred as well when the child was asked to draw. They showed no desire to measure and proportions appeared meaningless. Stage III (appearing at the age 7 or 8) showed spontaneous attempts to measure but the children did not realise that proportions rather than increase in absolute size were involved and the length of the rectangle was exaggerated. During substage IIIB the figure was enlarged by the addition of a length; only in the case of the ratio 2:1 were the correct answers given. It was only at stage IV that the child began to understand proportionality and his constructive thought outweighed perception.

An experiment using an open figure, where three lengths had to be enlarged, was also reported; the stages being in the same order but the advent of each a little later. The ability to use metric proportions on all three lengths being apparent at about age eleven. It is worth noting that even at stage IV on the closed rectangles, only one child of twelve years was quoted as multiplying by a non integer rather than trying to build up to an answer by using the difference in lengths of the two figures.

In Epistemologie et Psychologie de la Fonction (1968) Piaget and his co-authors dealt with proportionality in other settings. The experiments were quoted by Lovell (1971) in the N.C.T.M. booklet on Piagetian research. Lovell described the experiments and stated that the findings were suggestive "but they are in great need of confirmation" (p.138).

The first described was the presentation to the child of 'fish', five, ten and fifteen centimetres in length, the child was provided with firstly 'meat balls' and secondly 'biscuits' with which to feed the fish; they had appetites corresponding to their lengths. The questions asked were:

1. If fish A (5cm) gets one ball, how much must be given to B and C?
2. If B (10cm) gets four balls, how much must be given to A and C?
3. If C gets nine balls, how much must be given to A and B?

The questions were repeated using the continuous quantity of biscuits. Piaget and his fellow authors divided the children's responses into four main categories. The first or most naive being when the child reasoned that if the fish was larger he received more food but any extra amount would suffice. At the next level the child attempted to provide some regularity in the increase of the amount but gave only one more for the next largest in size. The third stage showed a more systematic increase in food, sometimes two units more for each of the larger fish: "Dir (7.4) donne 5, 7, et 9 : J'enleve 2 de 9 et j'enleve 2 de 7". This third stage was described by Lovell as the use of pre proportions "If the difference between A and B is a and that between B and C is b, then the child's preproportionality is of the form a is to a' as b is to b', but in which the equality of crossproducts is missing." (p.139). The fourth stage was the use of metric proportions. The questions using the biscuits were on the whole harder than those using the meat balls and each stage occurred later.

Lovell (1971) also called attention to four other experiments reported in Epistemologie et Psychologie de la Fonction each dealing with proportionality. These were:

1. Rectangles that have constant perimeter but in which the length of one side is decreased, leading to an increase in the adjacent side.
2. Serial regularities of diameters and positions of rings placed on rods of different lengths which were themselves set at fixed distances apart.
3. The distance travelled by a point on the rim of a wheel in relation to the size of the wheel.
4. The relationship of the size and frequency of rotation of wheels and the distances travelled by objects at the end of strings the other ends of which encircle the wheels.

Lovell summarised the findings by describing the first stage (lasting up to about eight years of age) as an inability to coordinate variables, i.e. a decrease in height of the rectangles does not bring about an increase in width. The second stage displayed the starting point of the appreciation of "all functional variations", an awareness of simple correspondence but a lack of comparison between absolute differences and no direct or inverse compensation. The third stage arose at about ten to twelve years of age and showed the beginnings of true proportionality, for example in experiment two above, it was realised that the distances between the rods must vary. It was not until stage four that the full understanding was reached, Lovell said "In Piaget's view, two conditions must be fulfilled before this stage is fully reached: the pupil must be able to both handle the boundary conditions of the variables and the ratios between the successive ordered values of the variables" (p.141). The child must in fact be capable of quantifying the proportion rather than describing it in qualitative terms.

Piaget (1958) included the beam balance experiment where the child was asked to place weights on either side of the fulcrum so that the bar remained horizontal. An effective performance was obtained at about age 10+ but the explanation was in terms of 'the heavier it is, the closer to the middle'. It was not until the early formal level that the proposition was given by the child in terms of $W/W' = L'/L$ (where W and W' are two unequal weights and L and L' the distances at which they are placed). This explanation truly belonged to the late formal stage but sometimes appeared earlier.

Piaget generalised from other experiments dealing with proportion and further described a characteristic view of the situation shown by children at all the angles are right angles, they provide less clue to the lack of similarity (which is apparent in non similar triangles). Using phrases such as "is the same shape but bigger" Piaget presented rectangles, the smallest of which was 1.5cm x 3.0cm; the child was asked to choose those which were similar to the small one (see Figure 2.3).

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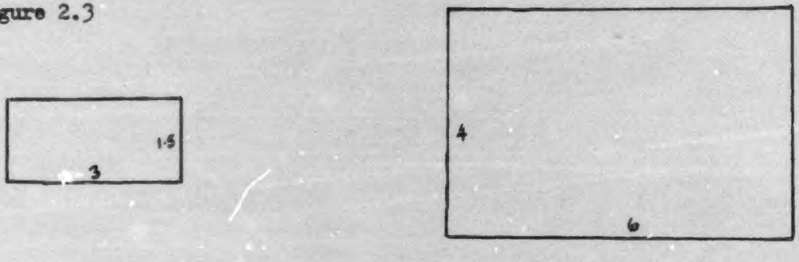

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Piaget generalised from other experiments dealing with proportion and further described a characteristic view of the situation shown by children at the late concrete stage (IIB).

> We should remember that an understanding of proportions does not appear until substage IIIA; this is true in all spheres and not only in the balance scale experiments. During substage IIB it has often been noted that subjects search for a common denominator of the two relations that they compare, but this common relation is thought to be additive. Thus, instead of the proportion W/W' = L'/L one would have an equality of differences W-W' = L-L'. (p.177).

Describing the different levels of performance on the task which has a truck (containing different weights) attached to a counterweight and balanced on an inclined plane, Piaget again gave a generalised description of concrete operational behaviour:

> As long as the subject is limited to using concrete operations of classes and relations, he cannot determine the law....
>
> The explanation is two fold; first the correspondences which must be empirically established are too complex, and second, the products of the multiplications of relations are in part indeterminate. (p.190).

He has already explained that at the concrete stage the child cannot simultaneously take the three facts (weight in truck, counterweight and slope) into consideration. The child might deal with them in pairs but forgets the third factor. At the formal stage the child seeks to coordinate the three factors into a single law. The proportionality statement comes about by substituting the height of the plane for its inclination and this does not appear until the late formal stage. In the interviews the substitution of height was suggested to one child who immediately afterwards formulated the law of proportionality, a second type of response came about because there was a search for proportionality and the factor of height arose as a consequence of the search. The realisation of what was needed was therefore fairly immediate in each case.

The distinctive feature of the late formal operational level on the task of placing different size rings at distances from a screen so that the shadows of two are identical was that the children had already formulated a hypothesis: "You have to put the largest the furthest away, and the ratio between the diameters of the rings and the distances has to be the same". The early formal replies took into account the distances from the screen of the first ring and not simply the distance between rings, but the children did not have a generalised hypothesis.

Generalising on the proportionality schema based on replies to the three tasks mentioned above and a fourth concerning balls of different weights placed at different distances from the centre of a spinning disc, Piaget said:

> Given two independent variables, the subject constructs the qualitative proportionality schema when he understands that an increase in one gives the same result as a decrease in the other. In all cases the structure of proportions requires an element of compensation. (p.219).

To find types of behavior at each of the levels of operational thinking one might signify late formal behaviour by the appearance of the use of hypotheses, an appreciation of the multiplicative relation required and an ability to deal with three variables. At the early formal stage the child can often deal with specific situations but not state a general law. At the late concrete stage the child does see there is a relationship but looks at differences (an additive relationship rather than a multiplicative one) and further faced with three variables ignores one and copes with just two.

The Research of Karplus

Karplus et al have published various findings on research dealing with children's understanding of proportion. In May 1970 Karplus and Petersen reported on research dealing with a version of the Mr. Short and Mr. Tall task which was later used in a different form in a wide scale survey. The 1970 version had two pin men each of which was measured by the experimenter demonstrating to the class of children. The first measurement was with large paper clips (biggies), Mr. Short being four biggies tall and Mr. Tall being six biggies in height. The children were then asked to measure a smaller version of Mr. Short using small paper clips (smallies). The problem was to predict the comparable height of Mr. Tall in "smallies". The replies were categorised in seven ways, the first (N) being no explanation. Category I involved intuition or guessing, category IC was assigned to children who used the data haphazardly and in an illogical way i.e. some inaccurate reasoning was present. Category A was assigned to those children who used all the data but who applied the difference rather than the ratio of measurements thus saying "the little man was 4 of his and 6 of mine so I added 2". Category S (scaling) used multiplication but not by the correct factor, 90 per cent of these children in fact doubled. Category AS (addition and scaling) involved a multiplicative strategy combined with an additive one e.g. "I think two smallies are as big as one biggie, so I added four smallies for the two extra biggies." Finally there was the category (P) for children who could solve the problem by setting up a ratio. Karplus did not order these categories and found that although the children from suburban homes achieved level P by the time they left school this was not true of the urban children in his study. Two years later Karplus and Karplus (1972a) used the same item with the same children to find out the progress towards level P of each child and in order to hypothesise an ordering for the responses, particularly levels AS, S and A. In the original experiment about one fourth of the children reaching level P in fact used a geometric method of solution, dividing Mr. Short into fourths by visual estimation and then extending Mr. Short's height by two fourths, the extended length being measured with their chain of 'smallies'. Karplus stated that levels P and AS were more advanced than the others because 28 per cent of those in other categories moved into these two over the two years, more than a third of the students showed no movement at all. Categories I and IC were regarded as the most naive types of response since 65 per cent of subjects in these categories moved into others over the two years. The study did not provide evidence for the ordering of the categories A and S since the same number of children moved from A to S as moved from S to A.

Karplus, Karplus and Wollman (1972b) extended the research using a second version of the Mr. Short and Mr. Tall problem, this time the measuring was done with buttons and paperclips. The sample was chosen from fourth to eighth-graders (ages 10 to 14). This second version of the task did not show Mr. Tall or the buttons and Karplus concluded that on this task the children relied on the measurements and not on their perception as they had done in the previous problem. The categories of response were extended and subdivisions of the AS and P responses described in the earlier research were provided. The following categories of response remained the same: N (no explanation), I, IC, S and A. AS was now defined as an explanation focussing on the excess height of Mr. Tall and using a scaling up of the two excess buttons by a factor not mentioned in the data. AP was a similar explanation but the scaling factor for the two buttons was based on the data. PC was an explanation which used the relation that a button was about 1.5 paper clips but the relation was obtained by measuring a quarter of Mr. Short's height with paper clips (frequently shown by pencil marks on the paper). Finally category R provided an explanation where the scale factor was derived directly from the data and the factor was applied by multiplication. The results showed that category S responses were now very small in number compared with the percentage of children providing this reply on the first version of the task. An attempt was made to equate the responses to the stages used in Piagetian theory, these will be described later. Karplus, Karplus and Wollman (1972b) also commented on the effect of changing the ratio:

> Careful examination of the responses has convinced us that a student's ability to recognise and apply a 2:1 ratio is not sufficient evidence of proportional thinking. The 3:2 ratio involved in the task we have described is significantly more demanding. (p.5).

The Mr. Short and Mr. Tall task was used with another task which involved the control of variables (Karplus and Karplus, 1974), the sample being eighth-graders. This work was a pilot study for a larger cross cultural study carried out in Europe. The countries visited in the seven nation survey, (Karplus, et al, 1975) were Denmark, Sweden, Italy, United States, Austria, Germany and Great Britain. No claim was made by the researchers that the sample from each country was representative of the child population and each was described in terms of its socio-economic setting. The second version of the Proportional Reasoning task was used and the response categories telescoped to provide just four: I (intuitive) - not making use of all the data or using the data in a haphazard way; A (Additive) - the explanation focussed on the difference; Tr (Transitional) - the explanation showed only partial proportional reasoning or made reference to concrete comparisons; R (Ratio) - the explanation used a proportion or derived the exact scale, no concrete or iterative procedures were employed. A further proportion question was included: "Mr. Tall's car is 14 paper clips wide. How wide is Mr. Tall's car, measured in buttons?". The car question proved to be more difficult. A composite score was obtained from the pair of responses for each child, if the scores were different, category R combined with any other was called Tr, Tr combined with A was scored A, and either Tr or A combined with I was scored I. The percentages for 3,300 children aged 13.6 to 14.6 were: I, 28 per cent; A, 15 per cent; Tr, 32 per cent; and R, 25 per cent.

The task for assessing the students' reasoning which involved the control of variables (another concept identifying the stage of formal reasoning in Piagetian terms) described the collision of two spheres rolling on a track. After identifying the important variables the students could use the principle that only one variable at a time must be changed if an experiment is to be conclusive. The children in some countries found this very much more difficult than the proportional reasoning task.

The researchers concluded that about seven per cent of their total sample were formal thinkers, this was shown by the fact that they were fluent in the use of ratio and scored high marks on the control of variables task. The distribution of the responses varied from country to country and from one socio-economic group to another, in particular the incidence of the additive strategy appeared to be very low for some of the populations. It was therefore concluded that the additive strategy was not a part of the development of the understanding of ratio:

> The present data strongly suggest that additive reasoning does not lie on an invariant development sequence but is strongly influenced by instruction and represents an effort by students to deal with a task in an ad hoc rather than a systematic way. (Karplus, et al. 1975, p.65).

The British sample was drawn from schools in London; Grammar schools, Indepedent schools and Comprehensive schools - the first two types of school being highly selective. The children were completing their third year in secondary school. The number of students from each type of school was small, sometimes two classes only. The results are shown in Figure 2.4.

Summary

The Mr. Short and Mr. Tall problem was included in the CSMS(M) Ratio paper and was tried on a sample very much larger than that shown in Figure 2.4; the children chosen being representative of the English child population on I.Q., from areas other than London and in three different age groups. The fish question of Piaget was also adapted for inclusion on the Ratio paper, both in a form similar to that used in Geneva and also when different ratios were involved. The enlargement of an open figure was also included and the enlargement of a rectangle appeared on the CSMS(M) Piaget Test.

The analysis of their results by Karplus and Piaget pointed to a number of significant errors made by children. The analysis of the Ratio paper included the following specific points highlighted in their research:

a) the addition strategy

b) adding on one to make larger

c) the regular addition of a number more than one to make larger

d) doubling or halving when not appropriate

e) distinguishing the ratio 2:1 from other ratios.

Technical Terms and Skills

Some technical terms and notations are so important in mathematics that it is difficult (awkward and even confusing) to avoid their use, for example terms such as area or volume. In cases such as these either a trial item or a definition of the term was included on the test paper, for example:

Algebra. Trial items demonstrating the use of the arrow in $x \rightarrow 3x$ and the use of a in a + 4, 4a, were provided.

Rotations and Reflections. Practice questions on reflections were given, the child was asked to check his answers by folding; rotation practice was given together with the answers.

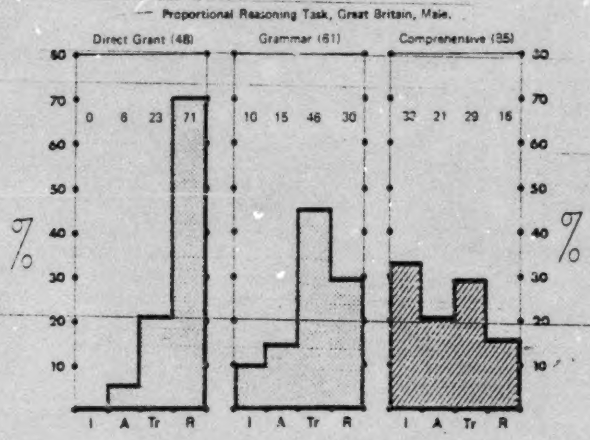
Measurement. The words perimeter, area and volume were defined with examples for the child (the answers were given).

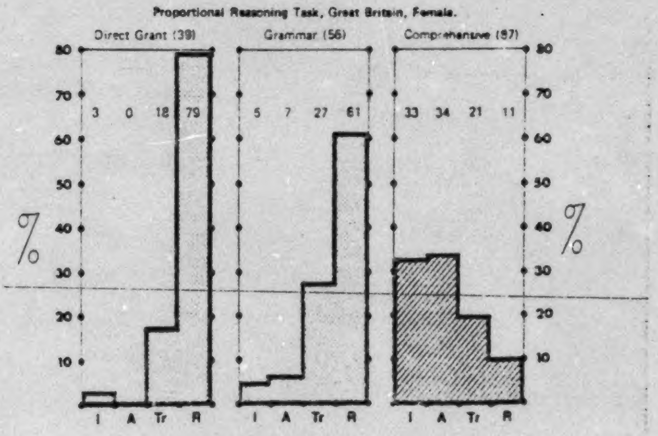
Number operations. A trial item was provided and discussed with the class for both the operations and stories. The children were reminded of the names and symbols for the four operations.

3~

Figure 2.4

Proportional Reasoning Data, Great Britain (Karplus, et al, 1975)





Vectors and Matrices. Each term was introduced.

Place Value and Decimals. The British standard notation was used, e.g. 30 000 rather than 30,000 and 4.29 rather than 4·29 was explained. A practice item explaining the naming system for the places with a picture to show the relationship between hundredths, tenths and a whole unit was provided.

Other terms commonly used by teachers seem not to be part of the child's vocabulary and so were replaced by more generally used terms even if these were less precise. For example, the word 'similar' has a very specific meaning in mathematics, but during the interviews it became obvious that to most children it meant 'approximately the same' so that all triangles were similar since they were all triangular in shape. 'Similar' then had to be replaced by other phrases such as 'the same shape but bigger'.

Computational skills often form a major part of assessment in mathematics; the child being asked to complete a computation which is written in exactly the same form as many already attempted in class being the usual testing strategy. Most problems in everyday life do not however come in this ready packaged form and the child has to decide which operation is needed before attempting the problem. In order to test understanding rather than the repetition of learned rules it was decided that the test papers should be composed of word problems where the numbers were simple and excessive computation could be avoided. Two exceptions to this were the tests on 'Decimals and Place Value' and 'Fractions'; in the first some addition and subtraction 'sums' were included and in the second a separate sheet of computations which mirrored the word problems was given to the children in order that a comparison could be made between performance on the computations and on the problems.

The analysis of the textbooks gave some indication of which parts of the topic were considered difficult by teachers and textbook writers - such material usually occurred late in the series of books. In addition the members of the research team had their own experience to guide them since all had taught mathematics to secondary school children. Attempts to analyse the topic and the problems a priori, gave no indication however of the degree of difficulty or how wide the gap was between the understanding of one section and another. For example, an analysis of the items which were selected for the final version of the CSMS(M) Ratio paper could be made in terms of the numbers involved and the steps required for solution, see Table 2.2.

An analysis such as this, carried out prior to the interviews was necessarily based on what the researcher thought was the demand of the item which in some cases was at variance with what the child considered to be required. For example in Table 2.2 a number of items are described by the operation of multiplication and yet during the interviews it was soon apparent that a ratio of 3:2 for example was not seen as a multiplier 3/2 of an amount 'a' but as an addition of a + $\frac{1}{2}$a.

Item Modification

The interview data were used for two purposes:

a) the identification of errors and methods (correct and incorrect) used by the children, which will be discussed in the next chapter, and

b) the type of modification which needed to be made due to certain misunderstandings.

Table 2.2

Analysis of Items on the Ratio Test in Terms of the Numbers Involved and Steps Required

No rate needed	Given Rate 1:x	Given Rate not 1:x	Find Rate 1:x	Find Rate not 1:x	Find rate requiring intermediate step	Operation
1, 2	7				16, 17, 15, 20	Whole numbers double or halve
2, 4						As above twice
	8		11, 6	9, 10, 27		Multiply whole numbers by integer
		25, 26		12, 13, 14, 22, 23	19, 21	x$1\frac{1}{2}$, x$2\frac{1}{2}$
					18	x3/5
5						Fraction operation

Note: The numbers quoted are not item numbers. Many items could be subdivided since questions of different degrees of complexity could be asked in the same problem situation using the same data. Each of these subsections was marked separately and each was assigned its own number, hereafter referred to as variable number (n). The number associated with each subsection of the questions is given in Appendix 1.

In considering (b), it was apparent that, firstly, there were problems in which the context of the question hid the essential mathematics. As an example consider the problem and interview notes given in Figure 2.5. In this situation it was clear that the problem for the child was not the mathematics but rather trying to understand that people might pay for coffee by the day and not by the amount actually drunk.

Figure 2.5

Original item:

In an office Mr. Adams comes in to work two days a week.
Mr. Brown comes in to work four days a week.
Mr. Carter comes in six days a week.
The bill for making coffee in the office for these three men is 240p.
How much should each pay for it to be fair?

Mr. Adams.......... Mr. Brown.......... Mr. Carter ....

Interview (Interviewer I, Child J)

J 10p, 8p, 6p.
I Does that come to 240p?
J That comes to 12 and I divided 12 into that which is 20, no that's wrong. 12 is the number of days they work a week.
I Why did you bother to do that?
J To tell you the honest truth I don't really know. Each one works two more days than the next one.
I So who should pay the most?
J Mr. Carter.
I Any relationship between Adams and Brown?
J Mr. Brown works two days more than Mr. Adams. It depends on how many times they have coffee a day.
I What can we assume?
J They have one cup a day. That has two cups of coffee a day, four, six, that makes 12 cups of coffee altogether in a week, 12 cups of coffee a day in a week.
I Can you have that?
J 12 cups a week then and one each day. It must be 20p a cup of coffee.
I You make the assumption they had one cup a day?
J Oh yes, they might have two cups a day.
I Would that make a difference?
J Yes.
I Do you think it would help if we knew how much a cup of coffee cost?
J It wouldn't really help. He might have three and he might have one.

Secondly there were problems in which the child made an incorrect assumption at the beginning but when questioned was prepared to think again. Such questioning would be impossible when written tests were given and so certain of these items were omitted. Figure 2.6 gives examples of such items along with excerpts from two interviews.

Figure 2.6

Questions

Tick which of the following are obviously true.
If not true state why.

a) Roger Bannister was the first man to run a mile in four minutes. He ran five miles in 20 minutes.

....

b) A Vienna loaf costs 12p, the shopper buys three, she pays 36p.

....

c) For one cake I use ten ounces of flour, for two of these cakes I use 20 ounces.

....

d) A duster takes 40 minutes to dry, the three dusters I have hung on the line should take two hours to dry.

....

Interview 1 (Interviewer I, Child A), Part a only.

A Yes. If he ran one mile in four min. 20 min is five times four min.
I Say if it was you.
A I'd get tired. Mathematically it's true, physically it's not.
I What do you mean by mathematically it's true?
A If it was a vehicle it would be true. People get tired.

Interview 2 (Interviewer I, Child B), parts a, b, c and d.

B True.
I Say if you were doing that?
B I was doing it mathematically not logically. If you were doing it mathematically you don't take notice of how much energy you need.
B If Vienna loaves were different weights and different prices, it would be different. (Correct)
B Because they're both the same cakes. (Correct)
B Because if you hang them all on the line at the same time, they'd all dry at the same time. One takes 40 mins so three must take 40 mins. (Correct)

A multiple choice format in which a number of answers were given was not often employed since it restricted the number of incorrect answers which might naturally arise if the children were left to solve the problems in their own fashion. On interview it soon became apparent that the methods selected for multiple choice items did not correspond to those normally used by the children and in order to choose one of them the child was reduced to working out the answer and then working out each of the options. Alternatively there were cases in which the child opted for one of the methods but was either unable to explain why or gave reasons not consistent with the computations. For example, consider the following item:

35 per cent of all adults read a newspaper. If a town has 25,300 adults, how many newspapers would one expect to be sold there?

(i) $\frac{35 \times 100}{25{,}300}$ (ii) $\frac{25{,}300 \times 35}{100}$ (iii) $\frac{25{,}300 \times 100}{35}$

(iv) $\frac{35 + 25{,}300}{100}$ (v) $\frac{25{,}300}{100 + 35}$ (vi) 35×253

James aged 14 knew why he opted for method (vi): "It's the simplest way. It has smaller numbers." Examples of children who worked out the question themselves or provided alternatives to the six methods provided were:

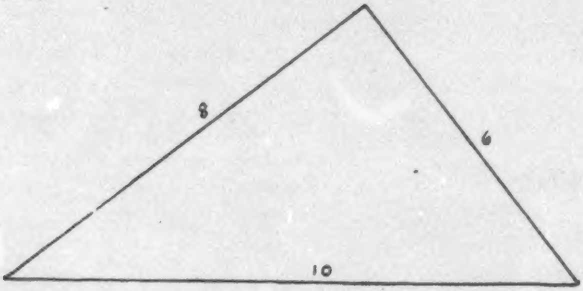
Mark: You want really $\frac{35 \times 100}{25,300}$

Tim: $253\overline{)25300}$ with 100 above — I've got to get my 100 then.

Patrick: 3500 per 10,000
3500 " "
1750 per ½(10,000)
105 3 times 35.

Another reason for rejecting or modifying a question was because the child had no way of checking whether his answer was correct. For example, one of the more open items on similarity required the subject to draw a triangle either larger or smaller (a choice) than that given, but of the same shape (see Figure 2.7). This introduced a factor of dependency on the child's skill in constructing the triangle.

Figure 2.7



You are asked to make a triangle which is the same shape as this one but which can be larger or smaller. Imagine you are making the new triangle with meccano strips, you must start with the one marked 12. Which other strips would you use? Draw a rough diagram. Are there any other possible triangles you could make, still starting with the one marked 12?

Meccano strips (lengths)

12 units
15 units
8 units
9 units
20 units
10 units
16 units
14 units

Interview (Interviewer I, Child J)

(When the children enlarged the triangle, the realisation that to simply enlarge the sides did not necessarily preserve the angles did not occur to many)

I Which would you start with?
J The largest one. Ten goes to 12, eight to ten and six to eight. If you want to make it to the scale of that but larger, if you add two

units onto the ten you ought to add two onto the eight and two onto the six.

I Say if you started with another side?

J Six to 12, ten to 16, eight to 14.

The results from the triangle questions and the difficulties the children had with the questions pointed to the need for an example on similarity where both the original and the enlarged figure were given.

It is not possible to report on all interviews, even for the single topic of ratio. However, the data were used to change approximately 30 per cent (on the average) of the items for each test. Many of the changed, or new, items were also tested in an interview context as a final check on their appropriateness to the research.

Class Tests

After the interviews, rewriting of items and discussion among the project researchers, a written test paper was tried on a number of children in selected London schools. A member of the research team administered the test to the children and was available to answer questions on terminology etc. It was also possible for the researcher to ask individual children which questions seemed to be hard and why. The papers were marked for correct and incorrect answers and the facility for each item computed. The children were ranked according to their total score and their performance on each test item coded as 0 (incorrect) or 1 (correct). A hard item was said to be discriminating well if it was successfully completed by only those children who had high total scores, an easy item should have been completed by children with low total scores as well. Items which appeared to be equally well done by either set of children were thought to be solved more by chance than understanding and were therefore subjected to further scrutiny. Such items were rewritten and tried again or omitted from the final test paper which was to be administered in the large scale survey.

The information used to test the discrimination of the items on the CSMS(M) Ratio paper is given in Appendix 2. The sample for administration of the initial version of the class test was 177 secondary age children (2nd- and 3rd-year) from four schools.

Note that each test was to be administered to more than one age group (see Chapter One) in order to provide data for a comparison of levels of performance, throughout the secondary school programme of mathematics. Thus the appropriateness of each test for a given age band was also a consideration in both the interviews and initial class testing of preliminary versions.

Assessing Levels of Demand from Results of Interviews and Class Tests

We have already mentioned that the analysis of a topic in terms of the Mathematics required does not necessarily match what is seen by the children to be the mathematical demand. Similarly if one asks a fellow researcher to a) rank the items in order of difficulty and b) state the cognitive level required in order to succeed, the reply is often based on an opinion on how this person would do the problems. Attempts were made to gather such information for a number of the tests, but no clear picture emerged since invariably when one considered the methods the children used, the opinions on levels had to be changed. For example in the Ratio question given in Figure 2.8 one might judge parts c and d, to be much more difficult than a and b, if one judged the method to be either multiplication by 3/4 or finding how much for one and then multiplying (i.e. division by eight followed by multiplication by six). Similarly one might hypothesise that e would be close in facility to c and d. A common method used by children in the interviews, however, was essentially additive. So that to find the amount of an ingredient for six people one used the answer obtained for four people and said 'the amount for four added to half the amount for four'. This made parts c and d seem nearly as easy as parts a and b, but it also made part e very much harder for the additive method involved (½ of ½) + ½(½ of ½).

Figure 2.8

Onion Soup Recipe for eight persons

8 onions
2 pints water
4 chicken soup cubes
2 dessertspoons butter
½ pint cream

I am cooking onion soup for four people.

a) How much water do I need? ....

b) How many chicken soup cubes do I need? ....

I am cooking onion soup for six people.

c) How much water do I need? ....

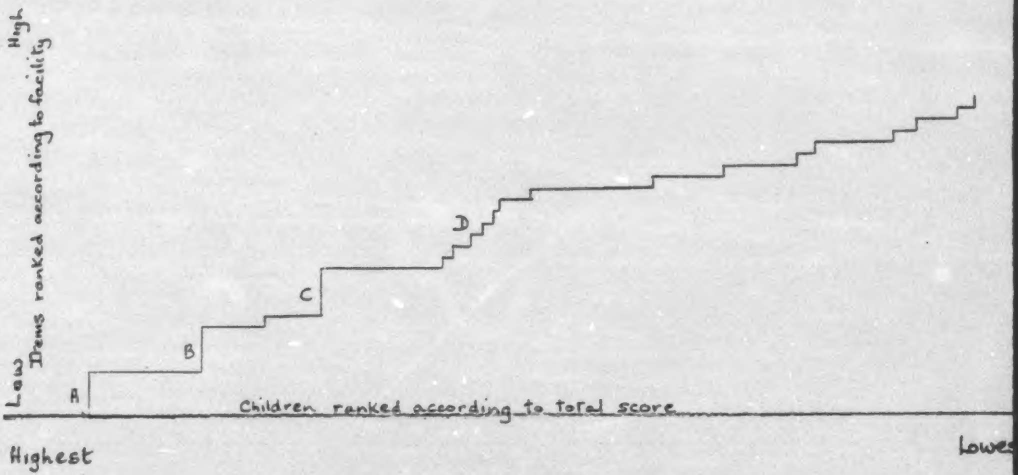
d) How many chicken soup cubes do I need? ....

e) How much cream do I need? ....

The results of the class tests enabled the researchers to rank items in order of facility and the discrimination data for the Ratio Test (see Appendix 2) suggests that there are 'plateaus' or cut offs between items - see Figure 2.9 which is a graphical representation of the data given in Appendix 2.

Figure 2.9

Plateaus in Ratio (Discrimination Graph)



A, B, C, D show steps in facility which correspond to groups of children

Another method of assessing the demand of the items was used on the Graphs paper, for which the results from testing 600 children were available. The items (and access to the results if desired) were given to three researchers familiar with the work of Piaget. They were asked to group items on a seven point scale (with 'demand' corresponding to stages from early concrete to late formal). The three opinions were not consistent in that even if one allowed an error of one stage, only 50 per cent of the items were judged to require the same level of thought by all three individuals.

Using the items on the tests which had been taken from the works of Piaget, an attempt was also made to match performance on these with general performance on the other items. Again however the children were not necessarily consistent or failed to respond in the manner described by Piaget.

The 'addition strategy' in ratio was identified by both Karplus and Piaget and is said by the latter to indicate late concrete operational thinking. The addition strategy used on the Mr. Short problem and on enlargement of an open figure was taken as an indicator of cognitive level and also a further item taken from Epistemologie de la Fonction (1968) was used. This additional question involved the sharing of numbers e.g. share 30 sweets between two boys so that one has six more than the other. It was not very successful in class test format as children who committed the error of halving and adding ($30 \div 2 = 15$, add 15 to 6 and subtract 6 from 15; answers 9 and 21) on the first part tended to do all the other parts of the question in similar ways. It was apparent that children who were performing at level IVA on the sharing task were using the addition strategy on the Ratio questions, this was therefore not a strategy adopted only by the children with a low cognitive level. The few children at level IVB (early formal) on the sharing question did not use the addition strategy. The performance of the subsample however was very inconsistent and no clear case could be made for designating groups of items as requiring particular levels of thought.

Grouping items according to facility (using the class test data) although providing information on likely candidates for group inclusion, did not take into account the commonality of the items i.e. were they completed by the same children? Grouping at this stage for purposes of formulating any initial or proposed hierarchy was therefore very tentative. However, the data obtained were supportive of the hypothesis that such a formulation was feasible and the final versions of the tests would be adequate for the task.

Summary of the Content of Each CSMS(M) Test

The aim of the research was to investigate secondary school children's understanding of mathematics. The mathematical topics chosen as the basis for the investigation are listed in Chapter One. A test was developed for each topic and final versions of these were administered to more than one age group in order to provide data for a comparison of performance between children of different ages. Items of different levels of difficulty were written for each test but obviously the entire school syllabus in any topic could not be covered. Since the papers were to be administered within the time available for the 'normal' mathematics lesson, none could require more than an hour for completion. The key concepts which provided the base for which items were written are described over. It is recognised that they are not exhaustive and do not cover all the secondary school syllabus.

Measurement

area

1. Conservation of area (complementary area)
2. Conservation of area and perimeter dissociation
3. " " " "
4. Measurement of area by unit reiteration ('tiling')
5. Effect on measure by reducing unit size
   Area compensation: dissociation 'square measure' and 'square shape'
6. Area measurement: recognition part-units
7. Measurement of area: whole and part-units
8. Development of formula (area by calculation):
   - (a) rectangular figures
   - (b) right triangle
   - (c) area by subtraction
   - (d) estimation
9. Construction rectangle with area and one side given:
   - (a) integral measure required
   - (b) fractional measure required
   - (c) construction rectangle with perimeter and one side given
10. Inverse operation, 'middle step': given area, base one rectangle, find area adjacent rectangle
11. Doubling the square:
    - (a) area
    - (b) perimeter

length

1. Dissociation 'length' and 'linear interval'
2. Length measurement: effect of reducing unit size
3. Comparison of length: need for common unit
4. Measurement of length: dissociation interval and endpoint
5. Measurement of length:
   - (a) non-zero origin
   - (b) whole and part-units
6. Length measurement: dissociation length and linear interval, length and area measure; additive lengths

volume

1. Notion of 'solid' volume by counting
   Inverse: given volume-by-counting and base, find height
2. Measurement by unit reiteration
   Effect on measure of reducing unit size
3. Measurement of volume: whole and part-units
4. Measurement of volume: cross-section x height
5. Development of formula (volume by calculation):
   - (a) rectilinear shapes
   - (b) triangular prism
6. Conservation of volume-by-counting
7. Conservation of complementary volume

Place value and decimals

1. Correspondence between 'name' and 'place'
2. The 'carrying' aspect of addition i.e. ten in one place is equivalent to one in the next place on the left
3. The 'borrowing' aspect of subtraction which is the reverse of (b) above i.e. one in one place is equivalent to ten of the next place on the right

4. Other relations between places, including comparison of size of two numbers.
5. Visual correspondence with fractions, both by length and area, as used in reading scales.
6. Significant figures and approximation.
7. The effect of multiplication by a multiple of a power of 10.
8. The effect of division by a multiple of a power of 10.
9. The notion that multiplication by a number less than one decreases the initial number, while division by such a number increases it.
10. The infinite nature of the set of real numbers.
11. Knowledge of the type of real situations in which decimals are normally used.

Ratio and proportion

1. Doubling or halving.
2. Multiplication by an integer.
3. Given a rate per unit, applying this rate.
4. Find a rate per unit and then apply it.
5. Enlargement of a drawing in ratio 2 : 1.
6. Enlargement of a drawing in ratio 3 : 2.
7. Enlargement of a drawing in ratio 5 : 3 etc.
8. Finding a ratio a : b using an intermediate quantity, c, i.e. given relationships a to c, b to c.
9. Using a fractional multiplier.
10. Simple percentages.

Vectors

1. Vectors as Ordered N-Tuples i.e. numerical column vectors either in a data storage "story" context or without any "meaning" attached. Simple addition, subtraction and scalar multiplication.
2. Vectors as Ordered N-Tuples Representing Movements and Position i.e. single and double translations, moves in two and three dimensions.
3. Free Vectors i.e. using the notation $\underline{a}$ $\frac{1}{2}\underline{a}$, structure and elementary geometrical applications.
4. Route Vectors i.e. using the notation $\overrightarrow{AB}$, $\overrightarrow{BC}$ for movements.

Matrices

1. The use of matrices to store data in a "story" context and the addition, subtraction and scalar multiplication of such matrices.
2. The manipulation of abstract (no "story") matrices of moderate order including the commutativity of matrix addition.
3. Elementary equations involving the addition and scalar multiplication of matrices containing some unknowns.
4. The logic of data storage including consistency of position and consistent units of measurement.
5. The introduction of matrix multiplication using a data storage approach.
6. The use of matrices to represent networks.
7. The introduction of matrix multiplication using networks.
8. The multiplication of abstract (no "story") matrices including order and the non-commutativity of matrix multiplication.
9. The use of matrix multiplication to answer "story" type problems.
10. Writing a "story" to make use of a given correct matrix multiplication.

11. The relationship between the elements of two matrices being multiplied and the link between this and what the answer matrix represents.
12. The use of matrices to perform rotations and reflections of a geometrical figure.

Algebra

1. Meaning of simple algebraic expressions.
2. Numerical substitution in algebraic expressions.
3. Comparison simple algebraic expressions:
   'like' operations (+ and -)
   'unlike' operations (+ and x).
4. Simplification of algebraic expressions.
5. Constructing algebraic expressions
   (i) by addition of constant
   (ii) by multiplication by constant.
6. 'Balancing' equations (performing same operation on both sides)
   (i) numerical operator
   (ii) algebraic operator.
7. Solution simple equations.
8. Numerical substitution in equations.
9. Algebraic substitution in equations.
10. Constructing generalised expressions/equations (e.g. for finding area/solving problems).
11. Solving simple equations in two variables for range of values.
12. Covariation - effect on one variable of changes in another.
13. Interpretation of algebraic expressions in two variables.
14. Construction of problem-solving equation in two variables.
15. Comparison algebraic statements which may vary re truth value.

Positive and negative numbers

1. Signed numbers viewed as shifts along the number line.
2. Positive and Negative cordinates.
3. The effect on co-ordinates of moving the origin (one and two dimensions).
4. Addition.
5. Subtraction.
6. Multiplication.
7. The use of negative numbers in recording temperature.

Graphs

1. Block graphs.
2. Interpretation of scattergrams.
3. The effect of changing axes on scattergrams.
4. Co-ordinates; whole numbers, $1\frac{1}{2}$ and decimal notation.
5. Finding the truth values for an open sentence given the graph of that open sentence.
6. Elementary ideas of rate.
7. Time-distance graphs.
8. Choice of correct scale (positive and negative).
9. Finding values for x, y, given a problem which must be interpreted in algebraic form. Plotting the graph.
10. Co-ordinates in an unfamiliar setting.
11. Gradient and parallelism.
12. Continuity.

13. Algebraic equations and their graphs.
14. Simultaneous equations in graphical form.

Rotation and reflection

1. Single reflections involving:
   (a) single points
   (b) flag heads and line segments
   (c) a triangle.
2. Reflections where the object, mirror line or image was off the page; the answers to be given in the form of co-ordinates.
3. Single rotations of a quarter turn; the image or centre of rotation had to be drawn.
4. Combinations of reflections and rotations and the resulting transformation.
5. Inverses.

Fractions

1. Recognition and labelling of a fraction as
   (a) part of a whole
   (b) subset of a set.
2. Sharing continuous quantity.
3. Equivalence of familiar ($\frac{1}{2}$) and unfamiliar fractions.
4. Ordering fractions according to size.
5. Comparison of fractions having
   (a) same numerator
   (b) same denominator.
6. Adding two fractions, same denominator and different denominators.
7. Subtraction from one.
8. Providing a fraction which lies between two given fractions.
9. Continuity.
10. Multiplication and division of fractions illustrated by area of a rectangle.
11. Multiplication arising from 'rate'.
12. Generalising laws of fractions.
13. Distributive law.

Number operations

Here, the aim was to examine children's understanding of the four basic operations (addition, multiplication and division) by finding to what extent children could both recognise which operations to apply in order to solve a 'word problem' set in the 'real' world, and supply an appropriate context for a formal computation 'sum'. On this test the numbers in the problems were restricted to whole numbers, but problems involving decimals were later included as part of the Place Value and Decimals test.

In Chapter One the topics were listed together with the age of child for which the test papers were considered suitable. In most cases the judgment of suitability was based on whether it could be assumed that the child was familiar with the notation and operations that perforce had to be used in the writing of the items. Some tests were appropriate for each year in the secondary school, others were written specifically for children of a particular age or because they had not been taught an essential operation. Two particular cases were the topics of Fractions and Matrices. It was thought that children who had not been taught multiplication and division of fractions would be unable to invent methods of finding solutions to problems involving these two operations and similarly those who had not been taught multiplication of Matrices would be unable to 'discover' the method. Two papers were written for each of Fractions and Matrices as follows:

Fractions 1. A subtest with problems requiring the recognition of fractional notation, addition and subtraction. A second subtest composed entirely of computations which to a large extent mirrored the problems. These two subtests comprised the Fractions 1 paper and were intended for the first and second year in the secondary school (ages 12 and 13).

Fractions 2. A subtest which included problems from Fractions 1 but also required the use of multiplication and division. A second subtest composed of computations (ages 14 and 15).

Matrices A. A paper which did not assume the knowledge of multiplication of matrices.

Matrices M. A paper which did include multiplication as well as problems in common with Matrices A.

Matrices A was given to those children (age 14-15) whose teachers stated that they had not been taught multiplication of matrices and Matrices M was given to those children whose teachers stated that they had been so taught.

Validation

Besides the phases of the research already described i.e. interviews prior to testing class tests and large scale testing using the written items, two other supporting investigations were carried out:

1. A longitudinal survey.
2. Interviews with children after they had completed a written paper.

Longitudinal Studies

The hierarchies in the chosen topics were to be based on the data obtained from testing a large number of children of different ages. A comparison of performance of different age groups could be made i.e. the performance of a representative (on IQ) sample of third years compared with a representative sample of second years but these two samples would contain different children. In order to obtain information on the progress of the same children, as they grew older, three small longitudinal studies were also carried out. Two hundred children who had taken the Ratio test in 1976, 200 who had taken Algebra and 200 who had taken Graphs, were tested again on these papers in the summers of 1977 and 1978. The results are discussed in Chapter Six.

Interviewing after the Test

Measurement. Certain items on the Measurement test paper had been adapted from Piagetian literature and given in written form with diagrams rather than with materials which could be moved and handled. It was thought possible that if children were provided with the materials, the errors they had made on the written paper, might not occur. Thirty children (age 14) who had completed the items in written form were given the written test paper two weeks later by the researcher. They were asked for their solution to the problems (questions 1, 7, 8, 18, 21, 24, on the Measurement paper). If their responses were incorrect they were then given the materials and asked the question again. Very little difference was observable between the performance on the written item the second time and when the materials were given. There was however often a difference in performance between the two attempts on the written paper, but this was not unexpected as the children had had an opportunity to think about the items and discuss them with each other.

Fractions. The same 30 children as above had also completed certain fraction problems and these were used as the basis of interviews some two weeks after the written test, in order to obtain more information on the methods of solution. The data obtained were used to assist in the classification of types of errors and inappropriate strategies (discussed in Chapter Three).

Piagetian Tasks

The Science wing of CSMS had adapted tasks used by Piaget and his colleagues in clinical interview, into class test format and had investigated the cognitive levels of a large number (10,000) secondary children using these tasks. The mathematics team attempted a somewhat similar task taking Piagetian work which appeared to be more mathematical than scientific in content. The aim was to match the mathematic topic hierarchies against Piagetian levels. Children were to be given both a mathematics test (Ratio, Graphs, etc.) and then a number of Piagetian tasks from which it was hoped their position on the early concrete-late formal continuum could be found. The plan therefore was:

1. to adapt some mathematical tasks found in the works of Piaget into a class test format;
2. to interview a number of children using these tasks and attempt to assign a cognitive level to their response to each task, using the descriptions provided by the Genevan researchers;
3. to refine the tasks until a form was reached which would be appropriate for testing about 500 children of different abilities and ages;
4. to give each of the sample a mathematics test;
5. to devise a marking scheme and then mark the Piaget tasks individually;
6. to assign to each child an overall Piagetian cognitive level (the basic assumption being that the responses would show a degree of consistency) and a 'level of understanding' on the mathematics test;
7. to cross tabulate mathematics performance against Piagetian level to provide a theoretical framework against which the hierarchies could be interpreted.

The first use of a single Piagetian task was Version 1 of the sharing number question, used at the stage of class-testing on the Ratio paper. It was anticipated that this would provide some evidence (in addition to the Piagetian items already in the paper) for grouping items. The item was given to about 30 children and it was found that many who made the initial error of dividing by two and then adding on the difference, continued to do so for most of the other questions.

The Piagetian tasks which were used on interview and then written in class test format (the CSMS(M) Piaget Test) were taken from six books of Piaget and his co-workers. Each task involved mathematical rather than physical science problems and had been described as requiring at least early formal thinking for its successful completion. The descriptions of behaviour indicative of thought at a level lower than early formal was also analysed in order that a marking scheme could be drawn up. The books used as source documents for the tasks on the six versions of the Piaget Test are given in Table 2.3.

Table 2.3

Source Documents and Stage Notation
for CSMS(M) Piaget Test Items

Sources	Stages mentioned			
	Pre-Concrete	Early Concrete	Late Concrete	Early Formal
1. *Child's Conception of Space*	IIA IIB	IIIA	IIIB	IV
2. *Origins of Chance*	1	2A	2B	3A 3B
3. *Child's Conception of Geometry*	IIA IIB	IIIA	IIIB	IV
4. *Epistemologie et Psychologie de la Fonction*	I	II	III	IV
Age		7-8	9-10	11-12

Note: The notation used in each book is matched on the basis of age of acquisition.

Version 2, containing ten questions was used on interview with children, the wording being changed and further examples and counter examples being built-in to try to explain similarity of rectangles and triangles.

Version 3 was comprised of five tasks selected from Version 2. This test was given to two classes of girls of below average ability (age 13+ and 15+) as a written test with two members of the team introducing the test and answering questions. A high ability group of 15 year-old girls was also tested in the same way. Then eight children from each of the first two classes were interviewed on a one to one basis to see whether their performance differed markedly on the two forms of testing, class test and interview. The high ability girls were interviewed in groups of four. There appeared to be little difference in performance in the two modes of testing, although the low ability girls found the questions very difficult in both forms. Ideas for re-wording and simplifying explanations were obtained from the interviews.

Various revisions of the five tasks on Version 3 were made and after two more revisions, a final form, Version 6 (see Appendix 3), was the one used for the purpose of crossmatching with performance on a mathematics test. Version 6 had five problems, one requiring the use of ratio, one needing combinations, another permutations, one in which the child had to copy angles and triangles and the sharing number question previously mentioned.

CHAPTER THREE

DEVELOPMENT OF A MARKING SCHEME

The marking scheme for each test paper was designed to take into account the specific errors and methods (both correct and incorrect) used by the children in their solutions to the items. The methodology employed both for identifying (defining) and clarifying these was to interview approximately 30 children for each set of topic items. It is the purpose of this chapter to present a general description of the interview protocols. This is then illustrated with specific examples from the topic Ratio, including the marking scheme.

Interview Protocols

As noted above, the interviews carried out on a one to one basis with children represent a critical component in the research programme. The interviews were used to

1) identify modifications needed in the terminology of the items
2) assess the suitability of the items for the given age groups
3) identify errors and the methods (both correct and incorrect) used by the children.

The information gained from the interviews was also used in forming and interpreting the hierarchies (Chapter Six); when the cut-off between two groups of items was not immediately obvious some items could be said to more naturally form a group since the same "child-method" was used on each. The general description given to the levels in each of the hierarchies could then take into account the methods used by children rather than merely relying on the mathematical demand of particular items.

Each set of topic items was used as the interview instrument with about 30 children; the interviewer was a member of the CSMS(M) research team. Children of different abilities (chosen by their teachers) were asked to talk their way through the problems and at each step to explain what they were doing. Each interview was taped and later transcribed.

Each interview took place in a quiet room made available by the school. The children were chosen by their teachers according to the interviewer's description of the type of child wanted, e.g. a third year child who is good at mathematics. Each interview took about an hour. Although not all children managed to solve the entire set of problems in that time it was considered that in-depth discussion of a subset of problems was more informative than a hurried account of all of them. Each child was informed about the nature and purpose of the interview, i.e. the subject would be asked some questions on how he or she did mathematics problems and that this was part of a research programme which was designed to help teachers in their teaching. The child was encouraged to ask questions and to point out words which were not understood. The interviewer took a neutral role and protocols were flexible.

Usually the interviewer started with what had been found to be an easy item and the child was asked to read it out loud, note was taken of words that appeared to be difficult. Sometimes the meaning of a word or expression was questioned by the interviewer, e.g. "Can you tell me what 'the same shape but bigger' means?". Then the interviewer provided two shapes and asked if they fitted the description and why they did. The child was then asked to do the example and explain what he or she was doing. The explanation was sometimes prodded by the interviewer with interjections such as "You have written two, would three be just as good?". The child was asked to defend an answer and although in some instances they were wooed away from their original answer by the interviewer's interjections, they were always asked finally to choose which one they preferred and why.

Exceptions to the Procedure

Fractions: The original ideas for the items on the problem paper for years 1 and 2 were obtained from a group of Bristol teachers who were attending an in-service course on the CSMS research. They interviewed the children in their own classes and reported back. A further five children (3rd year) were interviewed by the CSMS team prior to the testing and another 30 children (3rd year) after the testing.

Ratio: Eight children were interviewed in the language laboratory of their school. The interjections from the researcher were in this case less frequent than with the further 23 children interviewed on a one to one basis.

Errors

During the interviews various incorrect answers to items were apparent, if these arose from a recognisable incorrect strategy i.e. did not appear particularly idiosyncratic, they were coded when the written tests were marked. Other errors which appeared likely to occur but which were not apparent on interview were also coded and later during the marking if they did not appear on the written tests they were then rejected. In addition if during the first stages of the marking an incorrect answer appeared frequently, it was coded. The coding sheets and incidence of errors for each test paper appears in Appendix 1.

The following discussion includes excerpts from the interviews and selected quotations/comments made by the children who were interviewed on the Ratio test. The discussion also includes the research team's analysis of the child's reasoning. While complete transcripts are available these are not included due to space considerations. However the analyses do take into account the data obtained from a number of interviews and the selected excerpts are intended to be illustrative rather than comprehensive. With one exception all the interviewees quoted were 2nd and 3rd year pupils (ages 13-14).

Five questions (six if we consider each part of question two as a single item) have been included for discussion. The version presented for each question is that which appears in the final written paper (Appendix 1). This may be slightly different from that used in the initial interviews and any such deviations are noted. As stated previously the primary emphasis was on the correct and incorrect strategies employed by the children as these were used to develop the marking scheme described in Table 3.1 (and the same procedures were used to develop the schemes for the other papers).

Sample Interviews: Ratio

Question 1: Recipe (see Figure 3.1)

Figure 3.1

Onion Soup Recipe for eight persons

8 onions
2 pints water
4 chicken soup cubes
2 dessertspoons butter
½ pint cream

a) I am cooking onion soup for four people.

How much water do I need? ....

How many chicken soup cubes do I need? ....

b) I am cooking onion soup for six people.

How much water do I need? ....

How many chicken soup cubes do I need? ....

How much cream do I need? ....

Part a of this question essentially required halving. All 17 children interviewed used this method; none tried to find the amount for one person in order to use it as a rate, this would of course entail the manipulation of fractions. Part b produced a wider selection of methods especially the last part (cream) where often the method which worked on whole numbers was abandoned because it involved the manipulation of fractions. The most popular method for finding the amount of water and soup cubes for six people was to build up to an answer by taking half of a half, as in this reply:

> "Its a pint of water for four people, half of that for two people, add them its 1½ pints."

A lower level of reasoning was to say that six came between four and eight so the amount for six people should come between those for six and eight people. This is illustrated with an excerpt from one of the interviews.

Interview (I: Interviewer, A: Child)

A: Four for eight and two for four, it is three, that is halfway between two and four.
I: Two and a half is between as well, why three?
A: No I mean equally.

Finding a rate per person was used by a few children on part b; for example these two responses:

> "Eight people, a quarter of a pint of water was allotted to one person, its six quarters."

> "Half a pint for two people, multiply it by three."

Only two children of the 17 interviewed on this question immediately chose to multiply by three-quarters:

> "Its six instead of eight, that's three-quarters of everything."

When faced with the amount of cream for six people very often the method was changed, the child who multiplied by three-quarters said "For cream I did it differently, for eight people you need half a pint so that is 1/16 for every person. So for six people that is 6/16 or 3/8". The other child who had multiplied by three-quarters reduced the half pint to gills and then found three-quarters. Many of the children who had taken half-of-a-half continued to do so but could not handle the addition of two fractions:

Interview (I: Interviewer, B: Child)

B: Four people a quarter, its 1/4 and 1/8, 1/8 is half of 1/4.
I: How much is 1/4 and 1/8?
B: Is there another name for both of it? It was $\frac{1}{2}$ pint for eight people, half of that is 1/4 pint for four people, six is in between, its halfway.

Others immediately said the answer was a third because that fraction was between one quarter and one half. Some children who had been using the half-of-a-half method also changed when dealing with the cream question:

> "A half is four-eighths, one-eight of a pint for two people, three-eights for six."

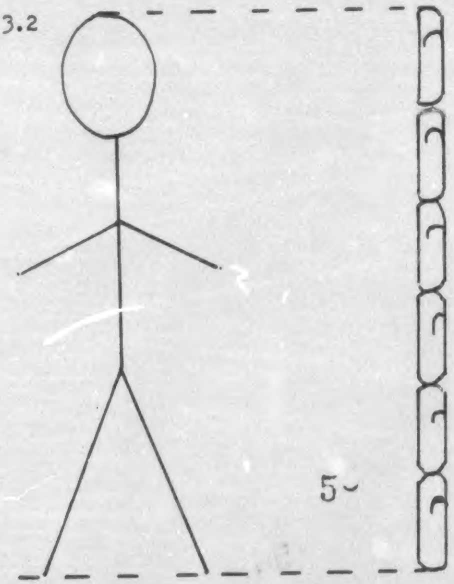
Others continued to use the half-of-a-half and successfully dealt with the addition of the two fractions. The cream part of the question caused considerable unease however. When asked by the interviewer why this part was more difficult one child who had been successful on all the other parts of the question said:

> "Its the fractions. The answer is 1/3, its going to have to be more than 1/4 and less than 1/2."

In the light of what the children did, question one essentially required the ability to halve, this procedure being used twice when the amounts for six people were required. The specific error investigated on the large scale testing was the answer "one-third" for the cream question in part b.

Question 5: Mr. Short and Mr. Tall (see Figure 3.2)

Figure 3.2



You can see the height of Mr. Short measured with paper clips.

Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:

Mr. Short's height is four matchsticks
Mr. Tall's height is six matchsticks

How many paper clips are needed for Mr. Tall's height? ....

Three sample interviews which illustrate correct solutions are given below:

Interview 1 (I: Interviewer, C: Child)

C: Each paperclips is one and a half matchsticks.
I: Why?
C: Six paperclips and four matchsticks. Four into six goes one and a half. Nine.
I: How did you get that,
C: Two matchsticks bigger. (long pause) It's 12.
I: Why?
C: If each paperclip is one and a half long, it's four matchsticks. Half is three paperclips. (pause) Nine. Just add six one and halves, makes nine.

Interview 2 (Child: D)

D: Nine. If Mr. Short is six paper clips and four matchsticks tall, it means paper clip is 2/3 matchstick. Six matchsticks is six times 2/3. You multiply that six by 3/2, you turn it upside down.
I: Why?
D: It's easier to work it out.
I: How many paperclips does one matchstick equal?
D: One-1/3. Same as 4/3, so it's eight. It's one and a half paperclips to a matchstick. It's four matchsticks and six paperclips. 6/4, that's the proportion, it's 3/2. It's 6 x $\frac{3}{2}$, which is $\frac{18}{2}$ or nine.

Interview 3 (Child: E)

E: That's one and a half. So it will be seven and a half. Mr. Short is four matchsticks and six paperclips. For every four matchsticks you add two paperclips. Four and half of four. You got six and half of six - nine.
I: Where did the one and a half at the beginning come from?
E: Six is one and a half times four.
I: Why did you give up that method?
E: It was confusing - that's where I got that seven and a half.

Other children started by using a fraction and then abandoned it. This is illustrated with excerpts from two interviews:

Interview 1 (I: Interviewer, F: Child)

F: Four matchsticks, six paperclips. 2/3. Divide your six by 2/3. No multiply. Four is 2/3 of six. So it's, no that's the wrong method.
I: What was the 2/3?

F: Four over six. You add half of that, half of four makes six, add half of that one makes nine. Matchstick 2/3, paperclip 3/3. Matchstick is one add a half which is equal to a paperclip. One-1/4 paper clips equal a matchstick. Four paperclips and two left over so that's a half. One and a half paperclips equal one matchstick.

Interview 2 (Child: G)

		P	M
I: (writes) as	Tall	?	6
	Short	6	4

G: Eight. Four to six and six to eight.
I: Any other ideas?
G: Four matchsticks doesn't go evenly into paperclips. Are we allowed to measure these? (Measures with ruler) Be about seven and a half.
I: Why?
G: It's almost that. I took the height and divided it by four because there's four matchsticks.
I: Why did you need a ruler?
G: Four won't go into six. I wanted to see how many matchsticks equalled paperclips.
I: Four matchsticks equals? One matchstick equals?
G: One and a half. Answer is nine.
I: Which answer do you want to choose?
G: I'll stick to nine. The amount of matchsticks doesn't go evenly into the amount of paperclips.

The most popular incorrect answer to question 5 was 'eight'. This was most commonly arrived at by employing an addition strategy; e.g.

> "Mr. Short is six paperclips but four matchsticks makes it two more. As he is six matchsticks, it should be two more."

or

> "Six equals four matchsticks. Six matchsticks equals eight paperclips. Two matchsticks higher so take two matchsticks away when you measure in paperclips is six. So it's six matchsticks, take two away, so eight paperclips. He's two more matchsticks."

Another child thought Mr. Tall should have double the answer and gave 12 as the answer immediately.

Interview (I: Interviewer, H: Child)

H: Twelve, no about 24. It's 12, for every two matches you have one paperclip. I don't know if that is true, I just thought.
I: Should you check?
H: If he were double his size then it still would be 12 paperclips.
I: Is he double?
H: No he's only two matches taller. Matches four and six... paperclips six and... It's about 12.

Karplus had identified the following strategies in the child's solution of the Mr. Short and Mr. Tall question: Intuitive, Addition, Transitional and Ratio. We have described the addition strategy, Intuitive was akin to guessing or not making use of all the data, it would include the doubling strategy mentioned above. The transitional category showed only partial proportional reasoning and would include the reversal of the match/paperclip relationship.

Question 6: Metal Alloy (see Figure 3.3)

Figure 3.3

In a particular metal alloy there are:

1 part mercury to 5 parts copper
3 parts tin to 10 parts copper
8 parts zinc to 15 parts copper

You would need how many parts mercury to how many parts tin?

.... parts mercury to .... parts tin.

You would need how many parts zinc to how many parts tin?

.... parts zinc to .... parts tin.

Few children on interview had a strategy which gave the correct solution, although none appeared to have difficulty with the word 'parts'. One child successfully used fractions. She firstly used the chain of reasoning, "One part mercury to five copper, ten copper to three tin, that is one mercury ot one and a half tin." For the second part she found a rate per five parts copper thus ten copper - 2 2/3 x two equals 5 1/3, so 5 1/3 parts zinc to three parts tin. Another subject tried to make the amount of copper uniform throughout but used 15 rather than 30:

Interview (I: Interviewer, J: Child)

J: You have to make them to get to 15. Multiply that one by one and a half. The amounts of that gives you same amount of copper. Four and a half tin to 15 copper. Three m to 15 copper. It's one to one and a half. So it's three m to four and a half tin. Eight parts zinc to four and a half tin. You can cancel the fraction out. You can't cancel zinc to tin. You could make them all halves.
I: Why did you use 15?
J: You need the copper to be the same to find the relationship between mercury and tin.
I: Could I have used five?
J: Yes. You could multiply by half and then by 1/3.

Other children knew that something had to be done to the amounts but were not quite sure what. Part one was more likely to be correct since doubling would provide the answer. Another incorrect method was to multiply both the amounts given thus satisfying the idea that something had to be done but demonstrating that the understanding of equivalent fractions was rather tenuous. Others doubled one amount and then halved the second amount:

Interview (I: Interviewer, K: Child)

K: Two mercury. If ten parts of copper that means there'll be two parts of mercury.
I: Two mercury to how many tin?
K: One and a half. I don't know why - took ten and then took the three to five. (obviously divided three by two)
I: So two mercury to one and a half tin?
K: Four and a half for tin (had increased three by half) six for mercury. (had tried to take 2/3)

One child used 30 parts copper throughout and was both quick and accurate.

Many of the children however completely ignored the fact that the two comparisons with copper were in fact comparisons with different amounts of copper, they concentrated on the two commodities that were required for an answer. Two samples are given below.

Interview 1 (I: Interviewer, L: Child)

I: What does that word 'part' mean?
L: It means one per cent mercury to five per cent copper.
One part mercury to three parts tin.
I: Doesn't it make any difference that you have five and ten?
Not really, no. Eight to three.

Interview 2 (Child: M)

M: One part mercury to three parts tin.
I: Why did you decide on those?
M: It's there, one part mercury and three parts tin.
I: Doesn't it make any difference that I have five parts copper and ten parts copper?
M: Yes it does I suppose.
I: What are you going to do about it?
M: Well you can't really work it out like that.

M: Eight parts zinc to three parts tin.
I: Does it matter that you have ten parts copper and 15 parts copper?
M: No.

Question 3: Office Lighting (see Figure 3.4)

Figure 3.4

In an office Mr. Adams comes in to work two days a week.
Mr. Brown comes in to work four days a week.
Mr. Carter comes in six days a week.
The bill for lighting the office for these three men is 240p.

How much should each pay for it to be fair?

Mr. Adams .... Mr. Brown .... Mr. Carter ....

The most popular method of solution for this question was to find the total number of days involved - 12 and then divide 12 into 240, the resultant of 20 was then multiplied to find the amount payable by each person:

> "The average cost we find by saying 12 into 240, that is 20p. 20p is the average for each man if he came in each day. So A pays 40, B pays 80 and C pays 120."

One child who multiplied by a fraction in the cream question used a similar procedure in this item (the version of this question used on interview had 'cost of coffee' instead of 'cost of lighting' - see reference in Chapter Two):

"Mr. Adams should pay 1/6 of 240p, that's 40p. There's 12 altogether. The number of days they come to work. If Mr. A comes into work two days of the 12 days that coffee is drunk, that is 2/12 or 1/6 of the total cost. Mr. Brown would pay double, Mr. C. triple."

Some children recognised that Mr. C came into work the same number of days as the other two men together so reasoned that he should pay half of the 240p, the distribution of the remainder was often a case of trial and error. When pushed some children produced reasons for their answers which were obtained after removing the original 120, e.g.:

Interview (I: Interviewer, N: Child)

N: Mr. Carter should pay half, because those two together make six. One hundred and twenty pence and then another 120p left over, which is 80p for Mr. Brown and 40p for Mr. Adams.
I: Why 40 and 80?
N: I thought if that would be three days it would be 60, half of it, but it's one more. 80 and 40 add up to 120.
I: So do 50 and 70.
N: He comes in four days a week, he should pay double his - because he comes in two days.

When the question involved coffee some children were convinced that one of the men must pay a pound even when they had obtained the number 80 by division:

Interview (I: Interviewer, O: Child)

O: Is that £240 for a week?
I: It doesn't say, perhaps they settle up the bill every now and then.
O: Oh do we divide by three?
I: Try it and see.
O: That's 80p each.
I: Is that fair?
O: No, Mr. C should pay more ... then it's 60, 80, 100.
I: Does that add up?
O: Yes, it's fair now.

Another subject concentrated on the difference between the number of days worked and tried to keep a pound as the amount paid by one of the men:

Interview (I: Interviewer, P: Child)

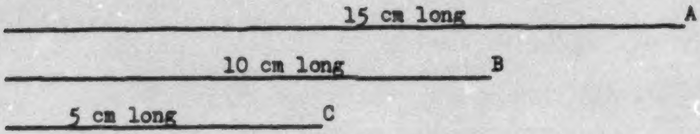
P: Would it come out equally?
I: The men don't go to work equally though, do they?
P: 80, 60, 40. I'll add it up. No £1, 80p, 80p.
I: Why?
P: Because there's two days in between each so that would be 20p in between.
I: Why not 25p?
P: Carter has to pay most. I thought he'd have a pound. Then what do you add to a pound to make 240p. They don't work the same amount. I started with a pound. Each 60, then 80.

Lastly a few children took no notice of the number of days each person was in the office and simply divided 240 by three.

Question 2a: Eels (see Figure 3.5)

Figure 3.5

There are three eels, A, B and C in the tank at the Zoo.



The eels are fed sprats, the number depending on their length.
If C is fed two sprats, how many sprats should B and A be fed to match?

Part 1: B .... A ....

If B eats 12 sprats, how many sprats should A be fed to match?

Part 2: A ....

If A gets nine sprats, how many sprats should B get to match?

Part 3: B ....

The first eel question had been modified from Piaget's original. The CSMS(M) first part gave the amount of sprats for the smallest eel which probably provided a clue for the method used by many of the children in the subsequent parts of the question. Part 1 was almost invariably solved by doubling and trebling the amount given to the smallest eel, e.g.:

> "C is two, times it by two and then times it by three."

Other children used a slightly modified form of this reasoning, talking more in terms of a rate for five centimetres of eel:

> "It's two, four, six... Five, ten, 15 goes up in fives. It's five up to there, add another five which is two, add another five - it's another two, so two, four, six."

Another subject provided a mixture of the two methods:

> "He's twice as big so four sprats. He's (A) twice as much and another five so six sprats. The five left over is another two sprats."

In the second part of the question the child is given the number of sprats for the ten centimetre eel so trebling cannot be used immediately neither can a rate since it has first to be found. A sophisticated method from one subject (who had been given the version of the paper in which B gets four sprats) was:

Interview (I: Interviewer, Q: Child)

Q: The answer is six, if he's another half of his size. Like another 1/3 really, let's imagine he (B) is 2/3, so add another 1/3 which would make it six.
I: Why thirds?

Q: Well they're both multiples of five and five goes three times into 15. So just imagine they are thirds. 1/3 would be half of his size (B) so that is two, so 3/3 would be six.

The finding of a rate is illustrated by the following two examples, both on the 'B has four' version of the items:

"C first, half of B gives C. A gets three times as much as C."

"Four for every ten, A has one ten and half of ten."

Another child seemed to be using the same method but he explained finally in terms of a number pattern.

Interview (I: Interviewer, R: Child)

R: Half of four and then add two more for that one.
I: Why did you add two?
R: Because you'd taken two for that one below.
I: Why did you not choose three?
R: Don't know, it goes up in twos, equal number.

The number pattern reply occurred less often when B was given 12 sprats. Piaget had identified the strategy of just adding one for larger amounts, two children on interview did this. Piaget also identified a later stage (stage III) where the child added a fixed amount for each increase in eel length, the amount being constant but incorrect. An increase of both two and one i.e. the answers 13 and 14 for the amount of sprats for A, were investigated in the large sample testing. The answer obtained by simply doubling (24) was also coded as was a parallel to the addition strategy identified by Karplus where the child reasoned the number of sprats was two more than the number of units of length of the eel. This particular error did not appear on this part of the question on interview.

On part three the amount fed to the largest eel was given and the child required to determine the amount for the middle sized eel (B). The child who increased by one for larger sprats now decreased by one for smaller sprats. One subject, who had been successful in the previous two questions now gave up: "I think it could be nine minus something." Another child who also had the first two parts correct now halved the nine then halved again, finally obtaining $4\frac{1}{2} + 2\frac{1}{4}$. The problem was decidedly more difficult than the two already posed.

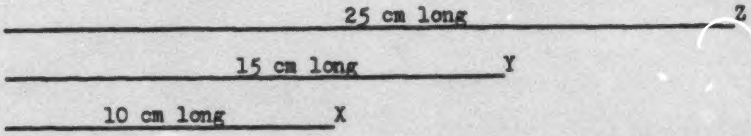
The most popular correct method was to divide nine by three to obtain the amount for the smallest eel and then to double to find the amount for B. This method degenerated into a number pattern reason for some of the children: "It just goes down in steps of three." One subject who had been using thirds already was the only child on interview to mention two-thirds explicitly:

"Each third would have three and he only has two thirds so that is six."

Question 2b: Eels (see Figure 3.6)

Figure 3.6

Three other eels, X, Y and Z are fed with fish fingers, the length of the fishfinger depending on the length of the eel.



If X has a fishfinger 2 cm long, how should the fishfinger given to Z be?

Part 1: Z ....

If Y has a fishfinger 9 cm long, how long should the fishfinger given to Z be?

Part 2: Z ....

If Z has a fishfinger 10 cm long, how long should the fishfingers given to X and Y be?

Part 3: X .... Y ....

This eel question was also adapted from the Piaget version so that the eel lengths were not integer multiples of the smallest. There was a greater variety of methods. Two children multiplied by two and a half to find the length of the fishfinger for a 25 cm eel given the amount for the 10 cm eel, but most who had the correct answer for this part of the question built up to the answer in other ways. Some used the fact that the length of X added to the length of Y was the length of Z:

> "Ten gets 2 cm. 15 is half again. The amount 15 gets plus the amount that ten gets is the amount 25 gets."

> "Y is one and a half times X, Z is X and Y together."

Others reasoned that 25 was a ten another ten and a half of ten giving:

> "The answer is five, two of those (X) would make 20 and then another five which is 1 cm fishfinger."

One child reasoned similarly but became confused over exactly what he was doubling:

> "Y is one and a half (times ten), Z is two and a half, you double it and half again, the answer is two and a half."

Finding a rate for five centimetres of eel was not as popular as previously:

> "2 cm to 10 cm, then its 1 cm to 5 cm eel. Five fives are 25."

The correct answer was also found from a type of number pattern very close to a rate argument:

> "Five into ten goes two and X has two; five into 15 goes three so Y had three; five into 25 goes five, so Z has five."

Incorrect methods were numerous, adding on one to give two, three, four occurred: guesses and doubling were also in evidence as illustrated in the the following interview.

Interview (I: Interviewer, S: Child)

S: It's five and eight. It just goes up more and more.
I: Why did you decide two, five, eight?
S: Well they're ten, 15, 25 doesn't go to 20 so guess it goes up more.
I: Why five and not six for Y?
S: Suppose it could do but five goes into that and that. Then you get another number for that one, more. You could have had ten.
I: You could have had ten but you chose eight.
S: They're more equal but either would be right.

The second part of the question presented more difficulties; apparent in the floundering that took place as children tried to produce a larger number than nine and a cogent reason for having that particular number. Three children did some form of multiplying by 2/3, however none multiplied by 5/3 (29 interviews). One child explained his use of 2/3 by:

> "I took away 1/3 of Y to get six and added 2/3 to get 15."

While another argued:

> "Ten is 2/3 of 15. 2/3 of nine is six. That is five plus ten so add those two amounts."

Two children used a rate by finding 'how much' for 5 cm of eel.

Another child produced this argument:

> "Times it by five, they both go into five. Three fives there. Three times could be three threes which is nine. Five would be five threes - 15. I'm not really sure why."

One child attempted to get a ratio by dividing the length of one eel into the length of another:

> "Y has 9 cm. Divide nine into 15, goes once with four left over. 1.4 into X goes seven which is 9.8. So it's 7.2 cm long."

Variations on the addition strategy also appeared, e.g. subtracting fishfingers from eels:

> "Nineteen. Take away nine from 15, you get six. If you take six from 25 you get 19."

Part three again gave the amount for the largest eel and required the length of fishfinger for the smaller eels. In one instance the child would have used subtraction:

Interview (I: Interviewer, T: Child)

I: Why are these end ones more difficult?
T: They're the minus ones.
I: Why do you say minus?
T: That's bigger so you really have to do minus to get there, but you could share.

Most of the interviewing was done with the fishfinger of length five centimetres and not ten as in the final version. In this context, two children dealt with the problem by using 3/5, others concentrated on the factor of five but often as a number pattern:

"Five into 25 goes five, five into ten goes two."

One child arrived at the correct answer (when Z had five) by a combination of methods:

Interview (I: Interviewer, U: Child)

U: I'll do X first it's a bit easier. Two and a half (X) of these go into that (Z). I'll divide by two first then a half, 25. I don't quite understand how I work it out but I think that this has two and this one three. If five goes into 25 five times, and that is 5 cm and five into 15 goes three so that should have 3 cm. It looks as if it's right.
I: How do you know?
U: That's (X) a bit smaller than that (Y) and it wouldn't be fair to give him one, so two and three.

Another subject started with the 25 and split it into ten and 15, the lengths of the other two eels:

Interview (I: Interviewer, V: Child)

I: Why not four, and one for the other two eels?
V: No, then X would be 5 cm long and Y would be 20 cm long.

Halving to find the amount for Y and then halving again for the amount for X also occurred. Many children did not attempt the question.

The methods used by the children involved addition to a very great extent in that the children seemed to prefer to find segments of the answer and add them together rather than multiply by a fraction.

It was apparent that very few children stayed with a consistent strategy all the way through the two eel questions.

Marking

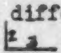
Interview data were analysed for each test paper. These data indicated the various incorrect strategies used by the children in trying to solve the problems. On those occasions when the methods led to a recognisable incorrect answer, these answers were coded in the marking scheme. This is illustrated in the next section again using the items from the Ratio paper, the section first includes a description of the errors and this is followed by the coding scheme which was employed in the large scale testing.

Ratio Items: General Error Descriptions

The interviews with 30 children provided insights into how the children might attempt to solve a problem, both correctly and incorrectly. In the discussion of the items we will try to describe the methods used by the children and also outline what seems to be the mathematical demand of the question. Some incorrect methods of solving the problems on test paper R occur so often that we will describe them in general terms.

Adding on 1. The child faced with an enlargement will reason that the resultant must be larger but one unit larger will suffice; similarly to obtain a smaller amount a subtraction of one unit is sufficient (Code 4).

Adding on 2. This is similar to the above but the child is reasoning that extra units are needed; he has little idea how to provide those extra units. Similarly for providing a smaller amount (Code 5).

Addition strategy. The child concentrates on the difference a - b rather than the ratio a/b. For example asked to enlarge  so that the new base line is five units, he will say five is two more than three so the upright is two more than two, answer four units (Code 6).

Doubling. The child regards all enlargement as a doubling situation and similarly halving as the process for making something smaller. The code 7 is used when this is not what is required in the problem.

Other errors. Certain errors arose from incorrect computation. These will be described for each item.

Ratio Items

Each item is described in terms of the methods children used and errors they committed - as well as the codes used for the classification.

Question 1: Recipe. The results for 1a are whole numbers. The answer was most often obtained by halving.

In 1b, part (a) has a fractional answer. The most common method used by the children interviewed was that of saying "How much for four people? How much for two people. Add the two amounts." In part (c) considerable difficulty was found since this method resulted in the addition of two fractions. Many children opted for "What is the fraction halfway between $\frac{1}{4}$ and $\frac{1}{2}$?", but chose $\frac{1}{3}$ as fulfilling this condition. (Code 8)

Question 2: Eels. This question was taken from the work of Piaget (*Epistemologie et Psychologie de la Fonction*, 1968) with the modification that the numbers were different and the question was in written form.

Part 2a involves discrete amounts and if using a rate for 5 cm of eel, only multiplication by two and three is required; sometimes the child added three times instead of multiplying. We have allowed coding for the addition strategy seen as taking the difference in eel lengths, converting this into sprats and adding to the original number of sprats. Doubling in part one (given food for C find it for A) is seen as completing the number pattern 2(C), 4(B), 8(A). There were some children who simply added the length of eel (cm) to the number of sprats, so this error is coded 8.

In part 2b the eel lengths are no longer integer multiplies of the smallest. The amounts of food fed are no longer discrete. The most common correct method used on interview was to find how much for 5 cm of eel and multiply or add. Adding fishfinger lengths to eel lengths is coded 8. To show doubling the answers coded are those which would arise from consistent doubling (X, Y, Z).

Question 3: Office Lighting. This item requires the child to find a rate of lighting per day and then multiply by two, four and six. Most children on interview did this or multiplied the rate by two; then doubled this amount for four and then added to obtain the lighting bill for six. Other children decided that each man should pay the same amount (80p, 80p, 80p) yet others required the bills to differ by a fixed amount, the total being 240p, e.g. 60p, 80p, £1 (code 3). Still others found the correct amounts to be paid but reversed the order (Code 2).

Question 4: Enlargement. The enlargement for 4a is 2:1 but the child must find this by comparing the two base lines. The use of a centre of enlargement is difficult since it comes off the page. The vertical and horizontal gaps should also be enlarged in the ratio two to one, however it was found on the trials that both children and adults forgot this. In order to be able to compare results with those obtained by CSMS, the gaps have been retained, the important points being investigated however are a) given two line segments can the child find the factor of enlargement and b) can they then double the length of a line either by multiplying by two or by taking the line and then repeating it. The addition strategy would result in the child saying the difference between base lengths is 6 cm so the difference between lengths of upright is 6 cm, answer 8 cm (coded 6) similarly for the gaps. An omission of a gap is coded 3, leaving the gaps as they were in the original diagram is coded 8.

The enlargement of the line segment in 4b is in the ratio 5:3. This does not lend itself very readily to the method quoted in item 1 of finding two amounts and adding (an enlargement of 3:2 could be achieved this way). Usually the children who succeeded attempted (2 x 5)/3 by some method. The answer 3 cm was also coded, it could arise from adding one, or from "The 3 cm base is contained 'one and a bit' times in the 5 cm base, the new upright is 'one and a bit' times 2 cm" (Code 4).

Question 5: Mr. Short and Mr. Tall. This item was taken from the work of Karplus (Proportional Reasoning and Control of Variables in Seven Countries, 1975). The correct methods of solution are a) using some form of (6 x 6)/4, b) finding a rate matchstick:paperclip and then multiplying or c) finding the equivalent to two matchsticks in paperclips and then using (2 + 4) matchsticks and their equivalent. The problem requires the child to recognise that ratio is needed, ascertain a correspondence in some form and then find a solution.

Question 6: Metal Alloy. This problem requires the child to use a third commodity in order to find a relationship between two other commodities. Many children ignored the first commodity (copper) since it did not appear in the requirements of the answer (coded 8), or doubled the one and three which were given (coded 3).

Correct strategies for mercury : tin were a) doubling or b) saying "one part mercury and another part mercury correspond to ten parts copper". The zinc, tin ratio could be obtained by multiplying by one and a half or a more popular method was to say "Five parts more copper would give me one and a half parts more tin so (3 + 1½) tin to eight zinc". Some children (few) decided to work with 30 parts copper throughout and adjust the other commodities accordingly.

Question 7: Similarity. The child must recognise that a ratio is needed by inspecting the two similar figures and then compute the factor of enlargement. Incorrect strategies mentioned previously have been coded and in addition on part 2 some children correctly ascertained the ratio 3:2 but enlarged the figure UVRST (coded 8).

Question 8: Percentage. The symbol % is introduced and each part of the question is solvable by using "parts of a hundred" without recourse to excessive computation. On part c many children half remembered a rule and used (800/24) x 100 = 192 (coded 8). Part d produced children who forgot to subtract the pound and gave it as an answer (code 2). Others who interpreted 5% as one fifth gave the answer four or subtracted four from 20 to give 16. Another version was to simply state five as the resultant or to subtract five from 20.

Summary

The interviews were used to identify errors and strategies (correct and incorrect). The written tests were marked according to codes which were derived from the interview data and from information obtained during the marking of the final written papers (the large scale survey - described in Chapter Four). Interview data were also used to modify items and in some cases these modifications were intended to reflect commonly recurring child strategies.

Coding schemes which reflected particular errors arising from identifiable strategies were formulated for each topic and these appear in Appendix 1. Note should be made that the final coding schemes which accompany the published versions of the test (NFER) are slightly different in that adjustments were made to facilitate use by teachers. The published version of the coding scheme for the Ratio paper is given in Table 3.1.

The coding schemes enabled the research team to identify consistent patterns and in the case of the large scale survey estimate percentages of children for a given age range making the particular type of error. It was also felt that this information would be useful in establishing links between items in the hierarchy - on the basis of method used as well as the mathematical demand.

Table 3.1

Marking Scheme for Ratio Paper

Code		1	2	3	4	5	6	7	8	9
Meaning		Correct Answer			Adding on 1	Adding on 2	Addition Strategy	Doubling	Specific error described in "Items"	Other wrong
Items		Result*								
1a	water	1								
	cubes	2								
b	water	1½(3/2)								
	cubes	3								
	cream	3/8(1/4+1/8)							1/3	
2a	B	4			3	-	7	-	12	
	A	6			4	-	12	8	17	
	A	18			13	14	17	24	27	
	B	6			8	7	4	4½	19	
2b	Z	5			4	6	17	8	27	
	Z	15			10	11	19	18	34	
	X	4			8	6	-	2½	20	
	Y	6			9	8	-	5	25	
3	A B C	40, 80, 120	120, 80, 40	Equal steps e.g. 60, 80, 100					80, 80, 80	
4	Vertical line	4		-			8		-	
	Vertical Gap	2		no gaps shown			7		1	
	Horizontal Gap	3					7½		1½	
		3.2-3.5			3		4		2	
5	Mr. Tall	9			7		8	12		
6	Mercury:Tin	2, 3 1, 1½		2, 6) or other multiple			1, 8		1, 3	
	Zinc:Tin	16, 9 8,4½		16,6)			8, 8 or 3, 13		8, 3	

Table 3.1 continued

Code	1	2	3	4	5	6	7	8	9
Meaning	Correct Answer			Adding on 1	Adding on 2	Addition Strategy	Doubling	Specific error described in "Items"	Other wrong
	Results*								
7 RS	13½			10		13	18	–	
DE	12			17		14	9	27	
8 a	4								
b	15								
c	3							192	

*The answer given was interpreted as an indication that the child was employing that particular strategy.

CHAPTER FOUR

SAMPLE AND ADMINISTRATION OF THE WRITTEN PAPERS

The Samples

Four samples of children were tested during the research:

1) The sample interviewed on a one to one basis prior to the wide-scale testing using written test papers (approximately 300 children).

2) The sample of London children to whom the written test was given prior to the survey (approximately 200 children for each paper).

3) The sample of children (from schools mainly outside London), who took the written tests in the Summers of 1975, 1976 and 1977 (approximately 10,000 children).

4) The three longitudinal survey (1976-1978) samples; children who took a test paper three times (approximately 200 children in the initial testing in each of three topic areas; Algebra, Ratio and Graphs).

Samples 1) and 2) above usually came from schools within London. The schools were asked to provide children for each phase of the testing. The selection of the children was made by the class teacher or head of mathematics according to requests from the research team e.g. a group of third year pupils who were thought to be good at mathematics. A range of ages and ability was sought in each case.

Sample 3) was formed on the basis of teachers' willingness to participate. The teachers in these schools were attending in-service courses when they offered their help and did not feel threatened by an assessment taking place in their classes. It might be assumed therefore that they would be among the more confident of mathematics teachers. An attempt was made to include both rural and urban schools within the sample and to include the full ability range. No attempt was made to control for socio-economic background of the pupils, size of school or the curriculum used in the school.

Sample 4) was a subset of the second year children in sample 3) who had been tested initially in 1976.

Sample in the Survey

There were three periods of testing using printed tests; in the summers of 1975, 1976, 1977. The procedures for the selection of the sample differed from year to year.

Summer 1975

Only the test paper 'Number Operations' was sent to schools, this being the only one written by July 1975. The paper was given to top primary classes and first or second year secondary children (see Table 4.1).

Summer 1976

Four testpapers were available by the summer of 1976, Algebra, Ratio, Vectors and Graphs. The Graphs test paper had already been used by D. Kerslake in the spring of 1976 on 600 Bristol school children, as part of her M.Phil. work. The Graphs sample tested by CSMS in addition to the Bristol sample was chosen so that the two together satisfied the criteria applied to all the other test samples.

Table 4.1

Number Operations Administration in 1975

Age	Number of children
11	497 (all ability levels)
12	247 (all ability levels)
13	132 (all ability levels, but only two schools)

In order to select a sample that was representative of the normal distribution of IQ in the British child population, the NFER Test DH (Calvert Non-Verbal) was given to the second year (age 13+) in each school. No satisfactory non-verbal IQ test was found for older children. The teachers in each volunteer school were asked whether their school status, catchment area or recruitment procedure had changed over the previous four years and whether they considered the spread of ability in the 2nd year was comparable to that in the 1st, 3rd and 4th years. If the teachers thought the intake had changed markedly then only those children for whom IQ scores were available were included in the sample. If there appeared to be no reason why the four year groups should not have an approximately equal spread of IQ scores then the other year groups were considered to have the same IQ distribution as the 2nd year. The research team marked the Calvert tests and the results were also returned to the schools.

For the most part the schools used were outside London and usually comprehensive schools. At least five schools were used for each age group on each test. In 1976 entire year groups from each school were tested. In each age range the IQ scores of the intake from the schools was matched against the normal distribution by using the Komogorov-Smirnov one sample test of goodness of fit (see Table 4.2 and Table 4.3 for the ratio sample). It sometimes happened that there was a lack of children with high IQs in a particular subsample; in this case selective Grammar schools were asked to provide children for testing (their IQ scores were also obtained).

Each child was asked to complete two test papers, the final number of children taking each pair of papers is shown in Table 4.4. The table indicates the approximate numbers of children who took two, three, or four of the Algebra, Ratio, Vectors and Graphs tests.

Since entire year groups were used, each 1976 sample was large. If (as in the Ratio sample) a school did not test a particular year group until after the long summer vacation, it was omitted from the analysis. The type of school and the number of children from each school who did each test are given in Figure 4.1 and Table 4.5 respectively. Note that Table 4.5 includes all three years and all tests - the 1977 testing is described in the next section.

Summer 1977

By the summer of 1977 a further eight test papers had been written: Fractions 1 and 2, Fractions 3 and 4, Reflections and Rotations, Measurement, Decimals and Place Value, Positive and Negative Numbers, Matrices A (no multiplication), Matrices M (with multiplication). The Matrices papers were

Table 4.2

Distribution of CALVERT Scores for Ratio Subjects, 1976.

2nd year

CALVERT results

		75	80	85	90	95	100	105	110	115	120	125	
Name of School	- 72	73-77	78-82	83-87	88-92	93-97	98-102	103-107	108-112	113-117	118-122	123-127	128 +
(05) 1 class omitted	9	7	21	22	19	30	18	14	11	7	9	3	0
(07) 1 class omitted	0	2	1	11	11	10	13	6	6	9	6	2	0
(08)	2	4	12	9	17	15	20	21	7	7	6	1	0
(01)	1	0	3	2	3	5	3	3	3	2	4	1	0
(16)	9	8	14	27	39	42	39	32	29	29	16	6	2
(17)					1	0	6	4	14	22	23	14	10
(20)							5	7	10				
(22)													8
OBSERVED TOTAL (814)	21 (2.58)	21 (2.58)	51 (6.26)	71 (8,72)	90 (11.05)	102 (12.53	104)12.78	87 (10.7)	80 (9.83)	76 (9.34)	64 (7.86)	27 (3.32)	20 (2.46)
EXPECTED	27	27	44	65	85	101	107	101	85	65	44	27	27
% Ideal Distribution	3.34	3.34	5.48	8.08	10.61	12.53	13.24	12.53	10.61	8.08	5.48	3.34	3.34

Table 4.2 continued

3rd Year **CALVERT results**

Name of School	-72	75 73-77	80 78-82	85 83-87	90 88-92	95 93-97	100 98-102	105 103-107	110 108-112	115 113-117	120 118-122	125 123-127	128+
(08)	2	4	12	9	17	15	20	21	7	7	6	1	0
(10)	2	1	6	12	12	11	15	15	10	5	6	7	7
(17)					1	0	6	4	14	22	23	14	10
(18)	3	7	8	18	17	23	26	22	18	24	16	7	1
Minus remedial (19)	10	12	17	20	17	18	8	11	5	4	5	0	0
(04)	10	4	12	19	17	16	27	13	12	11	3	1	1
OBSERVED Total (787)	27	28	55	78	81	83	102	86	66	73	59	30	19
EXPECTED	26.29	26.29	43.13	63.59	83.5	98.6	104.2	98.6	83.5	63.59	43.13	26.29	26.29
% Ideal Distribution	3.34	3.34	5.48	8.08	10.61	12.53	13.24	12.53	10.61	8.08	5.48	3.34	3.34

Table 4.2 continued

4th Year

CALVERT results

Name of School Hall year	-72	75 73-77	80 78-82	85 83-87	90 88-92	95 93-97	100 98-102	105 103-107	110 108-112	115 113-117	120 118-122	125 123-127	128 +
(06)	1	4	6	8	11	9	12	17	12	9	8	2	1
(11)	5	3	7	12	19	17	24	20	9	8	4	4	3
(14)	11	3	15	19	33	23	25	30	11	17	9	2	0
(01)	1	0	3	2	3	5	3	3	3	2	4	1	0
(15)	0	0	0	4	3	9	8	16	11	17	13	1	6
(16)	9	8	14	27	39	42	39	32	29	29	16	6	2
(22)									7	5	0	3	10
OBSERVED TOTAL (868)	27	18	45	72	108	105	111	118	82	87	54	19	22
EXPECTED	28.99	28.99	47.57	70.13	92.1	108.8	114.9	108.8	92.1	70.13	47.57	28.99	28.99
% Ideal Distribution	3.34	3.34	5.48	8.08	10.61	12.53	13.24	12.53	10.61	8.08	5.48	3.34	3.34

Table 4.3
Kolmogorov-Smirnov Test for CALVERT I.Q.

1976 Sample - Ratio (Raw data are given in Table 4.2)

2nd Year	IQ≤72	73≤IQ≤77	78≤IQ≤82	83≤IQ≤87	88≤IQ≤92	93≤IQ≤97	98≤IQ≤102	103≤IQ≤107	108≤IQ≤112	113≤IQ≤117
	.0334	.0668	.1216	.2024	.3085	.4338	.5662	.6915	.7976	.8784
	.0258	.0516	.1142	.2014	.3119	.4372	.5650	.6720	.7703	.8637
	.0076	.0152	.0074	.0010	-.0034	-.0034	.0012	.0195	.0273	.0147

The last 3 IQ groups IQ 118-128+ give E=98, O=111 number of children.

Largest difference .0273

$\alpha = 0.05 \quad D = \frac{1.36}{\sqrt{814}} = \frac{1.36}{28.53} = .0477$

$\alpha = 0.01 \quad D = \frac{1.63}{\sqrt{814}} = .057$

3rd Year	.0334	.0668	.1216	.2024	.3085	.4338	.5662	.6915	.7976	.8784
	.0343	.0699	.1398	.2389	.3418	.4473	.5769	.6861	.7700	.8628
	.0009	.0031	.0182	.0365	.0333	.0135	.0107	.0054	.0276	.0156

The last 3 IQ groups
E = 95
O = 108 number of children.

D = .0365

$\alpha = .05 \quad D = \frac{1.36}{28.054} = .0485$

$\alpha = .01 \quad D = \frac{1.63}{28.054} = .0581$

4th Year	.0334	.0668	.1216	.2024	.3085	.4338	.5662	.6915	.7976	.8784
	.0311	.0518	.1037	.1866	.3111	.4320	.5599	.6959	.7903	.8906
	.0023	.0150	.0173	.0158	.0026	.0018	.0063	.0044	.0073	.0122

The last 3 IQ groups
E = 105
O = 95 number of children

Largest D = .0173

$\alpha = .05$ level $D = \frac{1.36}{29.462} = .0462$

$\alpha = .01 \quad D = \frac{1.63}{29.462} = .0553$

Distribution approximates to the normal distribution.

The Kolmogorov-Smirnov Test for goodness of fit (Siegel, 1956) compares the distribution with that of the normal curve. Let $F_o(x)$ be the theoretical frequency distribution function i.e. the distribution obtained from the normal curve.

$S_N(x)$ the observed cumulative frequence distribution

Maximum deviation D = maximum $|F_o(x) - S_N(x)|$

Table 4.4

Children Who Took Two or More Tests (1976)

	2 tests, or more						3 tests, or more				4 tests
	Algebra Ratio	Algebra Vectors	Algebra Graphs	Ratio Vectors	Ratio Graphs	Vectors Graphs	Algebra Ratio Vectors	Algebra Ratio Graphs	Algebra Vectors Graphs	Ratio Vectors Graphs	Algebra Ratio Vectors Graphs
2nd year	670	50	250	50	160	50	50	50	50	50	50
3rd year	680	730	250	600	320	360	530	250	190	260	180
4th year	500	360	90	310	20	380	250	20	90	20	20
All years	1850	1140	590	960	490	790	820	310	320	320	250

Figure 4.1

School Code	Type	School Code	Type
01	Mixed Comprehensive (Bristol)	28	Mixed Comprehensive (Somerset)
02	Mixed Comprehensive (Bristol)	29	Middle School (Sheffield)
03	Mixed Comprehensive (Bristol)	30	Middle School (Sheffield)
04	Mixed Comprehensive (Herts.)	31	Mixed Comprehensive (Sheffield)
05	Boys' Comprehensive (Herts.)	32	Middle School (Sheffield)
06	Mixed Comprehensive (Herts.)	33	Mixed Comprehensive (Sheffield)
07	Boys' Comprehensive (Herts.)	34	Boys' Comprehensive (Kent)
08	Mixed Comprehensive (Herts.)	35	Mixed Comprehensive (Kent)
09	Mixed Comprehensive (Herts.)	36	Boys' Comprehensive (Kent)
10	Mixed Comprehensive (Herts.)	37	Girls' Comprehensive (Kent)
11	Mixed Comprehensive (Herts.)	38	Mixed Comprehensive (Kent)
12	Mixed Comprehensive (Herts.)	39	Mixed Comprehensive (Leicester)
13	Mixed Comprehensive (Glos.)	40	Mixed Comprehensive (Essex)
14	Mixed Comprehensive (Glos.)	41	Boys' Comprehensive (Coventry)
15	Girls' Grammar (S. London)	42	Boys' Comprehensive (S. London)
16	Mixed Comprehensive (Coventry)	43	Mixed Secondary Modern (Suffolk)
17	Girls' Grammar (Plymouth)	44	Girls' Comprehensive (S. London)
18	Mixed Comprehensive (Notts.)	45	Mixed Comprehensive (Manchester)
19	Mixed Comprehensive (Notts.)	46	Mixed Comprehensive (London)
20	Middle School (Leeds)	47	Mixed Comprehensive (S. London)
21	Girls' Grammar (Private) (S. London)	48	Mixed Primary (Edinburgh)
22	Girls' Grammar (Herts.)	49	Boys' Comprehensive (London)
23	Boys' Grammar (Private) (London)	50	Mixed Comprehensive (Humberside)
24	Mixed Comprehensive (Salford)	51	Middle School (Surrey)
25	Girls' Comprehensive (Leeds)	52	Mixed Comprehensive (Middx.)
26	Mixed Comprehensive (Somerset)	53	Middle School (Surrey)
27	Mixed Comprehensive (Somerset)	54	Mixed Comprehensive (London)

Schools used in 1975, 1976, and 1977.

Table 4.5

Sample Breakdown: 1975, 1976 and 1977

Number Operations Test, 1975, n=976

Year	School Code	Number	
-1	20	102	
0	20	118	
	22	32	
	38	110	
	39	25	
	41	38	
	44	147	
	48	27	497
1	20	113	
	40	31	
	43	38	
	49	65	247
2	45	87	
	50	43	130

Algebra Test, 1976, n=2923

Year	School Code	Number	
2	5	182	
	6	220	
	7	82	
	8	123	
	15	86	
	16	276	
	20	133	
	23	26	1128
3	4	140	
	5	160	
	6	202	
	8	133	
	17	92	
	18	212	
	23	22	961
4	1	21	
	6	92	
	9	98	
	11	134	
	14	142	
	16	220	
	23	24	731
5	6	45	
	15	58	103

Ratio Test, 1976, n=2257

Year	School Code	Number	
2	1	30	
	5	177	
	7	79	
	8	119	
	16	277	
	17	89	
	20	21	
	22	8	800
3	4	141	
	8	121	
	10	106	
	17	92	
	18	190	
	19	117	767
4	1	22	
	6	86	
	11	126	
	14	137	
	15	66	
	16	228	
	22	25	690

Vectors Test, 1976, n=2210

Year	School Code	Number	
2	20	52	
	21	68	120
3	3	107	
	4	142	
	5	163	
	6	202	
	8	121	
	10	103	
	15	61	
	18	220	
	23	21	1140
4	1	22	
	3	103	
	6	191	
	9	88	
	12	124	
	13	80	
	16	224	
	17	95	
	23	23	950

Table 4.5 continued

Graphs Test, 1976, n=1396

Year	School Code	Number	
2	1	26	
	2	9	
	3	96	
	6	189	
	17	60	
	20	9	389
3	1	85	
	3	112	
	10	101	
	18	206	
	19	80	
	22	10	594
4	1	15	
	3	107	
	9	98	
	12	118	
	13	75	413

Reflections and Rotations Test, 1977, n=1026

Year	School Code	Number	
2	9	24	
	11	23	
	12	38	
	20	23	
	21	14	
	25	30	
	26	56	
	31	58	
	33	27	293
3	7	19	
	8	64	
	14	50	
	15	15	
	16	144	
	17	33	
	18	48	
	24	76	449
4	4	37	
	12	37	
	13	51	
	25	29	
	28	83	
	38	47	284

Matrices Tests, 1977,
Test M, n=535; Test A, n=257

Year	School Code	Number	
3	9	63	
	18	40	
	21	67	170 M
4	34	101	
	35	139	
	36	54	
	37	71	365 M
3	5	66	
	9	12	
	18	57	135 A
4	34	52	
	36	48	
	37	22	122 A

Fractions Test 1 and 2,
1977, n=555

Year	School Code	Number	
1	17	22	
	21	70	
	29	25	
	30	26	
	32	48	
	34	55	246
2	9	23	
	11	26	
	12	37	
	20	29	
	21	17	
	25	30	
	26	53	
	31	70	
	33	24	309

Table 4.5 continued

Fractions Test 3 and 4, 1977, n=523

Year	School Code	Number	
3	7	19	
	14	80	
	15	16	
	16	68	
	17	30	
	18	95	308
4	4	35	
	12	32	
	13	23	
	25	23	
	28	46	
	38	56	215

Measurement Test, 1977, n=986

Year	School Code	Number	
1	17	21	
	29	27	
	30	28	
	32	42	
	34	51	169
2	9	51	
	11	57	
	12	77	
	20	52	
	21	26	
	25	27	
	26	102	
	33	52	444
3	7	21	
	8	56	
	14	43	
	15	21	
	16	74	
	17	37	
	18	47	
	24	74	373

Decimals Test, 1977, n=950

Year	School Code	Number	
1	17	21	
	29	27	
	30	25	
	32	44	
	34	53	170
2	9	23	
	11	28	
	12	39	
	20	30	
	25	29	
	26	58	
	31	64	
	33	23	294
3	7	18	
	8	25	
	14	48	
	15	19	
	16	74	
	17	31	
	24	32	247
4	4	35	
	12	35	
	13	22	
	21	15	
	25	29	
	28	48	
	38	55	239

Positive and Negative Numbers Test, 1977, n=818

Year	School Code	Number	
2	9	25	
	11	34	
	12	34	
	20	31	
	25	60	
	26	56	
	31	70	
	33	24	334
3	4	37	
	7	21	
	8	31	
	14	49	
	15	22	
	16	71	
	17	34	
	24	37	302
4	12	35	
	13	24	
	25	27	
	28	49	
	38	47	182

Table 4.5 continued

Graphs Test, 1977, n=503

Year	School Code	Number	
2	9	30	
	12	40	
	25	31	
	26	48	
	33	21	170
3	7	17	
	14	49	
	15	18	
	16	68	
	17	10	162
4	4	34	
	12	35	
	13	28	
	25	27	
	28	47	171

Ratio Test, 1977, n=743

Year	School Code	Number	
2	9	24	
	11	65	
	12	35	
	20	57	
	25	31	
	26	57	
	33	27	296
3	7	41	
	14	49	
	15	36	
	16	74	
	17	4	
	18	53	257
4	4	34	
	12	68	
	13	23	
	25	23	
	28	42	190

Algebra Test, 1977, n=730

Year	School Code	Number	
2	9	24	
	11	65	
	12	35	
	20	53	
	25	31	
	26	57	
	33	27	292
3	7	40	
	14	46	
	15	30	
	16	74	
	17	4	
	18	52	246
4	4	36	
	12	64	
	13	23	
	25	26	
	28	43	192

Vectors Test, 1977,
Sample I, n=467
Sample 2, n=454

Year	School Code	Number	
3	8	58	
	9	70	
	18	49	
	24	71	248 [1]
4	4	73	
	12	34	
	13	25	
	25	47	
	28	40	219 [1]
4	34	157	
	35	124	
	36	99	
	37	74	454 [2]

developed as part of the related research undertaken by G. Ruddock and the sample selection was rather different from that for the other papers. In the summer of 1977, the first six papers above and Graphs, Ratio, Algebra and Vectors were tested (the samples for all tests are given in Table 4.5). Additional schools had volunteered and the sample for each year and each test paper was selected on the same criteria as noted previously except that entire year groups would have demanded more resources for marking than were available to the research team. Instead of entire year groups, a quarter of each year group in any school was used for any one test. Thus, in School A, 2nd year, the teachers were asked to give four test papers to each class on two separate occasions:

Session 1	Session 2
Algebra/Ratio	Measurement
Fractions 1 and 2	Graphs
Reflections and Rotations	Positive and Negative Numbers
Decimals and Place Value	Fractions 1 and 2

This entailed matching schools according to I.Q. prior to issuing the test papers. The spread of I.Q. scores for each school is shown in Figure 4.2. In some cases the I.Q. distribution was skewed to either right or left and an attempt was made to match schools in pairs so that together the I.Q. scores would approximate to a normal distribution. The Kolmogorov-Smirnov test was used and details are supplied in Appendix 4. Class lists were requested from each school and the selection for each test within the class was done by taking four sections alphabetically from the class list. All the test papers except Algebra and Ratio needed one hour for completion, these two papers took only half an hour each so they were sent as a package and every child who did Algebra also did Ratio. The matching of pairs of papers was established to provide as many pair combinations as possible, see Table 4.6, bearing in mind that four papers had already been crossmatched in 1976. Certain combinations of two papers were omitted but for each pair included different schools and different age ranges were assigned. Table 4.7 shows the number of children who finally took each pair of tests. The test papers were sent to the schools already labelled with the name of the child for whom they were intended.

The two versions of the Matrices paper differed in that one included multiplication and the other did not. Two samples were chosen on this basis, i.e. children who had and those who had not been taught multiplication of matrices. It emerged that those in the second sample (not been taught multiplication) tended to be less able in that it seemed likely that teachers continued teaching Matrices (as far as multiplication) only to those classes they considered potential examination candidates.

Sample for the Longitudinal Study

The research also investigated the understanding of Algebra, Ratio and Graphs in three successive years using the same children. The sample was chosen from four schools for each topic, as far as possible having an equal number of children who had completed the test in their second year (1976) in each of four I.Q. score ranges (see Table 4.8).

Figure 4.2

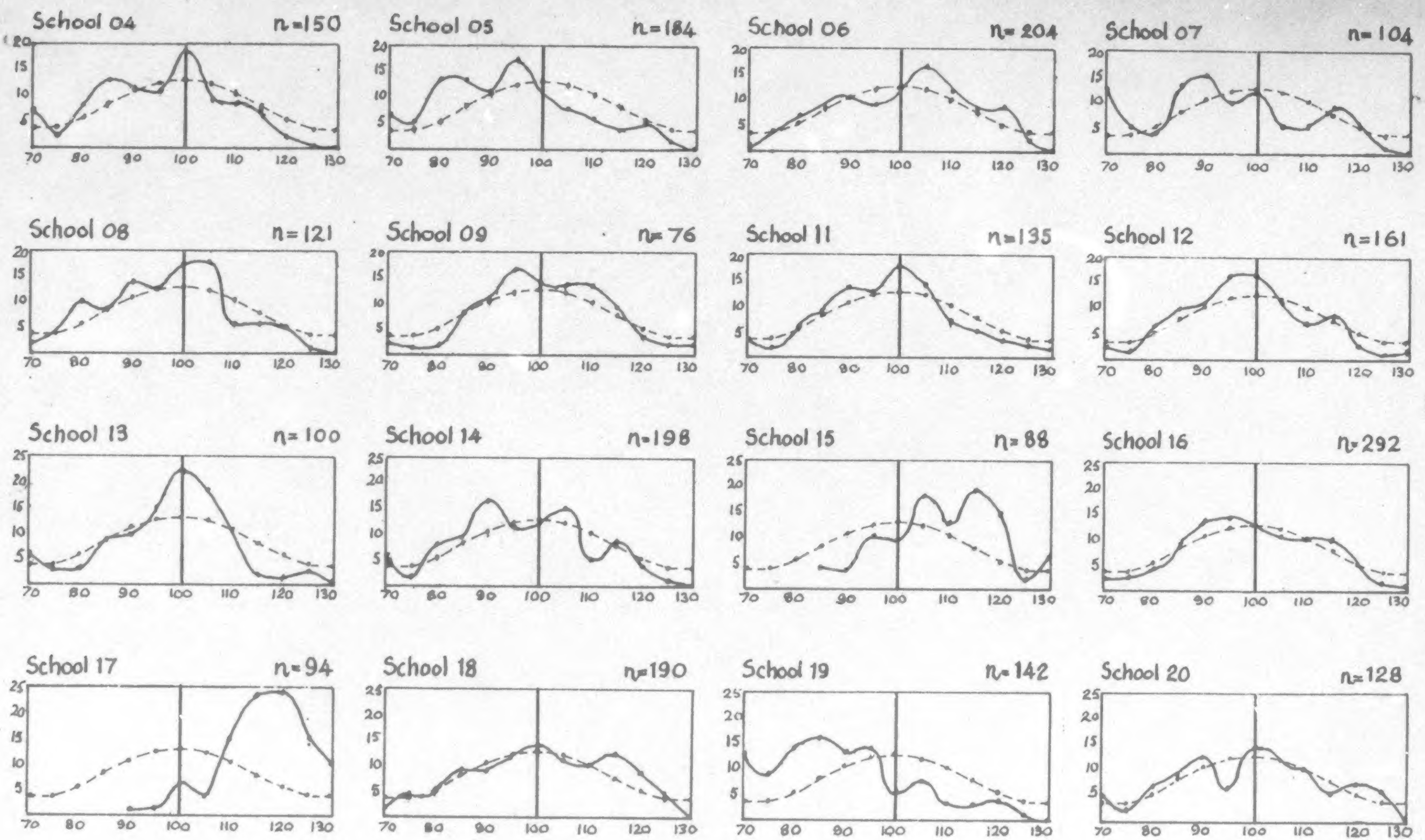
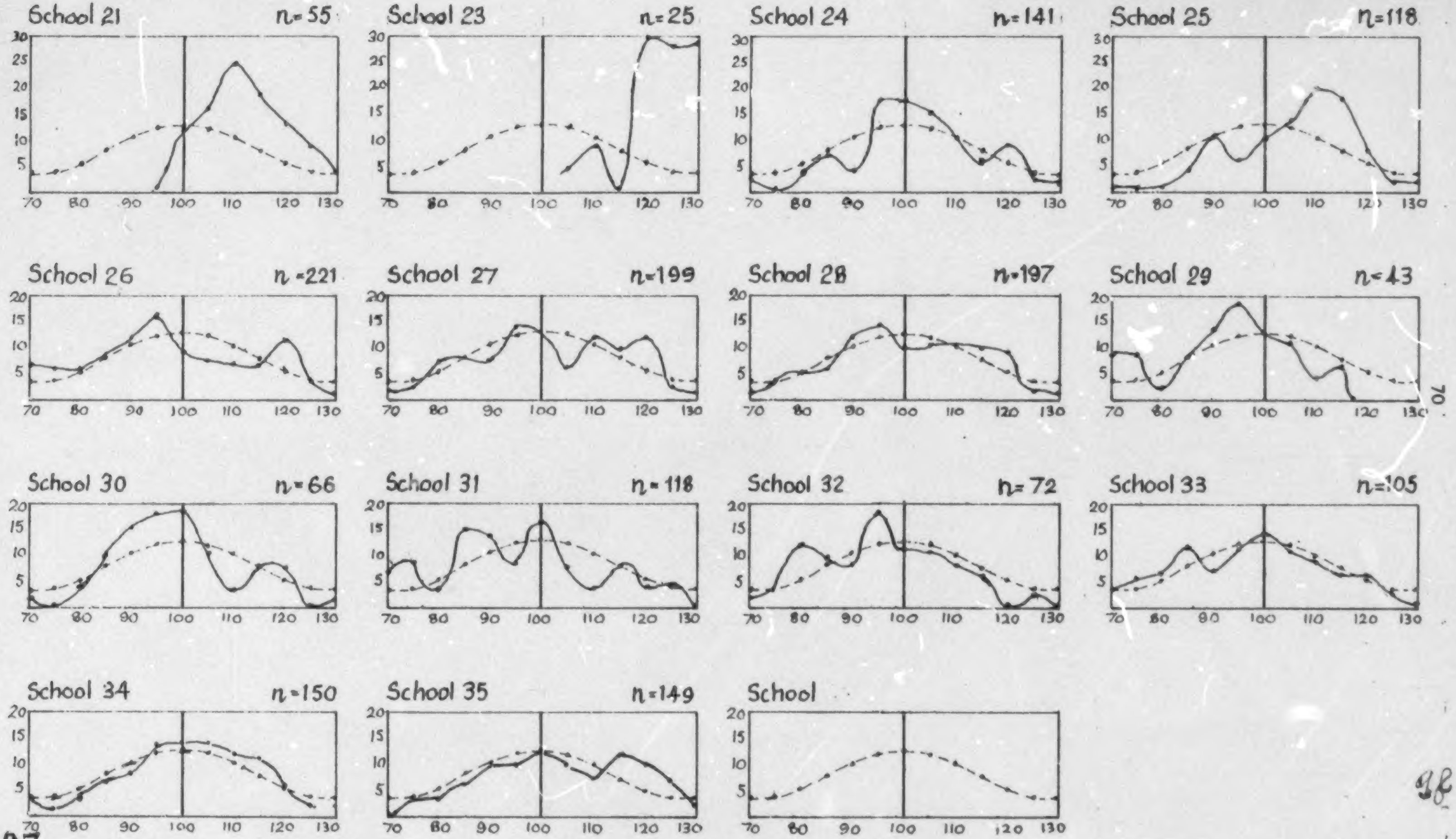

School 04
n=150
School 05
n=184
School 06
n=204
School 07
n=104
School 08
n=121
School 09
n=76
School 11
n=135
School 12
n=161
School 13
n=100
School 14
n=198
School 15
n=88
School 16
n=292
School 17
n=94
School 18
n=190
School 19
n=142
School 20
n=128
5
10
15
20
25
70
80
90
100
110
120
130

Figure 4.2 continued



Distribution of I.Q. Scores for Each School

Table 4.6

Cross Matching of Tests (Plan 1977)

	Reflections & Rotations	Measurement	Fractions	Decimals	Pos/Neg	Matrices	Number Operations
Algebra	1/4,1/4,1/4 1/4,1/4	1/4,1/4,1/4,1/4 1/4,1/4	1/4,1/4 1/4,1/4 1/4,1/4,	1/4,1/4 1/4 1/4,1/4	1/4,1/4 1/4,1/4 1/4,1/4	1/4	
Ratio	1/4,1/4,1/4 1/4,1/4	1/4,1/4,1/4,1/4 1/4,1/4	1/4,1/4 1/4,1/4 1/4,1/4	1/4,1/4 1/4 1/4,1/4	1/4,1/4 1/4,1/4 1/4,1/4		
Graphs	1/4,1/4 1/4 1/4,1/4	1/4,1/4,1/4,1/4 1/4,1/4	1/4,1/4,1/4 1/4,1/4	1/4,1/4 1/4,1/4			
Ref/Rot		1/4,1/4 1/4,1/4,1/4,1/4			1/4,1/4,1/4,1/4 1/4		
Measurement			1/3,1/3 1/4,1/4,1/4 1/4	1/3,1/3 1/4 1/4,1/4			1/3,1/3
Fractions				1/3,1/3 1/4,1/4 1/4,1/4		1/4	1/3,1/3
Decimals					1/4,1/4,1/4 1/4,1/4		1/3,1/3
Vectors	1/4,1/4 1/4,1/4,1/4,1/4				1/4,1/4 1/4,1/4,1/4,1/4	1/4,1/4,1	

Note: Each Fraction is from a different school
Each line is a different age group

Table 4.7

Number of Children Taking Two Tests 1977

	Algebra	Ratio	Graphs	Reflections and Rotations	Measurement	Fractions 1/2	Fractions 3/4	Decimals	Vectors
Algebra									
Ratio	718								
Graphs	476	439							
Reflections and Rotations	110	116	136		196				163
Meaurement	133	138	95	196					
Fractions 1/2	50	51	-		157				
" 3/4	64	68	148		92				
Decimals	189	184	79		134	93	72		
Positive and Negative Numbers	125	124	-	213				130	173
Matrices									165

Table 4.8

IQ Ranges for the Longitudinal Study

	School	IQ ≤ 89	90 ≤ IQ ≤ 99	100 ≤ IQ ≤ 109	IQ ≥ 110	
Algebra	07	10	10	10	10	
	05	10	10	10	10	
	06	10	10	10	10	
	16	10	10	10	10	
	Initial Sample (1976)	40	40	40	40	160
	Final numbers (1976-78)	23	22	28	32	115
Ratio	01	7	9	4	10	
	07	16	15	16	15	
	05	11	9	11	8	
	16	17	16	20	15	
	Initial Sample (1976)	51	49	51	48	199
	Final numbers (1976-1978)	21	28	23	27	99
Graphs	01	7	9	4	10	
	03	16	15	18	15	
	06	17	16	18	15	
	Initial Sample (1976)	40	40	40	40	160
	Final numbers (1976-1978)	2	15	19	25	61

Children were selected randomly for each IQ range in each school. The re-testing of the children in 1977 and 1978 caused many administrative difficulties for the schools and by summer 1978 the sample of children who had completed the test paper three times was considerably reduced (from an initial sample of 519 to 275 - see Table 4.8). This was not unexpected as the problems of absences, school changes and the fact that subjects were no longer together in the same classes accounted for this reduction.

Administration of the Tests

Testing was done in the mathematics classes and all the tests were administered by the mathematics teachers. Instructions for the administration of the test were given to the teachers. Most test papers were judged to need an hour for completion but the teachers who were administering them were told that the research team was interested more in the child doing as much as possible than in strict adherence to a time limit. Each school was visited by a member of the research team, who emphasised that we wanted the children to understand the questions and so parts could be read out by the teacher if it was known that reading skills were deficient. The test paper 'Number Operations' was read to all the children taking it.

A form was given to each class teacher; it asked which textbook series was used for teaching the topic, what additional material was used and whether the children had been taught the topic in the previous year. The information obtained showed that very few teachers kept strictly to a single textbook. (While some data are available, the research was not designed to compare the relative success of different teaching schemes.)

The results for all tests were also returned to the schools.

CHAPTER FIVE

FORMING A HIERARCHY - STATISTICAL METHODS

A major aim of the research was to provide hierarchies for ten mathematical topics which would be of use to teachers and developers of curriculum in their attempts to make decisions regarding the appropriateness and sequencing of teaching materials. The hierarchies were to be based on what the children seemed to understand rather than on a logical analysis of the topic in mathematical terms. An analysis of the content of the mathematical areas had of course taken place prior to the writing of the test papers. In Chapter One various interpretations of the word 'hierarchy' were mentioned and methods used by other researchers to test for the existence of a hierarchy were described.

From the start there was a desire to view the research within a Piagetian framework particularly since the work of the Science wing was almost wholly within this framework. Items on the mathematics test papers were often written to mirror the tasks Piaget has used, see for example the Measurement paper. Some of the fourth year children (age 15+) tested in mathematics in the summer of 1976 had been given the Science Piaget tasks in the autumn of 1975 and their "levels" on these tasks were available.

As described earlier, attempts to label the demand of items in terms of Piagetian levels on the basis of a subjective review by persons knowledgeable of the Piagetian work resulted in lack of agreement. The problem of using a Piagetian framework was formidable, in one sense it was too easy, in that ordering by facility and designating the hard items as requiring formal operational thought seemed simplistic and it was always possible that items were hard not because their solution required complicated thought processes but because the child was unfamiliar with the content. Using the results of testing the fourth year children with one science Piagetian task and then interpreting their mathematics test results in the light of this, prior to generalising to all others, also seemed simplistic since a task based on physical science might not be a fair indicator of a child's performance on a mathematically based Piagetian task. Although the unifying aspect of a 'stage' forms part of Piagetian theory, Piaget (1972) himself has stated that it is possible for a human being to use 'formal' thought in one area of his life and not in others.

Two alternatives considered by the CSMS(M) team were a) to assess the demand of the mathematics items in Piagetian terms (and the difficulties encountered with this approach have been mentioned previously) and b) to match performance on mathematics tests against performance on a test composed of tasks taken from the works of Piaget.

In using approach (b), the decision finally taken was to analyse the data from each mathematics test by statistical methods and then match the resulting levels in the topics against the results of testing children using a number of Piagetian tasks taken from original sources and which were mathematical in content. The basic assumptions in developing the Piaget Test were that:

1) One could describe the child's response in Piagetian terms.
2) A child's responses to a number of tasks would be relatively consistent so that the child could be said to 'be at' a stage of development.

3) The levels attained on a mathematics test given to the same children would correspond to their cognitive levels so there would be a clear distinction between the performance, say on Ratio, between the formal thinkers and those who were at the concrete operational level.

As reported in Chapter Two this use of the Piaget Test was finally abandoned as it did not yield consistent results. Thus while this may well have been due to faults in the test, in the marking scheme or because children are inconsistent, it was clear at this point that the condition imposed by assumption two above had not been met.

Thus the work on the hierarchies and the notion of stages was restricted to a consideration of the CSMS(M) mathematics tests themselves. A number of statistical techniques were investigated and this chapter discusses the various methods attempted and the procedure finally accepted as most appropriate to the purposes of the research.

Attempts to Form a Hierarchy

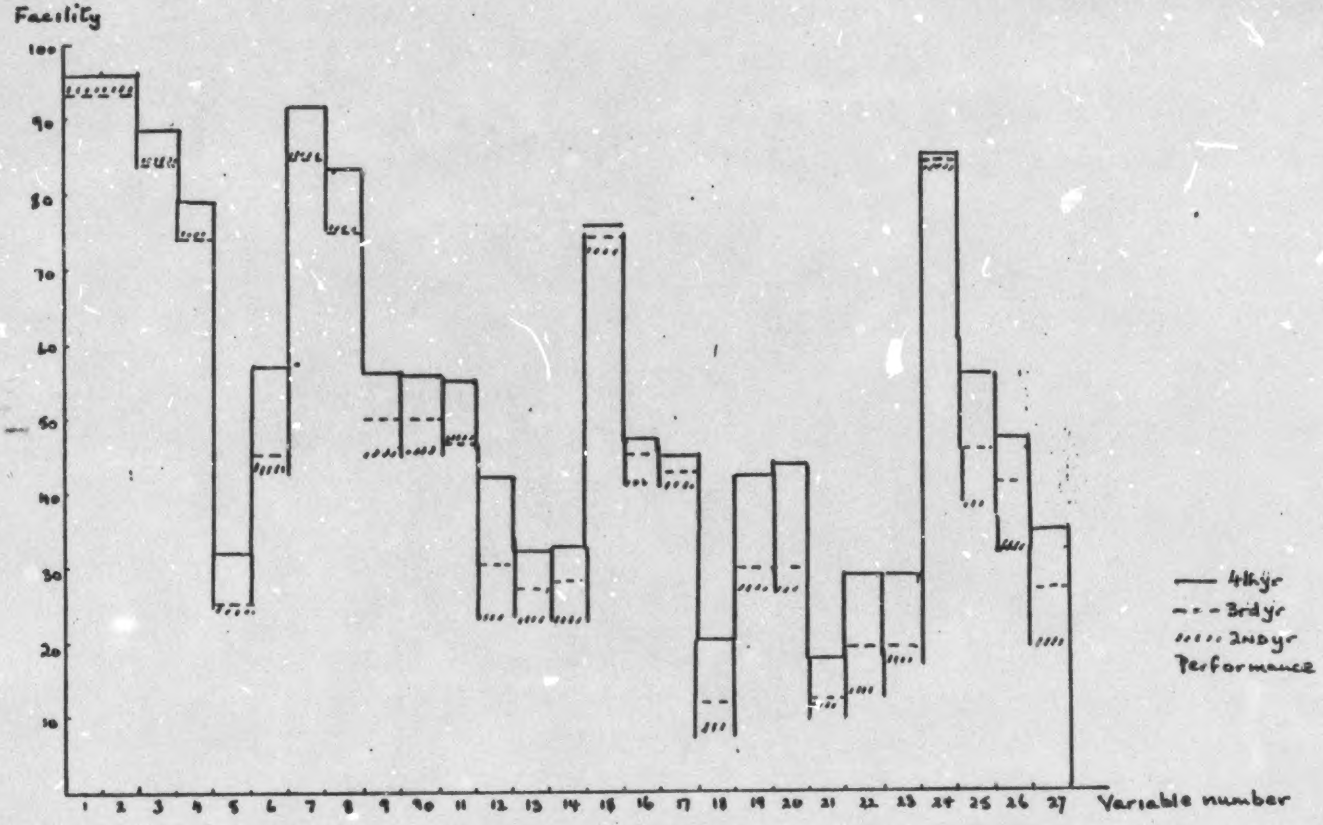
In the summer of 1976 the Graphs, Algebra and Ratio tests were given to 5,000 children, the data from this testing formed the basis for attempts at formulating a method to obtain a hierarchy, which could then be applied to the data obtained from all the other tests.

Preliminary Observations - Item Facilities

Preliminary attempts using statistics were tried on the 1976 Ratio data. This test paper had 27 items and was given to 2,257 children ages 13-15. An initial comparison of the performance on individual items between each age group is given in Figure 5.1 (facilities, or percentages answering an item correctly for each age group in the 1976 sample). It is readily apparent from Figure 5.1 that the very easy items were easy for each age level. On some items e.g. variables 25, 26 and 27 (all items dealing with percentages), there appears to be a regular increase in facility in that there is an increase of eight to ten per cent for each year tested. On other items e.g. variables 6, 19 and 20 there appears to be an increase in facility only for the fourth year, the data for the second and third year children being very similar. It was hoped that these facility patterns would give information to help in the grouping of items. The different facility patterns shown in Figure 5.1 are summarised in Table 5.1.

The items on the Ratio test paper spanned a facility range from ten to 90 per cent but there was a gap between 50 and 70 per cent in which no items appeared. The total scores obtained by the children in each year group did not vary very much, the mean marks being 45, 48 and 54 per cent. More of the fourth year obtained a high score than children in the other two years. The pattern of success, year by year varied according to the question set. The attempt to form items into groups came later but at this stage three items (variables 24, 16 and 17) seemed to be giving little information on the child's understanding of ratio and proportion. Variable 24 was simply a reiteration of the introduction to the percentage questions and thus needed a simple application of the definition given in the first sentence of the question. Fifteen per cent of the children could not do this. Variables 16 and 17 required the child to remember that in the enlargement of an open figure the gaps between the ends of the line segments also had to be enlarged. Some 20 to 30 per cent either forgot to enlarge the gap or omitted the gap altogether. This part of the question, although providing information on the child's perception of the figure and its enlargement does not tell us much, if anything, about the child's understanding of ratio.

Figure 5.1



A Comparison of Performance on Each Item

Table 5.1

Pattern of Facilities by Year (1976 Sample)

	Items (Variable Nos.)	Facility (per cent)
3 years close (easy items)	1, 2, 15	
2nd and 3rd year close with 5-7 per cent increase for 4th year	3, 4, 7, 8	70 - 80
5-7 per cent between each year	9, 10, 25, 26	45 - 55
2nd, 3rd years close with 10 per cent increase for 4th year	6, 11	40 - 50
2nd, 3rd years close with 10 per cent increase for 4th year	5, 19, 20	25 - 40
5-7 per cent between each year	12, 13, 14, 22, 23, 27	20 - 40
2nd, 3rd years close 7 per cent increase for 4th year	18, 21	15

A simple ranking of items according to facility is insufficient for a hierarchy since although items might be successfully completed by the same percentage of children, it does not necessarily follow that those individuals who succeeded on item (a) would be the same as those who succeeded on item (b), even though items (a) and (b) had the same facility. This could very likely be the case for two items each with a facility value of say 30 per cent. Thus, in attempting to establish a hierarchy the decision was taken that the following three criteria should apply:

1) Items should be grouped according to facility.
2) Items should be grouped when the same children appeared to do them successfully.
3) There should be some link between groups on the easy/hard continuum in that children who were successful on the hardest items should also be successful on the easier ones.

Note also, that in the early stages of the research (e.g. the interviews), an attempt was made to identify features common to particular items - for example the methods used by the children or the mathematical description of the items.

Cross Tabulation and Factor Analysis

A further investigation of child performance (1976 preliminary data) was made by crosstabulating pairs of items (sometimes triples). The Ratio items which gave the most consistent crosstabulation patterns i.e. performance on them did not appear to be random and they were of comparable difficulty were the following:

Variables 12, 13 and 14. Of those who had variable 14 incorrect 79.8 per cent also had variables 12 and 13 incorrect. Of those who had variable 14 correct 62.2 per cent also had variables 12 and 13 correct.

Variables 6, 9, 10 and 11. Of those who had variables 10 and 11 incorrect 65.8 per cent also had variables 6 and 9 incorrect. Of those who had variables 10 and 11 correct 67.9 per cent also had variables 6 and 9 correct.

Variables 27, 22, 23 and 13. Of those who had variables 23 and 13 incorrect 82.6 per cent also had variables 27 and 22 incorrect. Of those who had variables 23 and 13 correct 62.2 per cent also had variables 27 and 22 correct.

The crosstabulation of all other sets of Ratio items showed no substantial number of children consistently passing or failing.

A factor analysis of all the Ratio items was carried out using the varimax rotated factor matrix. The factor loadings on six main factors seemed to be influenced by the facilities of the items. Those items which had a loading of .5 or more on one of the six factors are shown in Figure 5.2. Figure 5.2 also shows possible ways of grouping or clustering items using each of the following as a criterion:

a) Similar age group performance pattern.
b) Similar performance by 'the adders' (see interview discussion in Chapter Three).
c) Similar pass/fail patterns from the crosstabulation.
d) High loading on the same factor (from the factor analysis).

Each method produced a different formation of groups and none of the methods described provided sufficient information on the link between hard and easy items on the same string of groups. No item had at this stage been omitted from the analysis.

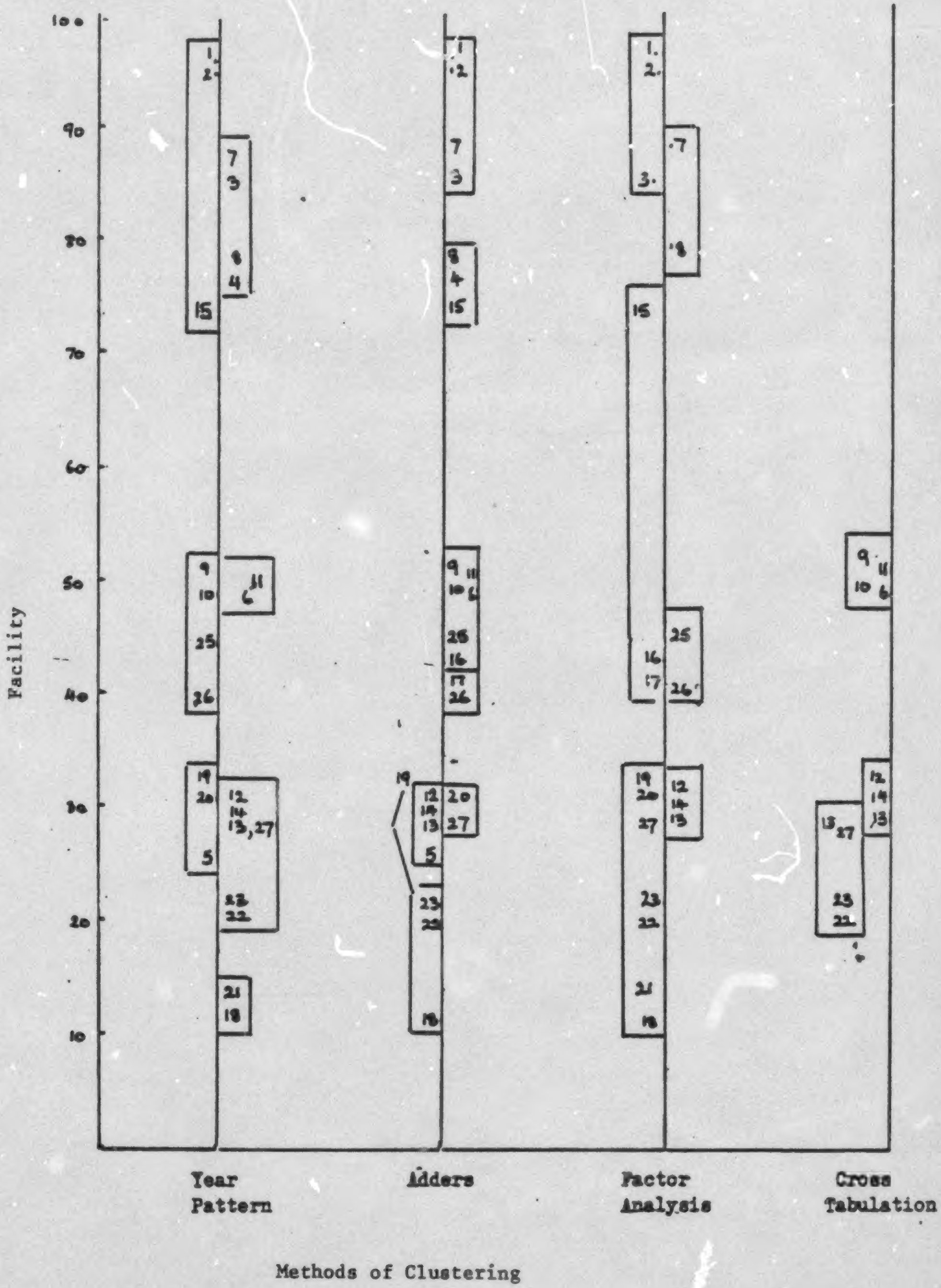
Discrimination of Groups of Items (Ratio Test)

A further attempt in which the discrimination of groups of items was assessed was made as follows:

> Firstly items with approximately the same facility were put together and then a pass mark of about two thirds correct was assigned; thus, for example, a child with three of the four items correct would pass the Alpha group. In order to find the measure of discrimination of each group, the number of children who passed each was matched against their total score.

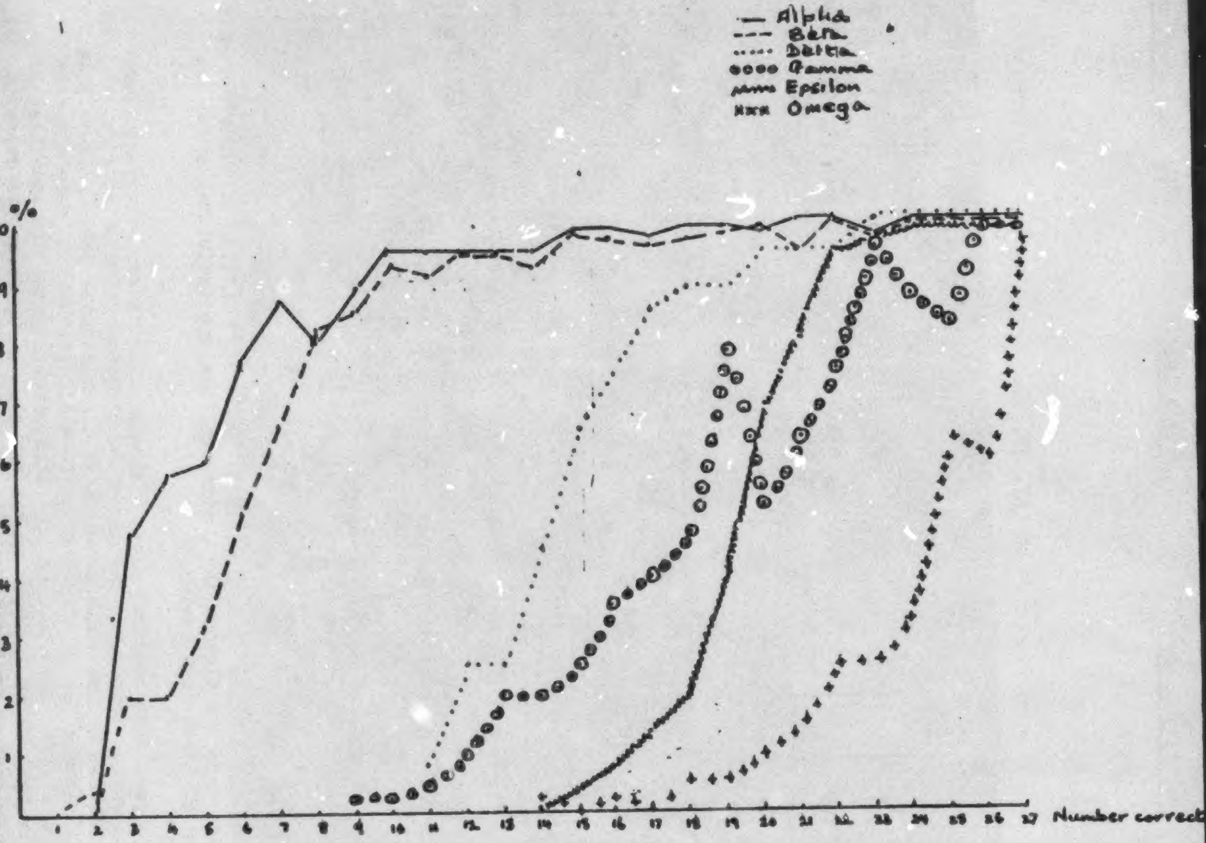
Figure 5.3 and Table 5.2 show the result of employing the discrimination procedure. Figure 5.3 shows the degree of discrimination of each group when matched against total score. Group Gamma did not discriminate well, one would expect a profile similar to that of the Delta group for good discrimination. A pass mark of 1/2 for the Omega group was obviously inadequate, the 2/2 pass mark gave a better profile in that it showed a clear distinction between children with high and low total scores whereas many children with low scores managed to correctly solve one of the items in group Omega.

Figure 5.2



The numbers listed are those identified as variable numbers for the items on the Ratio test.

Figure 5.3



Discrimination of Groups Alpha - Omega

Table 5.2

Groups According to Facility, Alpha - Omega (Ratio Test)

Group	Facility Range	Variables	Pass Mark Assigned
Alpha	75 - 90+	1, 2, 3, 4	3/4
Beta	72 - 85	7, 8, 15	2/3
Gamma	30 - 45	16, 17, 19, 20	3/4
Delta	40 - 50	6, 9, 10, 11, 25, 26	4/6
Epsilon	15 - 28	27, 22, 23, 12, 13, 14, 21	5/7
Omega	13 - 25	18, 5	1/2 and 2/2

The diagram shows which groups had good discrimination and although children with high total scores seemed more likely to succeed on the hardest group (omega) no information regarding their peformance on the other groups is available from this procedure. In order to find the extent to which the groups were scalable a Guttman Scalogram analysis was run on the six groups, the pass marks being those stated in Table 5.2 (2/2 being the pass mark for group Omega). The results of the scalogram analysis were as follows:

Errors 1414 n = 2257

Coefficient of Reproducibility 0.8956

Coefficient of Scalability 0.5641

The usual criteria for the acceptance of a scale of individual items are that the Coefficient of Reproducibility be above .75 and the Coefficient of Scalability be above .60. Although the values obtained were very close to these figures the number of errors was very large and it was obvious that the six groups did not satisfy the criterion that success on the harder groups of items entailed success on the easier groups. The method of grouping just described did not use information regarding the children who did the same questions. Methods of forming groups which take into account success by the same children are described in the next section.

Characteristic Curves

A somewhat similar investigation of the discrimination of items was made on the data from the Algebra test, although here the individual items and not groups of items were analysed.

For a test that measures along a single scale, a characteristic curve provides a way of displaying the degree of discrimination of an item, and the point (or level) on the scale at which it discriminates best. Figure 5.4a shows such a curve for an item with perfect discrimination (here the test measures on a six point scale and the item discriminates "between" levels 3 and 4, or "at" level 4, shown by the arrow). In practice the curve is likely to be more of an S-shape, or part of an S-shape depending on the item's facility. Figures 5.4 b-d show three such curves for items of high, medium and low facility respectively - the items are from the Algebra test, and the levels are for one of the early scales.

Figure 5.4a

Ideal Characteristic Curve

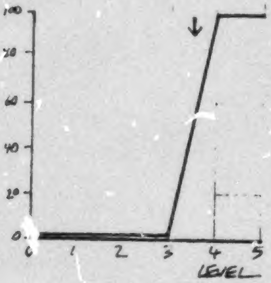


Figure 5.4b

High Facility Item (Item 5i on Algebra Test)

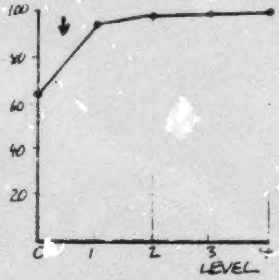


Figure 5.4c

Mid-facility Item (Item 9iv on Algebra Test)

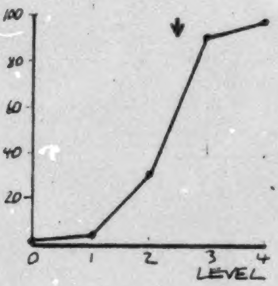
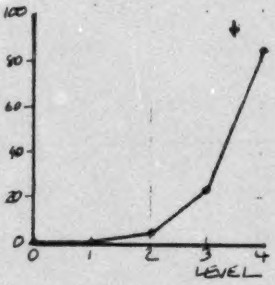


Figure 5.4d

Low Facility item (Item 7iv on Algebra Test)



The method developed by the CSMS science wing is an iterative one, in which the first step is to classify the items on a test into a number of levels. Next, scoring rules are devised, and children assigned to levels according to their performance on the test. (The most common rule, at least initially, was to assign children to the highest level for which at least 2/3 of the items had been answered successfully). Characteristic curves are then drawn for each item and in the light of these the item classification modified (by, for example, rejecting some items or classifying them at a different level). The scoring rules might also be modified at this stage (if, for example, too many items appear to be wrongly classified, or if a high proportion of children's performances do not "scale" in the sense of children who have been assigned to a given level failing at a lower level). The children are now reassessed, new characteristic curves are drawn and the cycle repeated until some kind of equilibrium is reached.

In practice the use of the characteristic curve suffers from a number of shortcomings. First, the method is circular, since all, or most of the items being investigated will have been used in determining children's levels in the first place. There is no completely satisfactory way of overcoming this circularity, and though it may not be too serious when a large number of items have been assigned to each level, this is frequently not the case. Second, though some poorly discriminating items can be readily identified by inspection (e.g. item 4iv shown in Figure 5.5a) it is difficult to find precise criteria for interpreting characteristic curves.

Figure 5.5a

Item 4iv Algebra

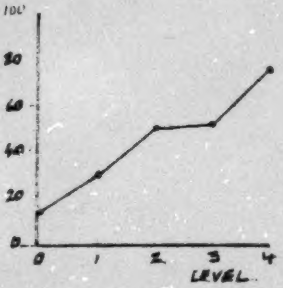


Figure 5.5b

Item 9iv Algebra

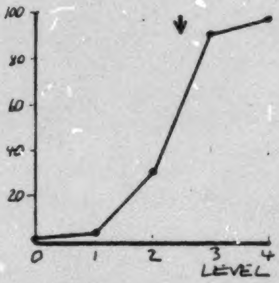
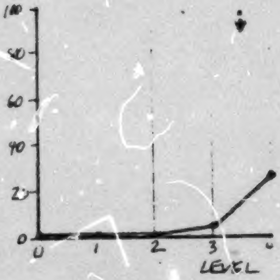


Figure 5.5c

Item 23 Algebra



Qualitatively, the middle portion of the characteristic curve should be "steep" and its end-point "high". However, it is not clear how these features can be expressed as a single quantitative measure, or whether they are even adequate. For example, though in these terms item 9iv (Figure 5.5b) is certainly a better discriminator than 4iv, it might be argued, despite appearances, that item 23 (Figure 5.5c) is at least as good, since about five times as many children at level 4 than at level 3 answer the item correctly.

Grouping Items Using Measures of Association

Various homogeneity coefficients are available and some have been used by other researchers in attempts to provide evidence of a connection between items. In this section the coefficients are calculated and applied on the ratio data. The facility of each item was available and the problem was to form groups of items. Homogeneity coefficients which used the fourfold table classifying n individuals according to their performance on two items I_1 and I_2, seemed suitable for grouping items. The indices considered were $\emptyset$, H_{ij} and Q, all of which are defined in Figure 5.6.

Figure 5.6

Items I_1 and I_2

		I_1	
		Fail	Pass
I_2	Pass	a	b
	Fail	c	d

a, b, c, d. number of children

a + b + c + d = n where n is the total number of children. In this notation the indices are:

X $$\emptyset = \frac{bc - ad}{\sqrt{(a+b)(c+d)(a+c)(b+d)}}$$ (Guilford 1965)

$$H_{ij} = \frac{bc - ad}{(b+d)(c+d)}$$ (Loevinger 1947)

$$Q = \frac{bc - ad}{bc + ad}$$ (Yule 1912)

(In the calculation of H_{ij}, I_1 must be the harder item i.e. $d > a$)

Grouping Items Using Loevinger H_{ij}. Perfect homogeneity i.e. a coefficient of one is seldom found when real data are used and an acceptable value of the coefficient has therefore to be decided. The coefficient was used in the following way:

1) Item / item Loevinger coefficients were found for all pairs of items.
2) The facility of the easiest item was plotted on graph paper and all items which had an H_{ij} of more than .64 with this item were also plotted.
3) Other easy items and those additional items which were connected to them by a high value of H_{ij} were also plotted.
4) All other connections resulting from a high value of H_{ij} were plotted.
5) Since groups of items at approximately the same level of facility were being sought, items which were connected only to others of very different facility were rejected.

This procedure when applied to the Ratio data resulted in the rejection of a number of items. The items plotted did not cluster in groups within facility levels except at the harder (greater than 50 per cent) level, see Figure 5.7.

The values of the coefficient obtained when the two items being compared were both easy, were very low. However if the criterion value of H_{ij} was made less stringent certain items within the 20-50 per cent range could be connected. A connection simply between items of the same facility was insufficient if the aim was to form a hierarchy, therefore when the coefficient was lowered to .55 it was decided that the items included on the diagram had to be connected to both the very easy items and some of the hardest. The connections between items using these new criteria are shown in Figure 5.8.

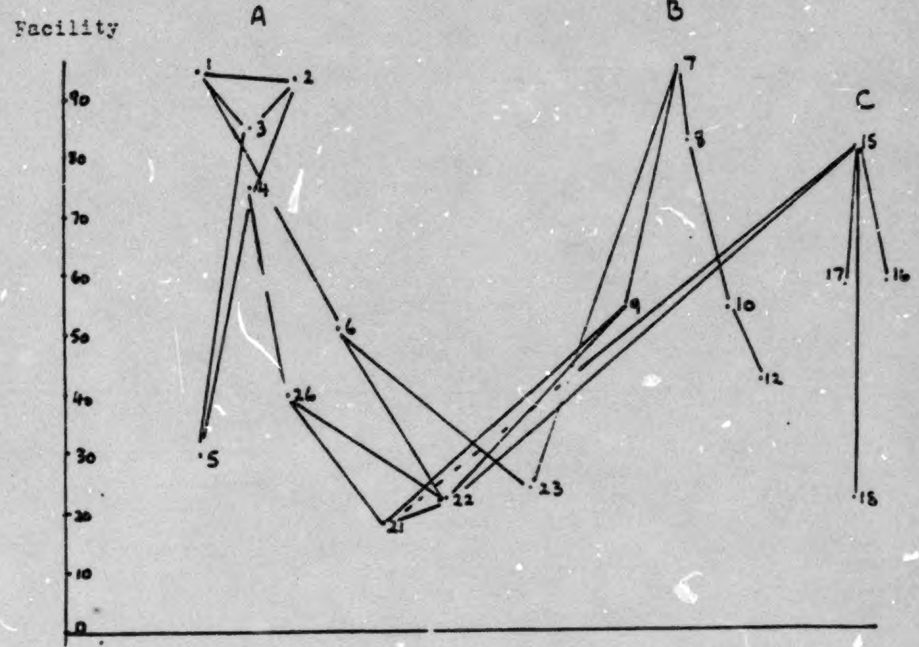
Nassefat (1973) argued that a greater degree of homogeneity between tasks was apparent when the children doing the tasks demonstrated a degree of stability in the cognitive level demanded by those same tasks. Extending this idea it could be argued that the data from the oldest children tested on the Ratio paper should produce the greatest values of the homogeneity coefficient item/item. Figure 5.9 shows the connection between items when the results of only the fourth year children were used; $H_{ij} \geqslant .64$.

All the above attempts to establish a hierarchy used the Loevinger coefficient H_{ij} with various restrictions. Other researchers, for example McQuitty (1937), and Bart and Krus (1973), have used homogeneity coefficients in a different way. The next sections discuss the application of the methods used by these researchers to the ratio data and include reasons for rejecting the procedure in the CSMS(M) analysis.

Linkage Analysis

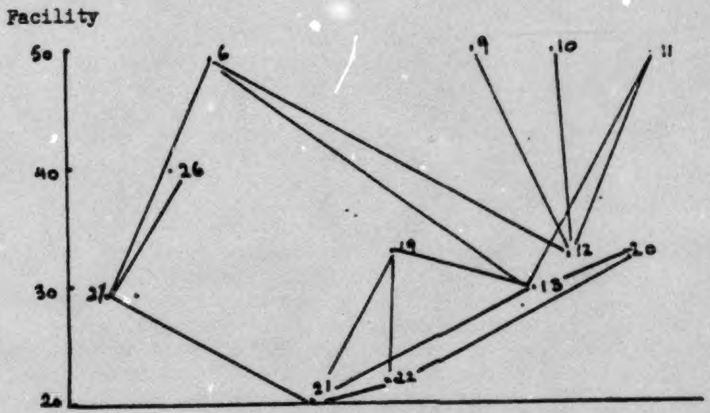
McQuitty used a comparison person to person in order to describe types of people. Applying his method to the ratio data one firstly listed all the item/item coefficients (Loevinger H_{ij}) and underlined the highest value of H_{ij} for each item. The two items which possessed the highest value of H_{ij} formed the first 'type'. In the coefficients from the ratio data these were variables 7 and 8. All items which had their highest coefficients with variables 7 or 8 were then appended. The process was continued to include those items which had their highest coefficients with an appendage. When type one was exhausted the pair of items with the highest coefficient as yet unused formed the second type and the process was continued. The ratio data coefficients showed five 'types', these are shown in Figure 5.10.

Figure 5.7



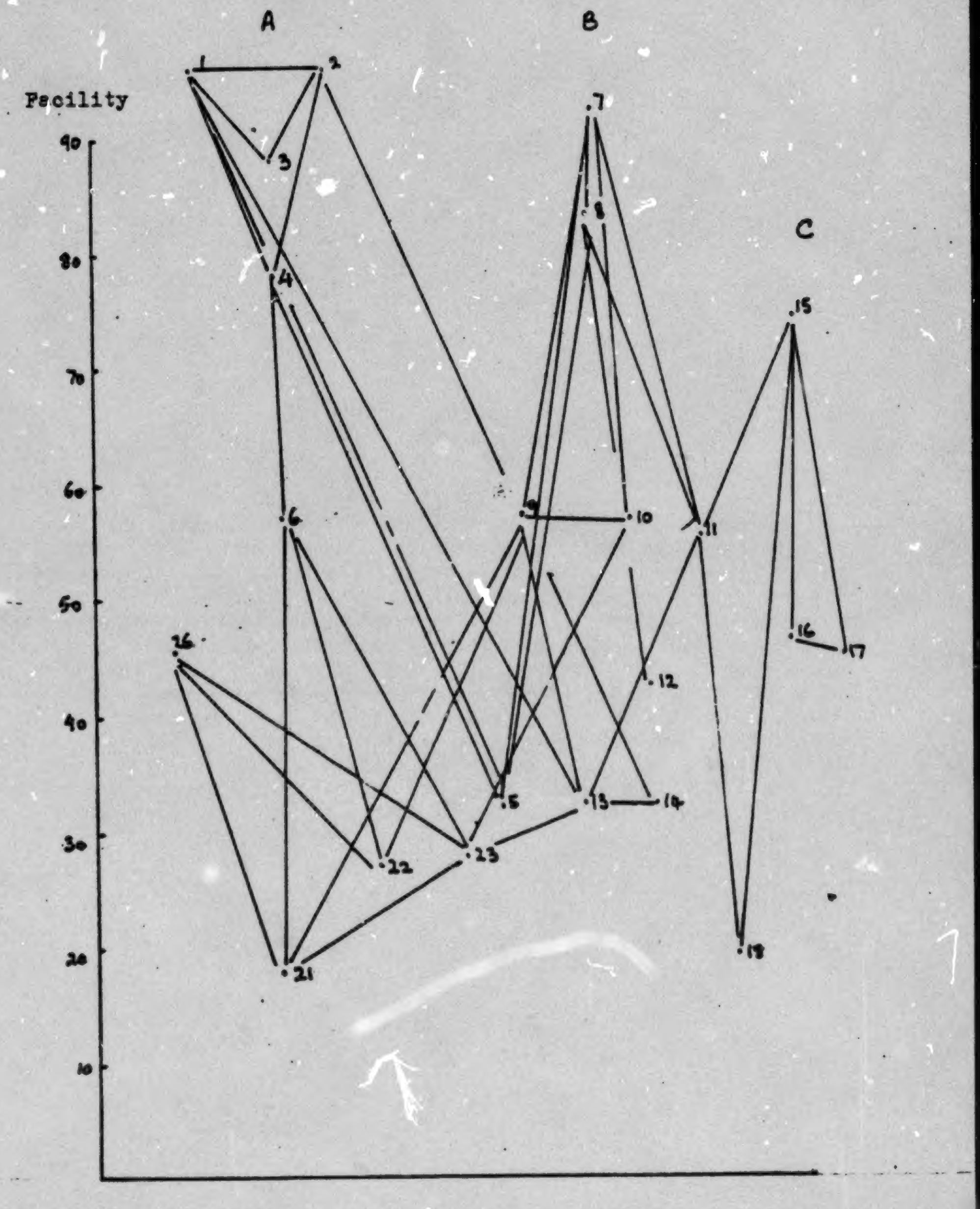
The Loevinger Coefficient $H_{ij} \geq .65$ Applied to 1976 Ratio Data (items adjusted to variable numbers)

Figure 5.8



Loevinger Coefficient (H_{ij} = .55) on Ratio Data (items adjusted to variable numbers)

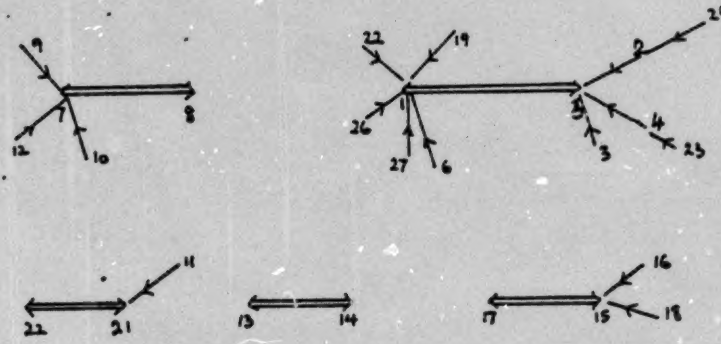
Figure 5.9



Loevinger Coefficient ($H_{ij} \geq .65$) 4th Year Data (numbers are variable numbers)

A drawback of the McQuitty method is that two items may be of the same type although they have a low value of the homogeneity coefficient as there is no examination of the relative values of the coefficients.

Figure 5.10

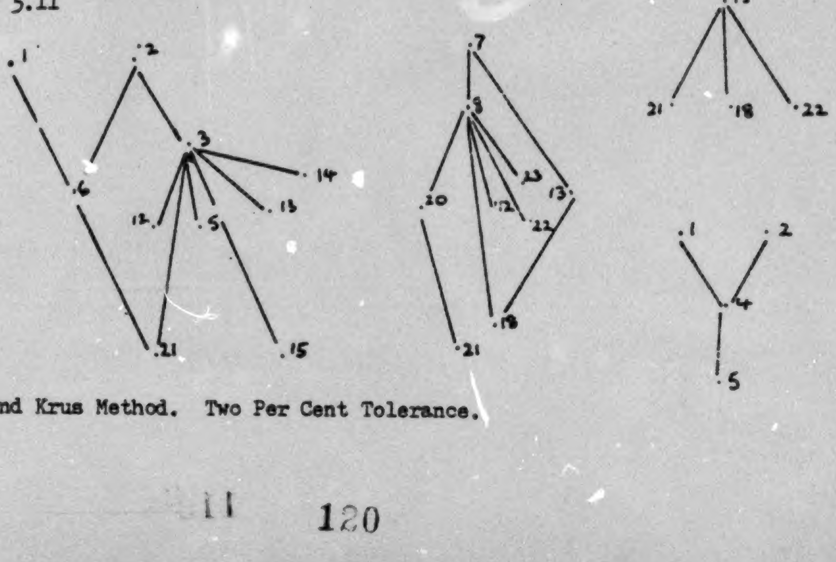


Grouping Items by McQuitty Linkage Analysis

The Analysis of Bart and Krus

Bart and Krus proposed another form of analysis, again involving a vertical chain of items hard/easy rather than facility grouping. They investigated two items i and j, if (0,1) meant (incorrect, correct) their interest was in the cell of the four cell matrix which displayed item i-0, item j-1. Item i was called a prerequisite for item j if a zero appeared in the cell, i.e. the cell was empty, which showed fail item i, pass item j. When the ratio data were analysed, no cell in the item/item matrices contained a zero entry. Entries close to zero had to be accepted, a cell entry of two per cent of the sample was called a two per cent tolerance level. At the two per cent level variables 1 and 2 were prerequisites for every other item except variables 7, 8 and 15. At the same tolerance level variable 3 was a prerequisite for nine items. Figure 5.11 shows the trees which were apparent when the two per cent tolerance level was applied. The only tree with more than three stages is that which connects variables 7, 8, 20 and 21.

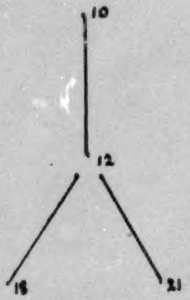
Figure 5.11



Bart and Krus Method. Two Per Cent Tolerance.

When a five per cent tolerance level was used, the distinction between the easiest items was lost and variables 1 and 2 became prerequisites for variables 7, 8 and 15. Very nearly all the items were prerequisite for variables 18 and 21 and many were prerequisite for variables 22 and 23. Linkages between items of medium facility were the most interesting but only one was apparent, see Figure 5.12.

Figure 5.12



Bart and Krus Method. Five Per Cent Tolerance.

All the above methods when applied to the ratio data produced links between hard and easy items but few connections between items of the same facility. The easiest items had no strong links and there appeared to be three strands A, B and C as shown in Figure 5.7. McCready and Merwin (1973) had addressed themselves specifically to clustering items within a facility band. They investigated 'item forms' with regard to diagnostic domain referenced tests. An item form was considered inadequate if a) the items within the form were not homogeneous, b) the items were not of equivalent difficulty or c) both of these. To study the nature of the relationship among items within an item form they used Loevinger's index of homogeneity H_t (Loevinger 1947), this index is defined as follows:

$$\text{Loevinger } H_t = \frac{\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} P_j \ (P_{i/j} - P_i)}{\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} P_j \ (1 - P_j)} \qquad (P_i \geq P_j)$$

where P_i is the probability of passing the i^{th} item

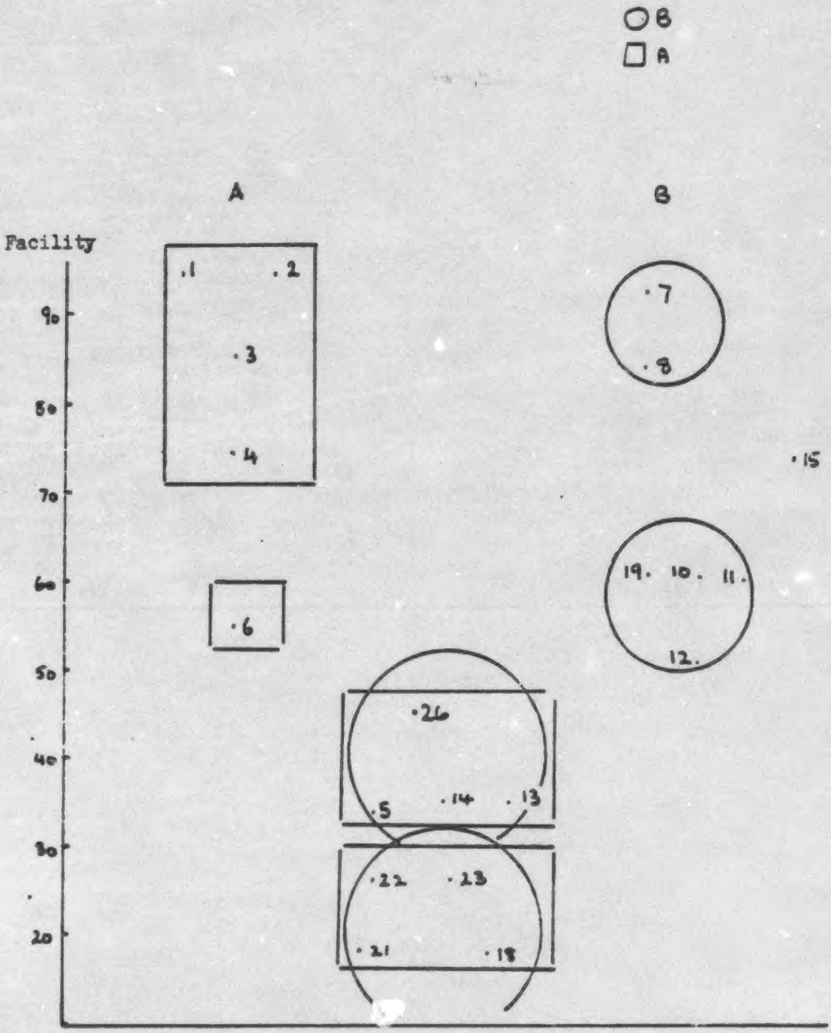
P_j is the probability of passing the j^{th} item

$P_{i/j}$ is the probability of passing the i^{th} item among those known to have passed the j^{th} item.

McCready and Merwin took $H_t \geq .5$ as an indication of relative homogeneity. When the ratio data were used, it was found that H_t could equal .5 when two items within a group of three had a high value of H_{ij} (item/item), the third item being connected at a low level with each of them.

The method outlined by McCready and Merwin was applied to the data obtained from the testing of the fourth years. It was applied only to items which occurred in strands A and B since strand C contained only three items, two of which were known to have caused difficulty (the gaps in the open figure question). The groups themselves are shown in Figure 5.13 and the values of H_t appear in Tables 5.3 and 5.4.

Figure 5.13



Groups Suggested from Fourth Year Data Using Method of McCready and Merwin

Table 5.3

Using Loevinger H_t Values for Grouping Items
(Fourth Year Data)

Group	Variables	Value of H_t
1 A	1, 2, 3, 4	.668
1 B	7, 8	.98
2 B	9, 10, 11, 12	.537
3 A, B	5, 26, 14, 13	.569
4 A, B	21, 22, 23	.628
Alternative group 4	21, 22, 23, 18	.566

Table 5.4

Using Loevinger H_t Values for Grouping Items
(Total Sample)

Group	Variables	Value of H_t
1 A	1, 2, 3, 4	.625
1 B	7 8	.96
2 B	9, 10, 11, 12	.506
3 A, B	5, 26, 14, 13	.505
4 A, B	22, 23, 21, 18	.501

Note: All the groups possess an H_t level within the stated criteria.

The sets of items were chosen according to facility and H_t found. The values of H_t when the total sample was used were lower than those for the fourth year sample although always over .5. In the above tables the groups are labelled by the strand in which they occur, if one assumed that the two groups of easiest items were distinct. An attempt was made to include item 6 in group 2B but this resulted in a value of .454 for H_t.

When the items on the ratio test paper had been grouped by using H_t as just described, the children who had formed the sample were assigned to a level of understanding on the basis of the hardest group of items they passed. This entailed the provision of a pass mark for each group of items and the institution of a stage zero which preceded stage one on either strand and meant that the child did not obtain a group one pass. A child was assigned to level 2B if he or she achieved a pass mark on levels 1 and 2 in the B strand, any partial success on strand A was ignored. If the child was assigned level 3 he or she could have been successful on levels 1 and 2 in strand A or levels 1 and 2 in strand B. In order to test scalability of the groups within each strand i.e. to test that success on a harder group entailed success on all easier groups, a Guttman scalogram analysis was carried out on each strand. Two pass marks for each group of items were tried. The results appear in Table 5.5.

Table 5.5

Guttman Scalogram Analysis Used on Two Strands (1976 Data)

Sample	Strand	Pass Mark				Coefft. of Reproducibility	Coefft. of Scalability
		Gp.1	Gp.2	Gp.3	Gp.4		
Total	A	3/4	1/1	3/4	4/4	.9759	.8848
n=2257	A	3/4	1/1	4/4	4/4	.9814	.9001
	B	2/2	3/4	3/4	4/4	.9688	.8494
	B	1/2	3/4	4/4	4/4	.9854	.9088
2nd year	A	3/4	1/1	3/4	4/4	.9781	.8803
n=800	B	2/2	3/4	3/4	4/4	.9706	.8365
4th year	A	3/4	1/1	3/4	4/4	.9701	.8667
n=690	B	2/2	3/4	3/4	4/4	.9360	.8431

By taking each strand separately for the scalogram analysis the error count on strand B3 (i.e. success on level 3 without sucess on all previous levels in that strand) would include those children who reached level 3 by the A route. The true error children were found to be 33 in number. Table 5.6 shows the performance of the total sample, the child was assigned to the highest level at which he obtained the pass mark, whether it was on strand A or on strand B. The less stringent pass marks shown in Table 5.5 were used. Those children who obtained pass marks on both strands appear under the heading 'Both'.

Table 5.6

Performance on Each Strand (Total Sample n=2257)

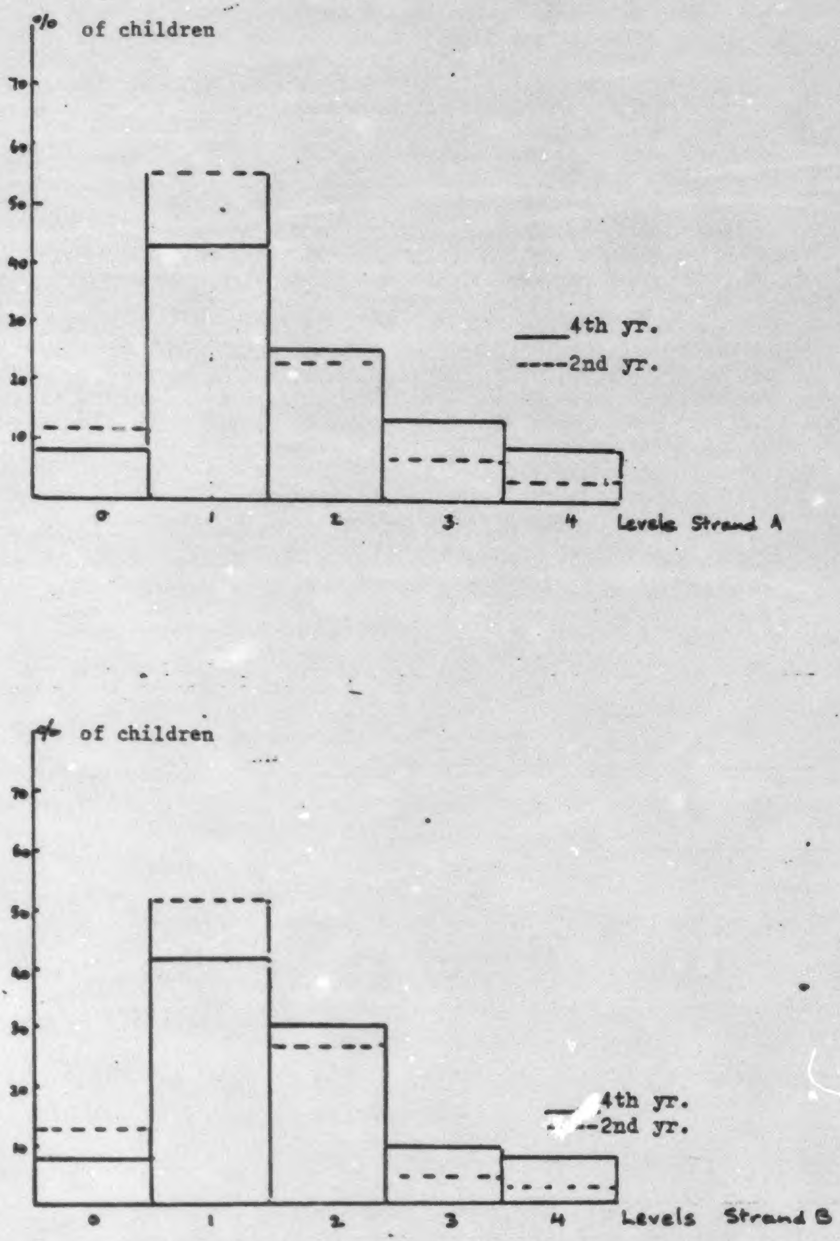
Strand A	Both	Strand B	
	92		Level 0
282	488	113	Level 1
403	245	158	Level 2
71	246	38	Level 3
6	73	3	Level 4

Error children - 33; 13 - 4th yr, 12 - 3rd yr, 8 - 2nd yr.

Although the great majority of children who obtained the pass mark on the hardest group of items had managed to obtain the requisite passmarks on all previous groups on both strands, it was possible to score on the hardest items when missing some of the easier items on one of the strands. A comparison of the performance of the second and fourth years on each strand is shown in Figure 5.14.

This method was not pursued since it was impossible to provide criteria for a comparison between the two strands i.e. whether level 2 Strand A was equivalent to level 2 Strand B.

Figure 5.14



Performance of 2nd and 4th Years on Strands A and B

Using Computer Programs for Rasch Analysis and Cluster Analysis

Both these forms of analysis were tried on the Ratio data. The Rasch analysis rejected nearly every item when the entire sample (n=2257) was used and when a very much smaller sample was analysed, it kept nearly every item. It therefore seemed heavily dependent on the size of the sample.

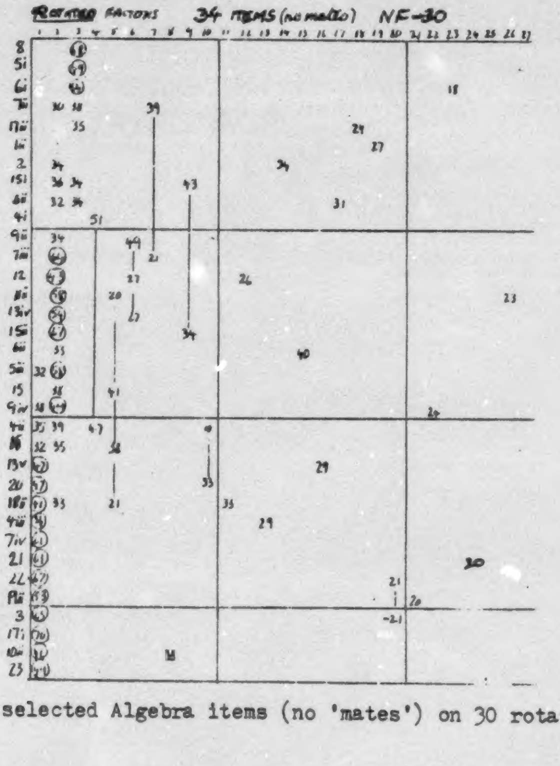
The Cluster program available at ULCC was not suitable technically for clustering items and so an attempt was made to cluster the children, this led to other difficulties and was not pursued.

Factor Analysis

The results of factor analysing the Ratio data and accepting high values of factors has been shown in Figure 5.2. There it can be seen that items taken from the same question (variables 1, 2, 3 are all from question 1) load on the same factor, which is not surprising. There also appeared to be a strong tendency for factors to be closely linked to facility. Factor analysis was further used on the Algebra, Graphs and Ratio data. In this second type of analysis the rule was devised that no two items from the same question ('mates') were allowed if they were close in facility; the item with the lower loading being rejected.

The factor analysis of the Algebra test is summarised in Figure 5.15

Figure 5.15



Loadings of 34 selected Algebra items (no 'mates') on 30 rotated factors which shows the loadings of selected items (no 'mates') on 30 factors after rotation. The items have been ordered in terms of facility, from which it can be seen that the first three factors are simply the facility factors (after rotation). The remaining factors are only showing substantial loadings on items from the same question which had not been removed because they were reasonably far apart in facility. The only factor with a loading above 0.3 which is not a facility or question factor is the fifth rotated factor on which items 14 and 16 have loadings of 0.41 and 0.38 respectively. However, more of the variance of these items is explained by the facility factors than by this factor and it is arguable whether much significance should be attached to it since it expresses very little of the overall variance of the test (and since only three of the original factors had eigenvalues greater than one).

As illustrated above, in a standard factor analysis procedure the first step is to calculate the intercorrelation matrix. If a factor analysis was to be performed on the items of a test the correlation to be used would be the inter-item phi-coefficients, the phi-coefficient being a special case of the Pearsonian correlation for dichotomous variables. If the test consists of items which are very different in facility then, because phi can achieve its maximum of one only between items of the same facility, the correlation matrix will have a peculiar form regardless of how similar in content the items are. Very easy and very difficult items cannot have high correlations so in general would not load highly on one factor even if one item was merely a harder version of the other. Thus, as Ferguson (1941) has shown, even if there were homogeneity of content in the items, three or more factors would arise, with the factor loadings being highly dependent on facility. If it was proposed to correct the phi-coefficient for facility differences, by for example using Yule's coefficient or ϕ/ϕ max, then similar problems arise because very easy items tend to have high coefficients with very hard items even when there is little support for homogeneity (see below) and factor loadings are again distorted by facility differences. Furthermore it is no longer clear to what extent factor loadings can be interpreted in terms of proportions of variance.

Method Selected for CSMS(M) Analysis

As noted in the previous section a variety of methods for establishing a hierarchy were subjected to preliminary investigation. These led to a consideration, and subsequent selection of the use of the homogeneity coefficient ϕ, clustering according to facility, and testing scalability of the groups of items by using Guttman scalogram analysis.

Choice of Homogeneity Coefficient, ϕ

Considering the homogeneity coefficients H_{ij}, Q and ϕ mentioned earlier let us suppose the two items are heterogeneous, that is passing one item does not affect the probability of passing the other. In this case all three indices have an expected value of zero. Negative values of all these coefficients are rare in the normal testing situation, since it would be rather unusual to include two items, which were inversely related, in the same test. As used here, the principal function of the indices would be to differentiate degrees of positive relationship.

Q and H_{ij} both have maximum values of +1, regardless of the perceived difficulties of the items and the maximum is attained when passing the harder item implies passing the easier. Phi, on the other hand, can attain its absolute maximum value of +1 only when the perceived difficulties of the items are the same, and in general has a maximum value which decreases as the difference in difficulty between the two items increases.

Indeed one formulation of H_{ij} is as $\frac{\phi}{\phi_{max}}$, where ϕ_{max} is the maximum attainable value of ϕ given the perceived difficulties of the items, and this fact supports the use of H_{ij} when a measure of the homogeneity of two items, very different in facility, is required. However, the constraint on the maximum value of ϕ would tend to support its use, when the aim is to detect groups of items, which are homogeneous and have similar level of difficulty.

Examples

1

	Fail	Pass
Pass	99	1
Fail	1	0
		101

$\phi = 0.01$

$H_{ij} = 1.$

$Q = 1.$

2

	Fail	Pass
Pass	22	39
Fail	38	2
		101

$\phi = 0.59$

$H_{ij} = 0.88$

$Q = 0.94$

In the first example a hard item has been compared with an easy one. The probability of the outcome shown for two heterogeneous items, is 0.99 and thus there is almost no support for homogeneity. H_{ij} and Q, however, have high values because there is no evidence against homogeneity, in that no-one has passed the harder item without having passed the easier. In the second example there is more support for homogeneity, the probability of an outcome as extreme as this being less than 0.001 for two heterogeneous items. ϕ has correspondingly increased, while H_{ij} and Q have fallen.

In fact, the absence of a high H_{ij} value between a very easy and a very difficult item may be more noteworthy since it implies that children who have passed the harder item do not have an improved chance of success on the easier. Clearly, any analytical procedure based on H_{ij} or Q would emphasise those relationships of least interest, namely those between very difficult and very easy items. In an agglomerative cluster analysis method, for example, if H_{ij} or Q were used then, very early on, hard and easy items would combine. This would tend to obscure any more informative clusters. Use of phi, on the other, would leave hard and easy items ungrouped at first. This is not to imply a criticism of the use of H_{ij} or Q in test analysis generally, but only to question their use, when it is the relationships between items of similar difficulty that are of prime interest.

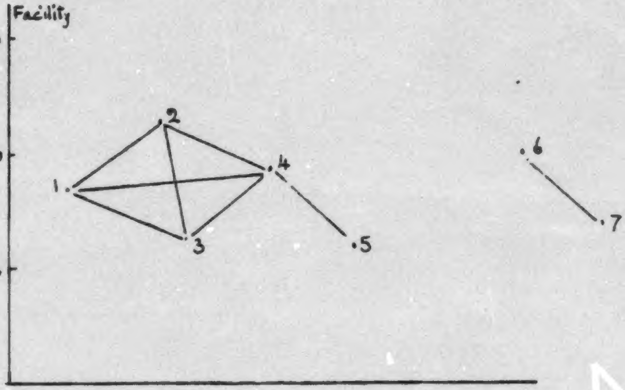
To solve the problem presented by each of the CSMS(M) tests the decision was taken to use homogeneity coefficients to form groups of items within a facility band. Although it was recognised that there might exist sets of items which formed hierarchies across the facility spectrum these would be, for the moment, ignored and an initial description of the data would consist of groups of items each of which was in a different facility range. A more detailed description could be provided later.

Some form of R-type cluster analysis would seem to be a suitable method for approaching the problem. A similarity coefficient inter-item would first have to be chosen and for the reasons outlined above the phi-coefficient would seem to be the most suitable. An attempt has been made to devise a method which will also give groups which are close in facility. The methods finally adopted are therefore slightly unorthodox in that they are designed to control for facility as well as operate on the phi-correlation matrix. Two methods were used, interrelated since they both used ϕ, the first was called 'The Plotting Method', the second 'Facility Ordered Complete Linkage'.

The Plotting Method

All the items are plotted on graph paper against an axis labelled with facility and then lines are drawn to connect all items which possess a ϕ of above .6. Next lines are drawn to represent ϕ = .5, .4 and .3. There will be some items which connect at a low level (.3) with many items, other which connect at a high level with only one other item. There will also of course be items which do not connect at $\phi \geqslant .3$ with anything, which are for the moment omitted. Each item is tested for its connection with harder items using H_{ij}, those retained have $H_{ij} \geqslant .6$ with at least one of the items of very different facility. Only those items are left which have a ϕ connection with similar facility items and an H_{ij} connection with very different facility items. Any item which connects with only one other is regarded as peripheral to the general structure or skeleton of the test, see Figure 5.16.

Figure 5.16



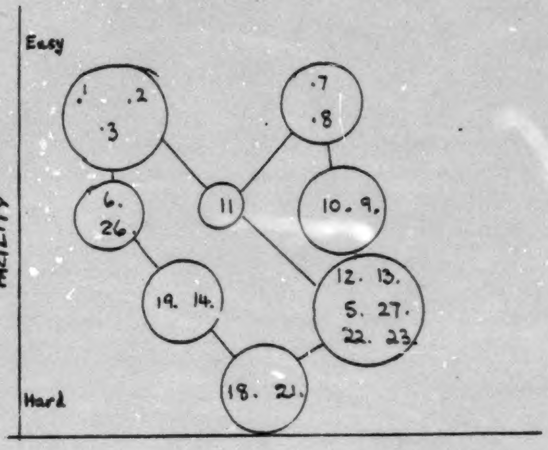
Lines show there is a ϕ connection

Items 6 and 7 will be regarded as not central to the test structure, item 4 will be regarded as in the "skeleton" with item 5 not in the skeleton but linked to item 4.

An example of how this form of analysis might develop is given using the 1976 Ratio data.

On the Ratio test we found many items were linked at the $\phi \geqslant .3$ level, some at a higher level. Looking at the values of H_{ij} as well, in order to see vertical linkage leads to the representation given in Figure 5.17. Items 1, 2, 3 were closely connected and were easy, similarly items 7 and 8. Since the intention was to provide just one group at a facility level these items were put together even though the ϕ coefficient item 7-item 1 was lower than between items 7 and 8. The final grouping with the mean ϕ value is shown in Figure 5.18. The problem of cut-off for groups is described later since it arises in both methods of analysing the items.

Figure 5.17



Within a circle ϕ is high. Strongest hard/easy links (ϕ or H) between circled items are shown by lines.

Figure 5.18

Facility per cent		Mean ϕ
80	1, 2, 3, 7, 8	.31
60		
40	10, 9, 11, 6, 26	.39
20	19, 12, 14, 13, 5, 27	.41
	23, 22, 21, 18	.41

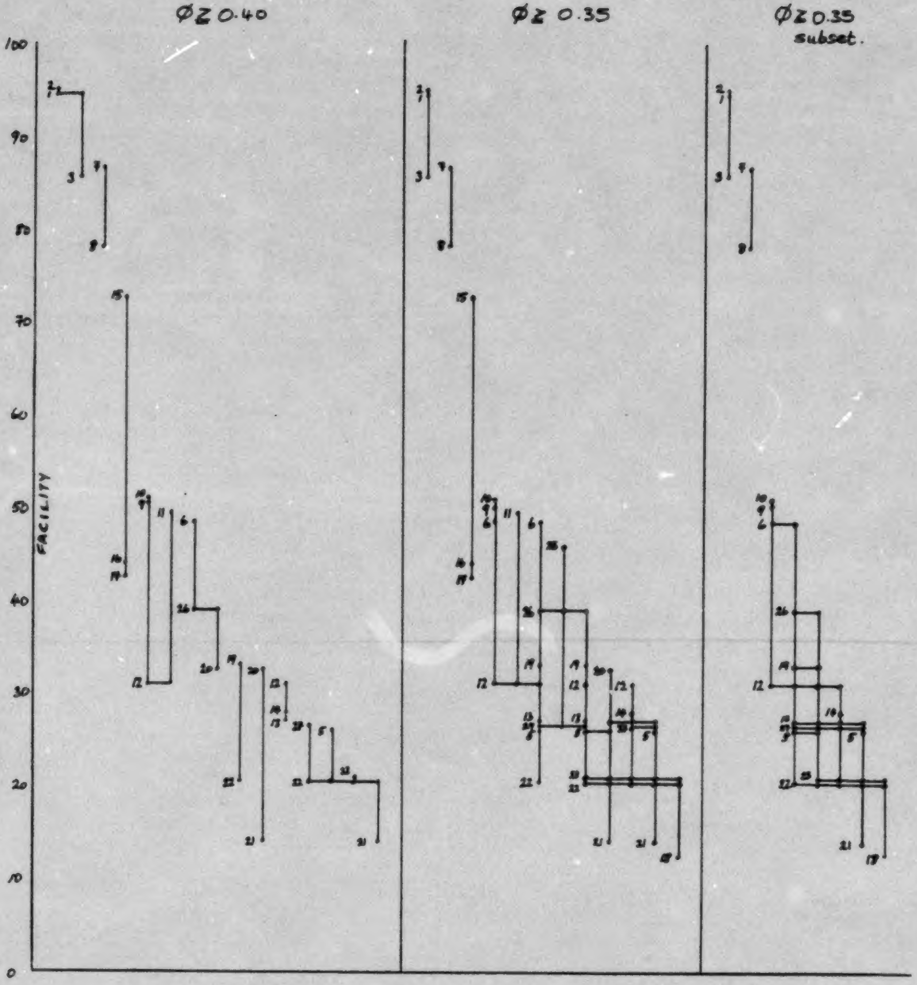
Facility Ordered Complete Linkage

A criterion value of phi is first chosen. Starting with the easiest item, the next most easy item with a phi value greater than the criterion is joined to it. The third item to join the group will be the next most easy item with phi values greater than the criterion with both items already included. Subsequent items are joined similarly, provided the phi values with items already included are all greater than the criterion. This continues until there are no further items achieving the criterion and the process is then repeated, starting with the second most easy item. And so on, to form a chain of items attached to each item.

In general however some of these chains may be subsets of others and may be ignored, and there may often be overlaps of chains which should be examined further. Also, an item may not necessarily achieve the criterion with any other item, which might throw doubt on whether it was performing as intended.

Clearly, the maximum value property of phi will permit the chains formed to span a difficulty range which is limited by the criterion chosen. It is sensible to repeat the method with different criterion values to balance the desire for reasonable sized chains and suitable difficulty range with the need to have the chains as homogeneous as possible.

Figure 5.19



Ratio Test (1976)

In the Ratio test the chains formed by the procedure outlined above could be plotted against observed facility level (see Figure 5.19). Items which appeared in more than one chain were joined, where possible, by a line parallel to the x-axis, the chains being signified by lines parallel to the facility axis. Items which did not enter any chain were omitted.

With a criterion value of 0.4 many chains were formed but each contained only a few items, and they often consisted of items which were different parts of the same question e.g. items 12, 13 and 14 which have a natural similarity. If the criterion is reduced to 0.35 the chains (as expected) spanned wider facility ranges, the groups were larger and there was considerable overlap. (The final group shown here would not strictly have been formed by the procedure above but was included as item 18 only just failed to reach the criterion with item 21). The diagram seems rather difficult to interpret and simplification was attempted by removing some of the items which did not appear to be performing well. Items 15, 16 and 17 are parts of a geometrical problem which, although highly interrelated (they require exactly the same operation), had very low correlations with all the other items on the test. Items 11 and 25 only appeared in small chains with other very similar items, and seemed therefore to be testing more the ability to cope with the particular question rather than the general subject matter of the rest of the test. They are both introductory items to their respective questions and were easier than the other items in their questions. Item 20 was also withdrawn at this stage because, although it had moderate correlations with the more difficult items it had low correlations with the slightly easier ones and therefore did not appear to fit the general pattern.

When the analysis was repeated using only the items that remain some of the confusion was removed. Apart from the very easy items which fell into two groups it appeared that in the remainder of the test the different chains had been formed on the basis of the differences in difficulty.

Both methods of plotting items which are connected by a ϕ of a specified level reduce the number of items being considered. Items which have few such linkages are ignored. A start was made therefore on grouping items, but the cut off between successive groups was however still undecided. If a large facility gap was apparent this was used as a cut-off point thus items 1, 2, 3, 7, and 8 seemed likely candidates for a group.

The easy items posed a special problem because they fell into two groups which had low correlations between them. However, since there were few items at this level they were assigned to a single group, which was used as a criterion for starting the remainder of the test. This action was supported by the fact that all of these five items had high Loevinger H_{ij} coefficients with the harder items. Item numbers 19, 12, 14, 13, 27 and 5 seemed to form a reasonable group. Their facilities were all similar and intercorrelations were good (the correlations between item 14 and item 5 (.348) and between item 14 and item 19 (.332) were the only ones to fail to achieve the criterion). Similarly at an easier level items 10, 9 and 6 combined, with item 26 somewhere in between. Item 26 was included with the easier of the two groups because its highest correlation was with item 6. Items 23, 22, 21 and 18 also formed a reasonably homogeneous group in the highest difficulty range (the only low correlation - between items 21 and 18 - was 0.322), although the separation of items 23 and 22 from the harder items in the group above was based on other criteria, i.e.

1) Mathematics coherence, and
2) Examination of the performance of children in terms of a particular error, e.g. items 18, 22 and 23 were often solved incorrectly because of the use of the addition strategy where, instead of using the ratio a/b, the child used a difference a-b and added it to obtain an enlargement.

Groups of just two items were amalgamated with others since two seemed to be too few to be describable in any general sense. To test the scalability of the groups arrived at, each child's performance on each group of items was assessed and if a child correctly solved 2/3 of the items within a group the child was assigned a score of 1 on the group, otherwise 0. A Guttman scalogram analysis was then carried out. If a preponderance of errors appears between two particular adjacent groups, these groups were further investigated to see whether the cut between them could be changed without denying any of the criteria already described.

A pass mark of 2/3 was an arbitrary decision, ½ seemed too low for the statement "has at least achieved this level" and 100 per cent seemed too strict a demand. The Guttman scalogram analysis was likely to result in no gross percentage error since items randomly solved by the children would have already been rejected on the basis of ø.

The Ratio test used as an illustration in the above discussion produced the hierarchy given in Figure 5.20.

Figure 5.20

Level 0	not of Group 1	Mean ø	Median ø
Level 1	$\frac{3}{5}$ items 1, 2, 3, 7, 8	.31	.17
Level 2	$\frac{3}{5}$ items 10, 9, 11, 6, 26	.39	.35
Level 3	$\frac{4}{6}$ items 19, 12, 14, 5, 27, 13	.41	.37
Level 4	$\frac{3}{4}$ items 23, 22, 21, 18	.41	.38

Child giving error responses 3.6 per cent (n=2257)

Coefficient of Reproducibility	.9816
Coefficient of Scalability	.9127

Using these groups of items on a sample of different children in 1977 the percentage of children giving error responses was 3.36, Coefficient of Reproducibility .98, Coefficient of Scalability .91.

CSMS(M) has produced 12 test papers on mathematical topics and analysed each by the method described above. The values of ø vary from test to test but an attempt was made to keep ø = .35 as the lower level for inclusion of items. Easy items tended to have lower ø values than those of about 50 per cent facility. A group of easy items had on every occasion been included however in order to provide a starting point for the description of levels in a particular topic.

A complete list of the ø values for each test appears in Appendix 1 and the mean and median ø's for each level in each test appear in Appendix 5.

CHAPTER SIX

THE HIERARCHY

In many cases the order in which mathematics is presented to children is dictated by the needs of mathematics e.g. linear equations before quadratics. In other cases there is no clear order of presentation apparent in the mathematics and decisions have to be made by the teacher, based on experience and the dictates of the school syllabus. The research being reported took individual topics and attempted to form a hierarchy in each based on what the children tested appeared to understand. The data from the wide scale testing were used to form the hierarchy in each topic and therefore all the hierarchies are dependent on the items which appeared in the tests. Simply ordering the items according to facility (hard to easy) would not satisfy the criteria for a hierarchy of understanding. Therefore a method was sought by which items on a test paper could be grouped to form a "type" and then the groups ordered to form a chain (easy to hard) in such a way that success on a harder group of items automatically entailed success on all easier groups. The coefficients of association that were used are defined in Chapter Five.

Items had to satisfy the following criteria before they were considered to form a group:

a) they should be of approximately the same level of difficulty i.e. the same facility;

b) the values of the homogeneity coefficient ϕ item/item should be at an acceptable level;

c) the items should be linked (ϕ or H_{ij}) with the items in both easier and harder groups;

d) there should be some measure of mathematical coherence to the items.

e) the groups should be scalable in the sense that a child's success (assessed as 2/3 correct) on a group entailed success on all easier groups. (Error subjects, or those violating the criteria, were not to exceed seven per cent of the sample in any one test.);

f) there should be no gross discrepancies when each age group's results were analysed in the same way.

In some topics there were obvious facility gaps between clusters of items, which suggested cut-off points between groups of items. In other topics there were no obvious cut-off points between groups of items and decisions were made on the basis of criteria d and e above. An acceptable level of ϕ depended on the test paper being considered; in some cases the easiest items were not highly correlated but were formed into a group in order that there should be some starting point to the hierarchy. The use of the criteria meant that several items on each test paper were rejected, usually because the correlation of a specific item with others of the same facility was noticeably lower. The values of ϕ were usually around 0.4 (1.00 showed perfect homogeneity); they are quoted in Appendices 1 and 5.

The gaps between levels in a hierarchy were very seldom equal and it was often the case that items in say level 3 were much more difficult than those in level 2 whereas the facilities of the hardest level 1 items were very close to those of the easiest level 2 items. In particular topics a child who obtained a 2/3 pass mark on level 3 items displayed a grasp of a type of mathematics different from that represented by items in level 2, e.g. in Graphs level 3 items were concerned with algebraic relations while level 2 items involved interpretation of data graphs. There is no suggestion that the step from level 2 to level 3 is equivalent to that from level 1 to 2 or indeed that the steps between any two levels can be regarded as equal. Because the items at a particular level were associated and because those same items were usually in problem form we have referred to the levels as 'levels of understanding'. They are of course measures of attainment on particular tests but since their formation required rather more than total scores and the attainment of each infers the attainment of all easier levels, the claim is made that they to some degree represent the child's grasp of the topic.

The Hierarchies for Each Topic Area

In order that the reader can see the contents of each level of each topic hierarchy; the items and their facilities included in each level for the different age groups tested are shown in Appendix 7. The numbers refer to parts of questions (variable numbers) which are printed on the test papers in Appendix 1. As noted previously, many items on each test were rejected because they did not show a sufficiently high level of association to be included in groups.

The contents of each level (a cluster of items) were described in general terms after discussion among the research team. For some of the tests teachers were also asked how they would describe the different levels. The descriptions were an amalgam of the mathematical content of the items and the methods children often used to solve them (obtained from the data). Thus on Ratio certain levels were described in terms of 'child methods' when these seemed popular but naive. The general descriptions for each topic hierarchy are given in Tables 6.1-6.12.

Table 6.1

Levels of the Measurement Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Area found by counting squares. Volume of a cuboid (diagram) when there is only one cube on each layer.
2	Volume can be found by counting cubes when not all of them are shown. Simple application of the area formula. (Examples of items at this level are 18b and 24.)
3	Volume of cuboids when dimensions but not cubes are shown. Formula for area of a rectangle needed. Area of a triangle. (Examples of items at this level are 18c and 14B.)
4	Application of area or volume formulae where the formula has to be adapted e.g. half units or half formula are used. (An example of items at this level is 20E.)

Table 6.2

Levels of the Graphs Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Plotting points (integer coordinates or halves). Interpretation of block graphs. Recognition that a straight line represents a constant rate. Interpretation of a scattergram.
2	Simple interpolation from a graph. Recognition of the connection between rate of growth and gradient. Use of scales shown on a graph and an awareness of the effect of changing the scale. Interpretation of simple travel graphs.
3	The relationship between a graph and an algebraic equation, for example $y = 2x$, $y = 2$, $x = 2$ and $x + y = 2$.

Table 6.3

Levels of the Ratio and Proportion Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	No rate needed or rate is given. Multiplication by 2, 3 or taking half. (Examples of items at this level are 1a, 2a(i), 2a(ii).)
2	The rate is easy to find or the answer can be obtained by taking an amount and then half as much again. (Examples of items at this level are 2a(iii), 2a(iv), 8c.)
3	The rate is harder to find than in level 2 questions, as in Mr. Short and Mr. Tall. Operations on fractions,e.g. 1b(iii).
4	The recognition that ratio is needed and that a 'multiplier' must be used. Non-integer enlargement, e.g. 5:3.

Table 6.4

Levels of the Fraction Problems Hierarchy (12-13 year olds)

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	The meaning of a fraction using pieces, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$. (An example of items at this level is 4a.)
2	The meaning of a fraction as a subset of a set. Naming a given configuration of pieces of a whole. Equivalent fractions obtained by doubling. Addition of two fractions with the same denominator. (Examples of items at this level are 2a, 14a, 19.)
3	Using equivalence to name parts with familiar fractions or when a diagram is provided. Equivalent fractions not obtained by doubling or when the fraction is less familiar. (Examples of items at this level are 4b, 14e, 22.)
4	More than one operation is needed, for example equivalence followed by addition or subtraction. (An example of items at this level is 20.)

Table 6.5

Levels of the Fraction Problems Hierarchy (14-15 year olds)

Level	Description of the Groups of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	The meaning of a fraction seen as part of a whole when no equivalence is needed. Equivalent fractions obtained by doubling. Addition of two fractions with the same denominator. (Examples of items at this level are 5a, 9.)
2	Equivalent fractions not obtained by doubling. Using equivalence to name parts with familiar fractions or when a diagram is provided. (An example of items at this level is 9d.)
3	More than one operation is needed, for example equivalence followed by addition or subtraction. (An example of items at this level is 15.)
4	Division and multiplication. (An example of items at this level is 22.)

Table 6.6

Levels of the Positive and Negative Numbers Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Integers used to describe positions on a number line. Integers treated as natural numbers.
2	Addition of integers when they can be modelled as shifts along the number line.
3	Multiplication when the rule 'Two minuses make a plus' can be used. Subtraction when the same rule can be used, as in ${}^{+}8 - {}^{-}6$. Subtraction when the integers are positive, as in ${}^{+}6 - {}^{+}8$.
4	Subtraction when both integers are negative, e.g. ${}^{-}2 - {}^{-}5$.

Table 6.7

Levels of the Vectors Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Elementary work with ordered n-tuples not representing movement or positions: (i) the use of ordered n-tuples to store data in a story context, in particular the addition of n-tuples and their interpretation, (ii) the interpretation of $a\underline{x}$ as 'a' lots of $\underline{x}$ where $\underline{x}$ was an ordered n-tuple.
2	Ordered n-tuples. Further work including elementary movements: (i) the formation and interpretation of scalar multiples of ordered n-tuples storing data in a 'story' context, (ii) the solution of equations involving scalar multiples of ordered n-tuples, (iii) an explanation of the need to use negative integers when writing down vector moves, (iv) writing a move in the form $\begin{pmatrix}a\\b\end{pmatrix}$ where a and b are positive integers.
3	Elementary geometrical work with vectors: (i) the expression of a given n-tuple in terms of $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ (help given with layout), (ii) the writing of a move with form $\begin{pmatrix}a\\b\end{pmatrix}$ where one element is a negative integer, (iii) a description of elementary scalar multiples of a given vector $\underline{a}$ such as $\frac{1}{2}\underline{a}$, $2\underline{a}$ and $-\underline{a}$ in terms of the magnitude and direction of $\underline{a}$, (iv) the completion of a simple vector triangle of addition where the vectors are already nose to tail and a similar example has been given, (v) the writing of a simple route vector chain as a single route vector and the expression of one route vector as a chain of others using a given diagram.
4	Elementary abstract movements: (i) the expression of a given n-tuple in terms of $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ and the expression of a n-tuple in terms of other bases when only simple simultaneous equations are involved, (ii) moves of the form $\begin{pmatrix}0\\-b\end{pmatrix}$, (iii) the drawing of the result of a simple translation, (iv) finding the vector for a double translation, (v) working out the result of a translation followed by its inverse where no diagram is available, (vi) the replacement of a string of vectors by one resultant vector when no diagram is provided, (vii) the marking of equivalent vectors when no distractors are present.

Table 6.7 continued

Level	Description of the Group of Items
5	Further abstract movements: (i) the representation of three successive movements on a diagram as an ordered n-tuple together with the addition of such movements (including negative integers), (ii) the vector for a double translation, (iii) the marking of scalar multiples of a given vector given a selection of vectors, (iv) the marking of equivalent vectors when strong distractors are present, (v) the provision of missing route vectors for a chain, given the resultant but no diagram, (vi) the finding of the inverse of a three dimensional move (no diagram).
6	Explanation, checking and commutativity: (i) the interpretation of the sum of three vector moves on a given diagram, (ii) finding a route and checking it by using vector equivalence, (iii) the explanation of the commutativity of vector addition, (iv) vector equivalence explained in terms of necessary and sufficient conditions.
7	Full use of vector equivalence: (i) the use of vector equivalence in either free vector or route notation (difficult items), (ii) the use of vector addition and vector chains involving vectors such as $-\underline{a}$.

Table 6.8

Levels of the Matrices Hierarchy

(Levels 1-4 are composed of items common to both A and M test papers. The sample of children for these levels is a combination of the A and M subsamples. Extra items contained in the M test and attempted by the M (brighter) sample enabled the hierarchy to be extended to levels 5 and 6.)

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Addition and simple interpretation of matrices: (i) the addition of matrices within a 'story' context or with no meaning attached, (ii) the interpretation of matrices given either row and column headings or the information in written form, (iii) writing data in matrix form, (iv) the extraction of data from a matrix and simple calculation on such data.
2	Further interpretation and work with elementary product matrices: (i) the interpretation of simple column matrices (no headings), (ii) the explanation of a simple matrix multiplication (data given in the test), (iii) the calculation of a single entry product matrix and the comparison of this with another single entry product matrix (data in the text), (iv) the calculation of one entry of a 2 x 1 product matrix when the other has been calculated and defined.
3	Interpretation of elementary product matrices and elementary route structure: (i) the identification and calculation of a number of routes from a given diagram where the number is small (3 x 3 = 9), (ii) the significance of the consistency of units of measurement in a data storage matrix, (iii) the interpretation of one element of a 2 x 1 product matrix when the other element has been calculated and defined, and the calculation of such an element when the other has not been defined, (iv) the extraction of information from a storage matrix and the performance of calculations on this to give elements of the product matrix.
4	Advanced interpretation, multiplicative route structure and the commutativity of addition: (i) the calculation of the number of routes in a route system (no diagram) and the number is large, (ii) the recognition of errors of position in a data storage matrix, (iii) the interpretation of one or both elements of a 2 x 1 product matrix, (iv) the recognition that the addition of matrices is commutative.

Table 6.8 continued

Level	Description of the Group of Items
5	Commutativity and complex classification: (i) the distinction between the non-commutativity of multiplication and the commutativity of addition, (ii) the drawing of transformed geometric figures, (iii) the accurate 2-way classification of a complex product matrix, (iv) the writing of a suitable story as a matrix multiplication, (v) the giving of the correct structure to a matrix so that matrix multiplication can be placed in a story context.
6	The relationship between the elements of a matrix multiplication: (i) the use of the correct row and column headings on a route product matrix, (ii) the understanding of the relationships between the elements of matrices being multiplied (including writing a story for a matrix multiplication).

Table 6.9

Levels of the Place Value and Decimals Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Place-value in whole numbers up to thousands.
2	Interpretation and use of 'tenths' place in decimals.
3	Interpretation and use of hundredths and thousandths etc. places.
4	In decimals, relating one place with the adjacent place on the left.
5	More complex relationships between places in decimals.
6	Decimals which result from division and the existence of an infinite number of decimals.

Table 6.10

Levels of the Algebra Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	The significance of letters need not be realised; letters can be ignored, evaluated or seen as objects.
2	Letters used as objects or evaluated but there is need to use mathematical syntax.
3	Letters as specific unknowns. Lack of closure must be accepted.
4	The manipulation of specific unknowns where this involves a coordination of operations or where the letters represent numbers of objects or their cost etc. rather than the objects themselves. Letters used as variables.

Table 6.11

Levels of the Reflection and Rotation Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Image can be constructed, by controlling direction and distance, coped with in a step by step manner.
2	Similar to level 1 but a more critical awareness of what is needed for completion is required.
3	Reflections where the object or mirror line is off the page so that the required distances have to be derived analytically rather than by direct measurement or counting. Rotations of 90° which require, at least mentally, that a vertical or horizontal line be constructed from the centre to the object.
4	Reflection of flags, etc. in slanting mirror lines, which requires a fully analytic approach. 90° rotation on a grid (which seemed to act as a distractor).
5	90° rotation when the object, or line from the centre of rotation to the object, is at any angle. Combination of reflections and rotations.

Table 6.12

Levels of the Number Operations Hierarchy

Level	Description of the Group of Items
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Recognition of subtraction. Construction of stories concerning subtraction.
2	Recognition of multiplication. Recognition of division.
3	Construction of stories concerning multiplication. Construction of stories concerning division.

Piagetian Test

The set of five tasks taken from the works of Piaget and his colleagues had been given in a class test format to 542 children in order that the mathematical hierarchies could be matched to the levels of cognitive development in the Piagetian theory.

Using the marking scheme devised by CSMS(M), the number of children whose responses (describable as a 'level of thought') were subsequently assigned to the same cognitive level on each task was small. The child's level of response seemed to be too task specific to assign a child 'an overall stage'. The values of the γ coefficient between the five tasks were very much lower than those between the mathematics tests (see Appendix 6 for comparison). The γ coefficients are given in Table 6.13. Note that the γ s were calculated for pairs of items rather than groups (as in the CSMS(M) tests) which might account for the different values.

Table 6.13

Piaget Maths Tests Inter-item γ Values

Task					
Q1	sample				
Q2	314 .269				
Q3	420 .373	314 .370			
Q4	375 .447	358 .250	449 .451		
Q5	358 .474	327 .325	419 .394	368 .407	
542	Q1	Q2	Q3	Q4	Q5

Task

Due to the difficulties in establishing a reasonable link between the Piagetian tasks, the analysis of the results has not been completed. Further work is being done and a paper will be produced at some time in the future.

Performance by Year

Each test paper was given to children from several age groups so that a comparison of performance on the same items could be made.

The general picture that emerges when one looks at the performance of say 13 year olds and compares this with success rate of older children, is that there is an increase of five to ten per cent in the facility of items over two years. In some topics the pattern is slightly different. For example in Algebra there is a large gap between the performance of the 13 year olds and the 14 year age group; the facilities of items for the 14 and 15 year old samples are rather closer together. The 13 year olds may have had little experience of generalised arithmetic in their mathematics lessons and the distinction between their results and that of older children is probably due to this. The distinction between year groups in Measurement appears to relate to the ability to use a formula for area and volume problems. The progress year to year is still of the order of five to ten per cent however. In Graphs, the 14 year olds were not significantly better than the children of 13; the third year in school seemingly being a consolidation period for this topic. Similarly the distinction between the 15 year olds and younger groups in the topic of Positive and Negative Numbers was small, and on some items the younger children giving a considerably better performance. This may be due to a desire on the part of the older children to rationalise the rules of operation e.g. minus times a minus makes a plus, the simple acceptance of which was sufficient for the younger age group.

In Fractions the older children were always better at solving problems but the 12 year olds showed considerable competency in addition computations.

Longitudinal Survey

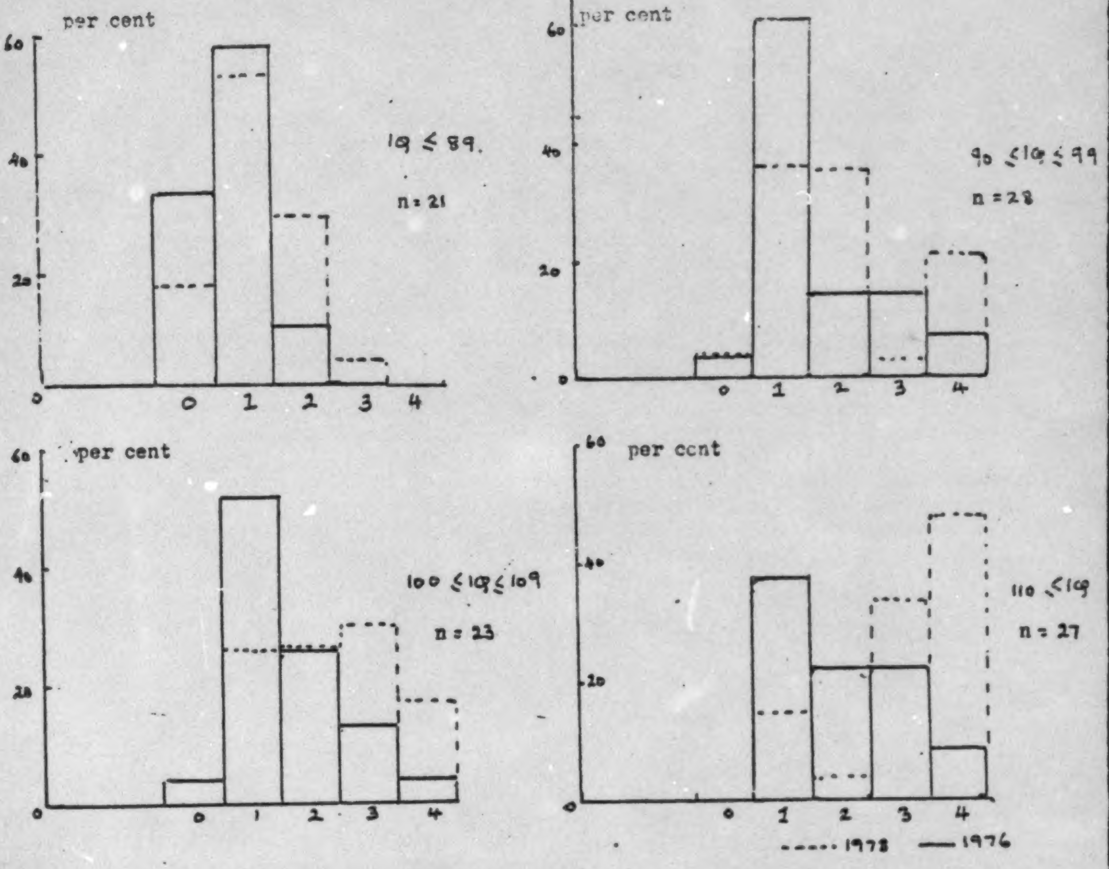
The picture presented so far is that of a comparison between sets of different children whose ages differ by one year. Further information is available from the longitudinal studies carried out over two years (three testings) in the topics of Algebra, Ratio and Graphs. Two hundred children were tested on each topic. The groups were from four schools with random selection from within four IQ ranges - IQ $\leq$ 89, 90 $\leq$ IQ $\leq$ 99, 100 $\leq$ IQ $\leq$ 109, IQ $\geq$ 110. By the summer of 1978 the number of children who had been tested three times had dropped to about 100 in each topic (absence, change of school or class being responsible for the drop out). It is possibly worth noting that there were many more absences within the lowest IQ band than within the other three.

The results of the three testings in each topic are described below. It should be borne in mind that the research team had no information on the child's performance in the classroom, on other areas of mathematics nor indeed on any aspect of learning except performance on the tests. The word 'progress' is used in this context to mean the attainment of the 2/3 pass mark on a level higher than that previously passed.

Ratio

The performance on the Ratio test appeared to be closely related to the IQ of the child; this was apparent from the start when no child if IQ less than 89 (standardised score) achieved levels 3 or 4 and only two were at level 2. In the group with IQ greater than 110, no 13 year old was at level 0 although many were at level 1. Figure 6.1 shows the percentage of children at each level in 1976 and 1978.

Figure 6.1



Change in Performance Levels, 1976-1978, in Four IQ Groups (Ratio)

Figure 6.1 indicates there was progress in each IQ group but for the lowest IQ group it did not go beyond level 3 while for those with the highest IQ the number who were at level 4 quadrupled in two years. Note however that not all of those with IQ greater than 110 had achieved the 2/3 pass mark in level 4 by the age of 15. In the IQ range 100 ≤ IQ ≤ 109 the children were dispersed equally between levels 1, 2, 3 with 17 per cent at level 4.

The pattern of movement level to level for individual children over the two years is shown in Table 6.14.

It should be noted that some children regressed at the second testing (1977) and then returned to or progressed beyond the position they held previously. Others were 'non scale' types (attained the pass mark at a higher level without achieving all lower levels). For example, in the IQ range 90 ≤ IQ ≤ 99 nine children apparently regressed to a lower level at either the second or third testing. Five of these however were 'non scale' types at some point in the testing. Three improved their marks on the easier groups of items or on the set immediately after that which they now failed (having succeeded on it earlier). One regressed in the second year and then improved in the third year and one regressed in the second year and stayed at that level.

Table 6.14

Movement Level / Level in Ratio (1976-1978)

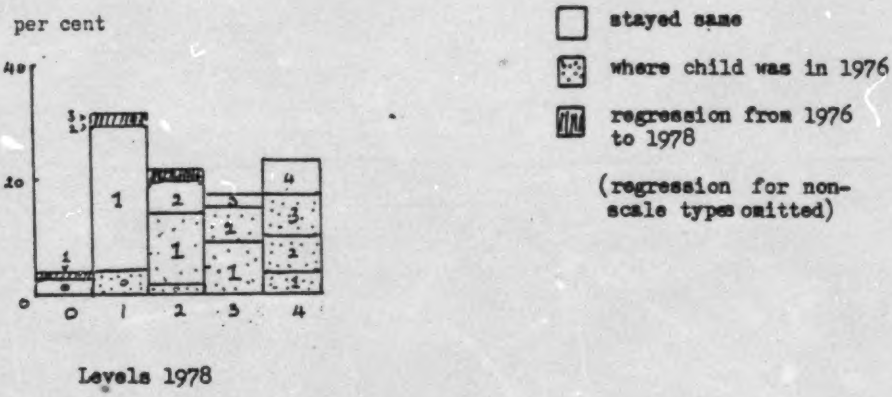
Moved	Total	IQ ≤ 89	90 ≤ IQ ≤ 99	100 ≤ IQ ≤ 109	IQ ≥ 110
		n = 21	n = 28	n = 23	n = 27
0 levels	41	12	13 (1 at L4)	6	10 (4 at L4)
1 level	26	5	6 (1 to L4)	9	6
2 levels	14	2	4 (3 to L4)	2	6
3 levels	2	0	0	1	1
Regressed	6	1	3	2	0

Notes: 1) A regression was a failure in 1978 to achieve the pass mark of 2/3 on a group of items which had been passed in 1976.

2) Ten 'non-scale' types (1978) were omitted.

Most of the fluctuation in levels occurred at the second testing rather than the third. Figure 6.2 shows the percentage of children at each level after two years and where those same children were in 1976 when they were aged 13.

Figure 6.2



Improvements and Regressions (Ratio Test, n = 99)

The progress was slow and many children reached only level 2 at age 15. This implies that the children could on the whole cope with doubling and halving and items in which they could build up to an answer by repeating an amount, but could not deal with items of greater complexity.

Graphs

The number of children with IQ less than 89 who did the Graphs test in 1976 had by 1978 been reduced by absence on one or more testings from 13 to 2. These two children were included with the next highest IQ group and the changes in performance, year by year are given in Figure 6.3.

The increase in the number of children who reached level 3 (the highest level) was most marked for the two higher IQ groups. The facilities for level 3 items for the general 13 year old sample were all between 20 and 7 per cent, for the 15 year olds the same items had facilities between 37 and 14 per cent. There was therefore an obvious improvement in attainment over the two years. Recall that the general sample was chosen to represent the normal distribution of IQ and the longitudinal sample described here was selected to represent IQ groups equally. The longitudinal group with IQ less than 99 showed some improvement but less than 20 per cent reached level 3. Two thirds of those at level 2 scored less than 3/11 on level 3 items. Of those children who made no apparent progress level to level over the two years, 11 stayed at level 2. On the other hand in 1978, 30 per cent of the group, 100 ≤ IQ ≤ 109, obtained full marks on the set of items in level 3 while half of those in the top IQ group did so. The movement of individual children is shown in Table 6.15.

Table 6.15

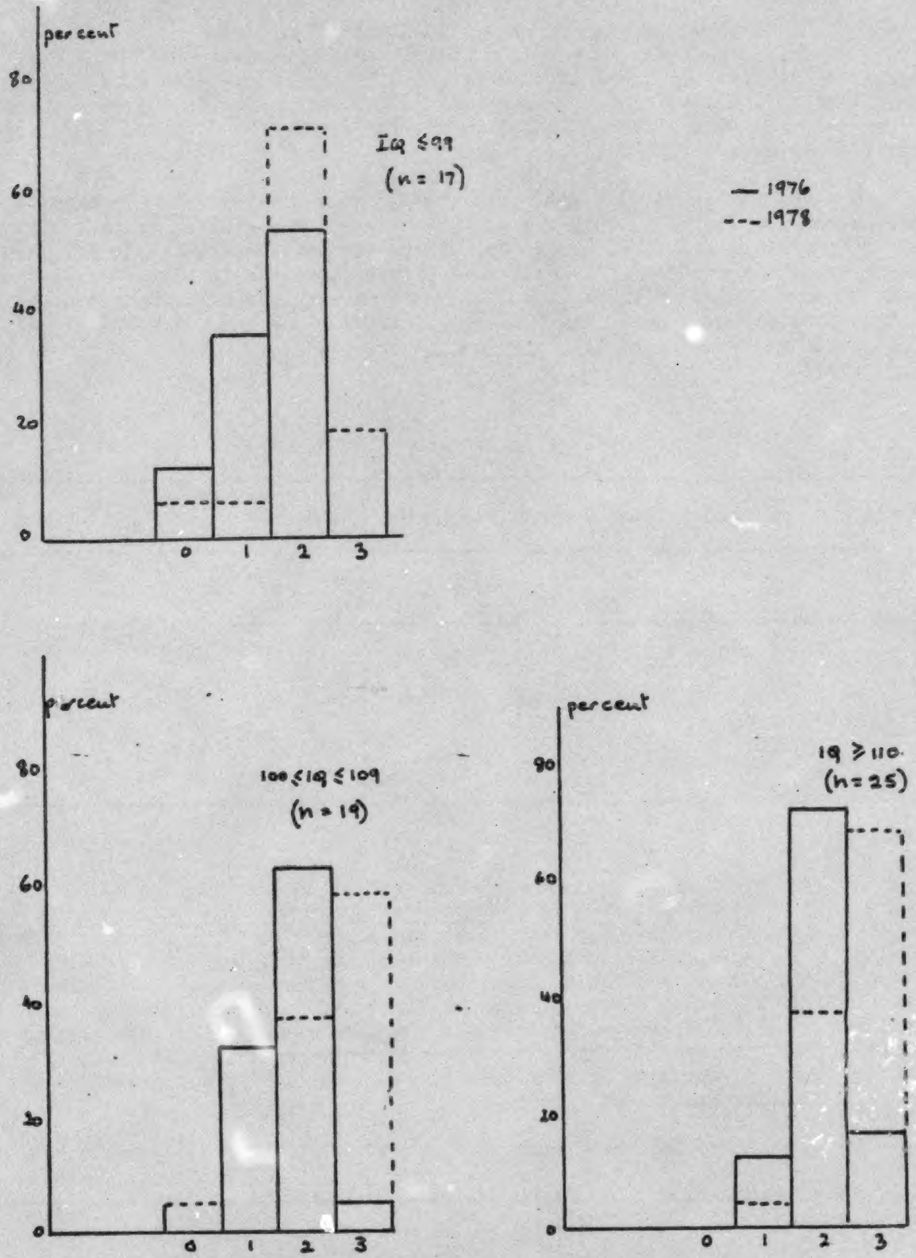
Movement Level / Level in Graphs (1976-1978)

Moved	Total	IQ ≤ 99	100 ≤ IQ ≤ 109	IQ ≥ 110
		n = 17	n = 19	n = 25
0 levels	22	8	4 (1 at L3)	10 (4 at L3)
1 level	36	8 (3 to L3)	13 (9 to L3)	15 (13 to L3)
2 levels	1	1	0	0
Regressed	1	0	1	0

Note: One 'non-scale' type (1978) was omitted.

Note that with only three levels in the Graphs hierarchy not a great deal of movement was possible. More children reached level 3 than might have been expected from the picture obtained from the wide scale testing.

Figure 6.3



Change in Performance Levels, 1976-1978, in Three IQ Groups (Graphs)

Algebra

The analysis of the test on generalised arithmetic resulted in four levels of understanding. There was a distinct difference in attainment (by the age of 15) between those with an IQ below 100 and those with IQ above 100. There were no children of IQ greater than 100 who were still at level 1 whereas 40 per cent and 17 per cent of those in the IQ ranges - $IQ \leq 89$ and $90 \leq IQ \leq 99$ respectively were still achieving only at this level, see Figure 6.4.

The progress to level 4 where the first true use of variables occurs, was most marked for the highest IQ group, this level of abstraction being beyond the attainment of the lowest IQ group. There was some progress in every group however over the two years, although the pattern level to level differred with each IQ range. Nine children (out of 105) moved two levels in two years, five of these had an IQ of more than 110. The details of movement for individual children is shown in Table 6.16.

Table 6.15

Movement Level / Level in Algebra

Moved	Total	$IQ \leq 89$	$90 \leq IQ \leq 99$	$100 \leq IQ \leq 109$	$IQ \geq 110$
		n = 23	n = 22	n = 28	n = 32
0 levels	42	9	14	13	6
1 level	51	10	7 (1 to L4)	14	20 (6 to L4)
2 levels	9	2	1	1 (to L4)	5 (4 to L4)
Regressed	3	2	0	0	1

Summary

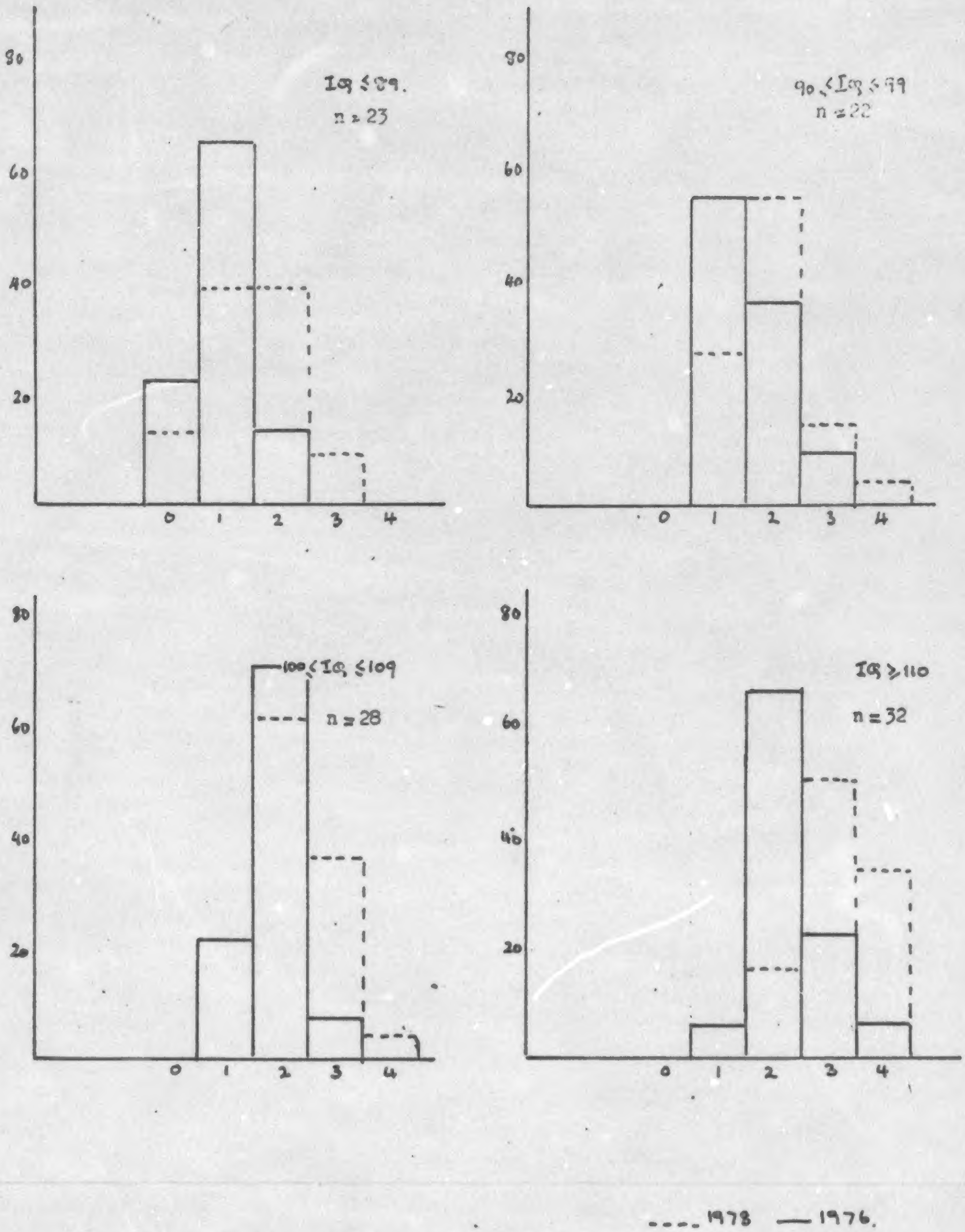
The results of the longitudinal study confirm the description of a slow progression of attainment year to year obtained from the topic hierarchies (based on data from wide-scale testing). The percentage of children at each level in a hierarchy was calculated on the attainment of the 2/3 pass mark and no allowance was made for a lesser mark in a higher group, so children may have achieved a greater degree of success than was apparent. In some areas the large facility gap between two levels may have accounted for the lack of movement from one level to the next. However, in general the lack of progress suggests the need for considerable consolidation and additional experience before a child can make such a gain.

Matching the Hierarchies with Each Other

After establishing a hierarchy for each topic an attempt was made to match the hierarchies with each other.

An exact match between topics was not possible for two reasons: 1) the number of levels in each topic hierarchy varies (from four to seven levels except for Graphs and Number Operations which have only three), and 2) the facility range of items within a level varies from topic to topic ; for example when attempting to match other topic hierarchies against that of Graphs we find

Figure 6.4



Change in Performance Levels, 1976-1978, in Four IQ Groups (Algebra) that two levels of Positive and Negative Numbers match Graphs level 1 and no Ratio level corresponds to Graphs level 2 (see Appendix 7). One reason for trying to match the hierarchies was to provide the teacher with subsections of the secondary mathematics syllabus made up of different topics, pieces of which seemed to correspond in the demands they make on children's understanding. The subsections would range from mathematical ideas which were relatively easy for most children to some which were quite difficult, i.e. successfully dealt with by a small number of children in the CSMS(M) research. A complete map would be invalid if there was no evidence to show that the topics were interrelated and that success on one topic was highly correlated with success on other topics.

The information available to support the formation of a total map (cross matching) of hierarchies was:

1) The facilities of the items on each topic paper for each year sample (albeit samples of different children for each topic).
2) The performance of the same children on two different papers.
3) The mathematical content of the items and the knowledge (from the interviews) of how children solved these items.

In the sections that follow, the items which appear in Appendix 7 will be described in greater detail (to show their content) and the difficulty (facility) level for each will be taken from the third year sample (age 14). Then the results obtained from the testing of subsamples of children on two test papers will be shown and an attempt made to match levels in different topics, on the basis of this information. The difficulty of dividing the total collection of hierarchies into seven or eight subsections is described in terms of the inconsistencies which are apparent when one looks at the percentage of children at each level in each hierarchy. There was a reasonably high correlation between success on one hierarchy and another, as indicated by the fact that the gamma coefficients obtained from comparing the performance of the same children on two papers were high. This crossmatching of levels is followed by a discussion of the four general subsections, referred to as general stages, obtained from all the hierarchies. Each of the four stages is described in terms of levels within topics and the mathematics required.

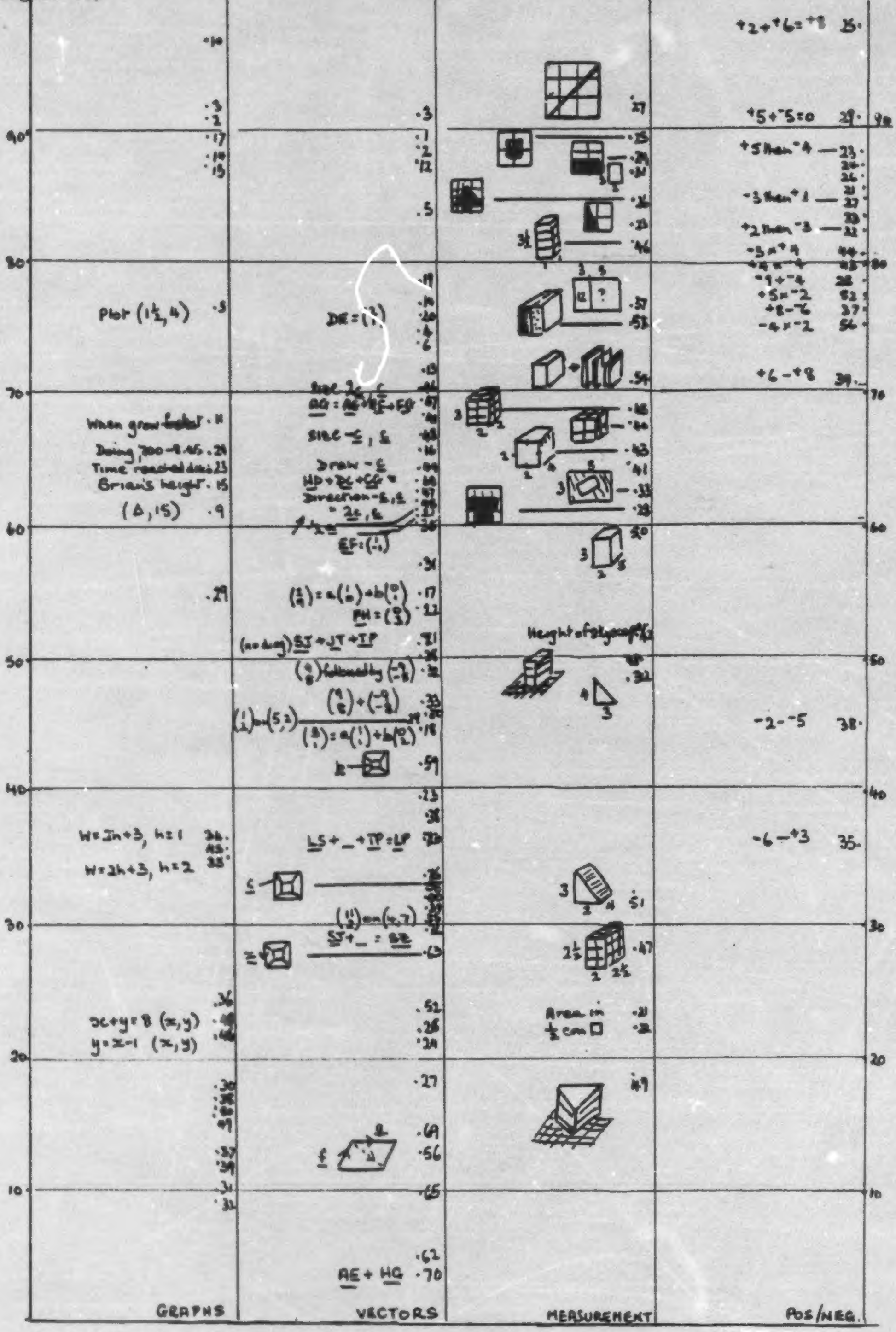
Third Year Sample

The items on each test which satisfy the grouping criteria given above and which thus formed part of the topic hierarchy are shown in Figure 6.5. (The Figure includes a brief description of the items which were included in the groupings and the actual test papers are in Appendix 1.) The Figure shows the items at each facility level for the sample of 14 year-olds (3rd year subjects).

Subsamples: Children Taking Two Tests

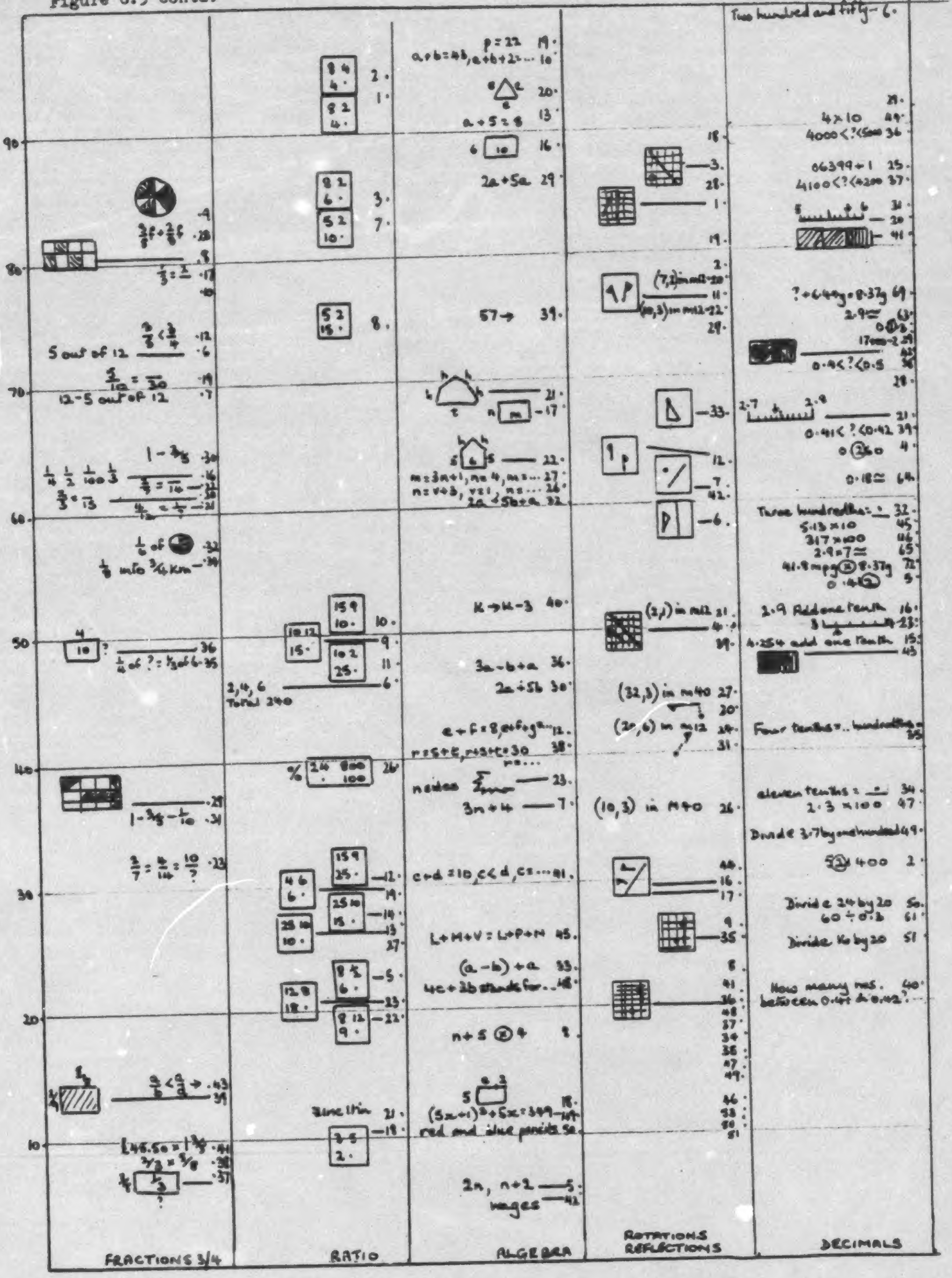
During the research the intention had been that each child should take two test papers. This was not always possible because of absence from school of a particular child or because the school could not find the time necessary for two sessions of testing. In the 1976 testing a large number of children took two test papers (of the four papers included in this testing). In the 1977 testing when 13 test papers were available the problem was that of obtaining a number of children who were asked to complete a particular combination of papers. To match every test paper with every other would have required an extremely large sample (there would have been 78 combinations) particularly if each pair of tests was to be completed by children from

Figure 6.5



General Sample - Third Year Facilities

Figure 6.5 contd.



different age groups and from different schools. In order to obtain a wide range of children taking a pair of papers, not all pairs were included; the total number of combinations was given in Table 4.7. The facilities of the items for each of these subsamples is given in Figure 6.6. In some cases the subsamples were small (n = 50). In addition when the combination of papers was suitable for one age group only (e.g. Ratio and Fractions 1 and 2) only this group was tested.

The percentage of children in each subsample at each level of a hierarchy is shown in Figure 6.7. The height of the block (labelled 0, 1, 2, 3, 4 etc.) in each column provides the percentage of children at each of those levels. For example in Figure 6.7 Ratio/Graphs, approximately 46 per cent of the subsample were at level 1.

Matching Subsample Marginal Distributions

The marginal distributions in Figure 6.7 indicate the percentage of children attaining a 2/3 pass mark on a particular group of items and all easier groups. Each subsample which took the Ratio paper has a slightly different distribution of levels but those for Ratio/Graphs, Ratio/Algebra, Ratio/Decimals, are approximately the same and all come from the same age group (13, 14, 15 year olds). Starting with the distribution for Ratio and then linking this up in turn with the corresponding Graphs, Algebra, Decimals distributions as shown in Figure 6.8, we find that the resulting Decimals/Graphs pattern is very similar to that obtained from the appropriate subsample which took Decimals and Graphs. Extending the figure, each time choosing a pair of tests for which the distribution of levels has already been plotted, we obtain a matching for six of the tests (recall that the test papers in Fractions for the 12-13 year olds was different from that for the 14-15 year olds). Based on the distribution of levels, five possible subsections of the match between six hierarchies were suggested - these are given in Figure 6.8 indicated by arrows (←).

Total Sample

The same type of comparison i.e. a scrutiny of the distribution of levels in each hierarchy, was made using the results of testing the large sample in each of four age groups. Figures 6.9-6.12 show the percentage of children at each level in each topic tested. The percentages are given as points on a scale and are cumulative since very nearly all children who 'passed' a harder level had also succeeded on all easier levels. For example in Figure 6.9, 91 per cent of the Fraction sample were at level 1 or higher, 67 per cent at level 2 or higher etc. By comparing the position of the figures for one year group e.g. Ratio 2 is below Graphs 2 and above Fractions 3 in the 3rd year, with the position for another year group, certain inconsistencies show up. In some cases this shows there was marked improvement in performance of one year group, i.e. the 3rd year sample had a much improved performance in Algebra, over the 2nd year. In other cases the inconsistencies lend weight to the opinion that attempting to impose more than rough divisions or stages on the total picture would add a refinement unsupported by exact matching between levels; for example if eight stages were imposed the contents for each would differ year to year.

Figure 6.6

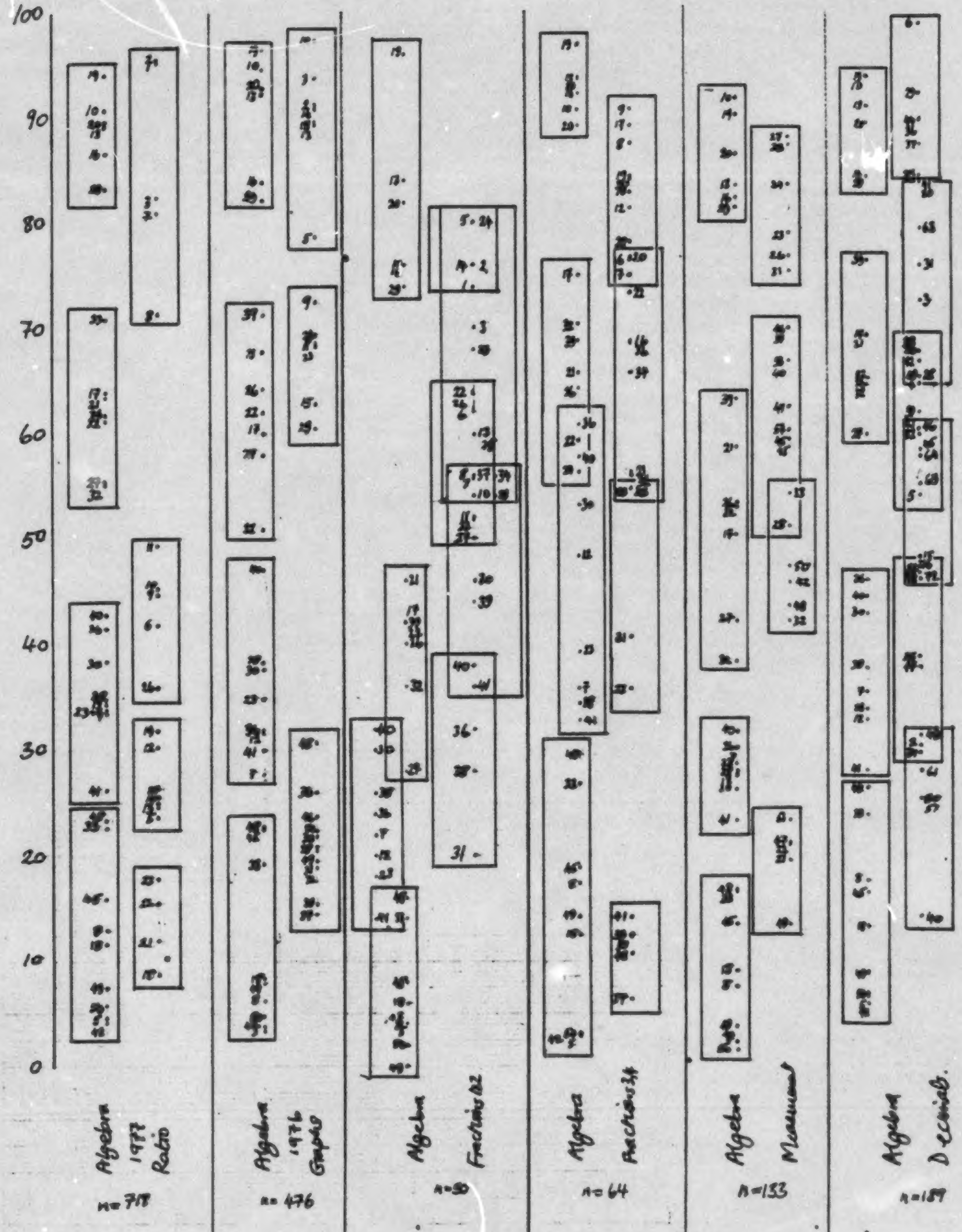

100
90
80
70
60
50
40
30
20
10
0
Algebra
1977 Ratio
n=718
Algebra
1976 Graphs
n=476
Algebra
Fractions 22
n=50
Algebra
Fractions 24
n=64
Algebra
Measurement
n=153
Algebra
Decimals
n=189

Figure 6.6 continued

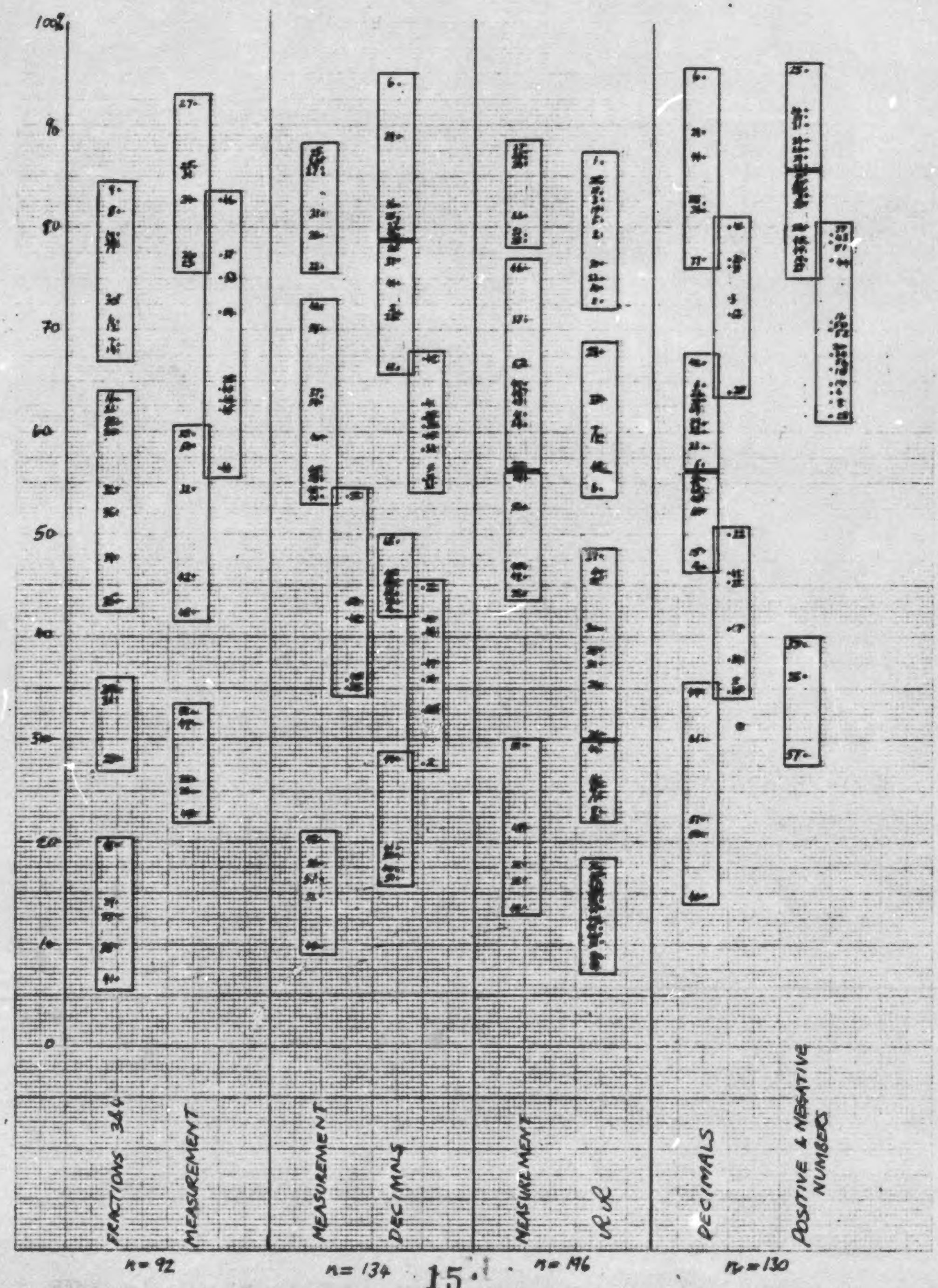


Figure 6.6 continued

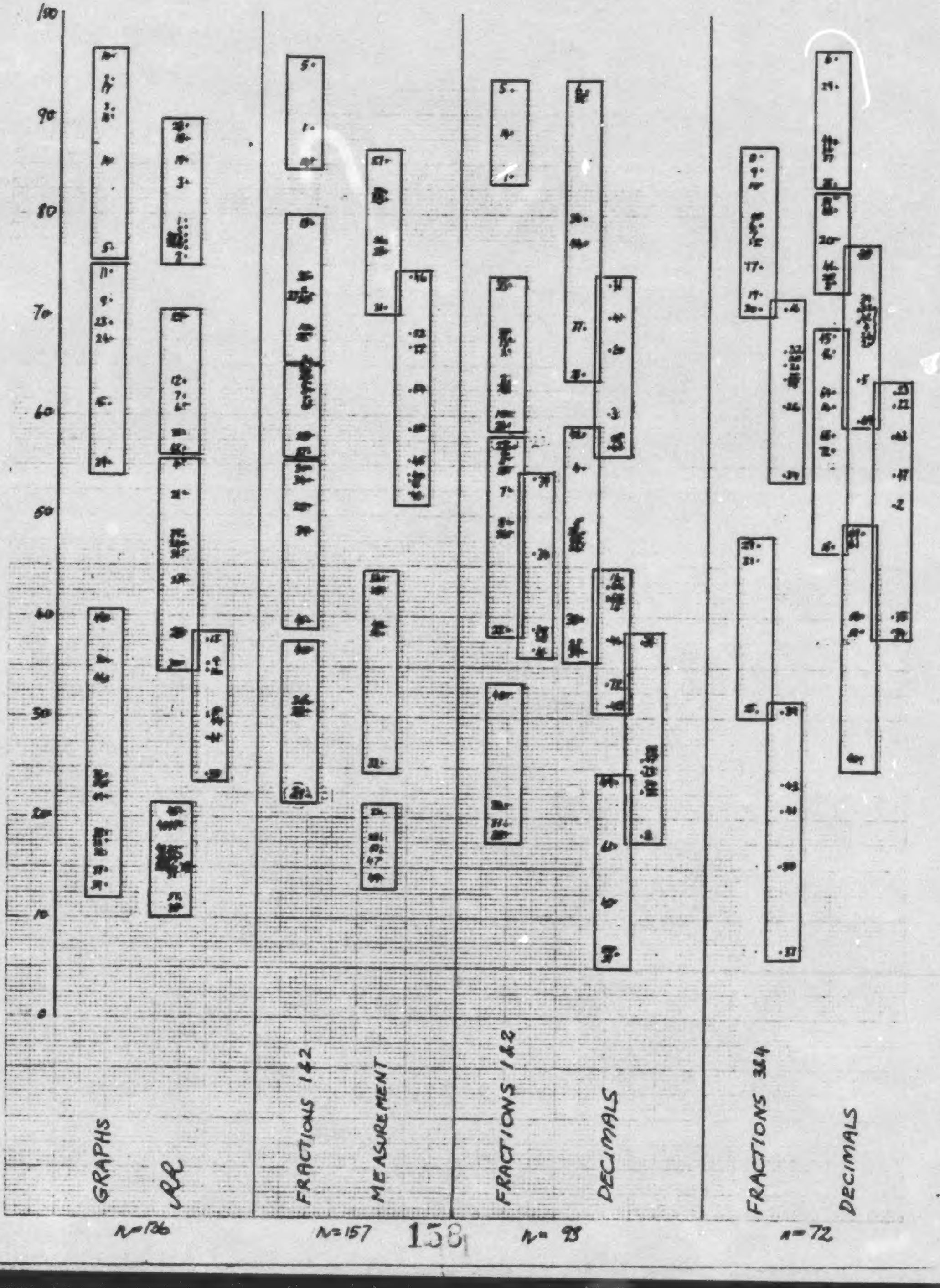


Figure 6.6 continued

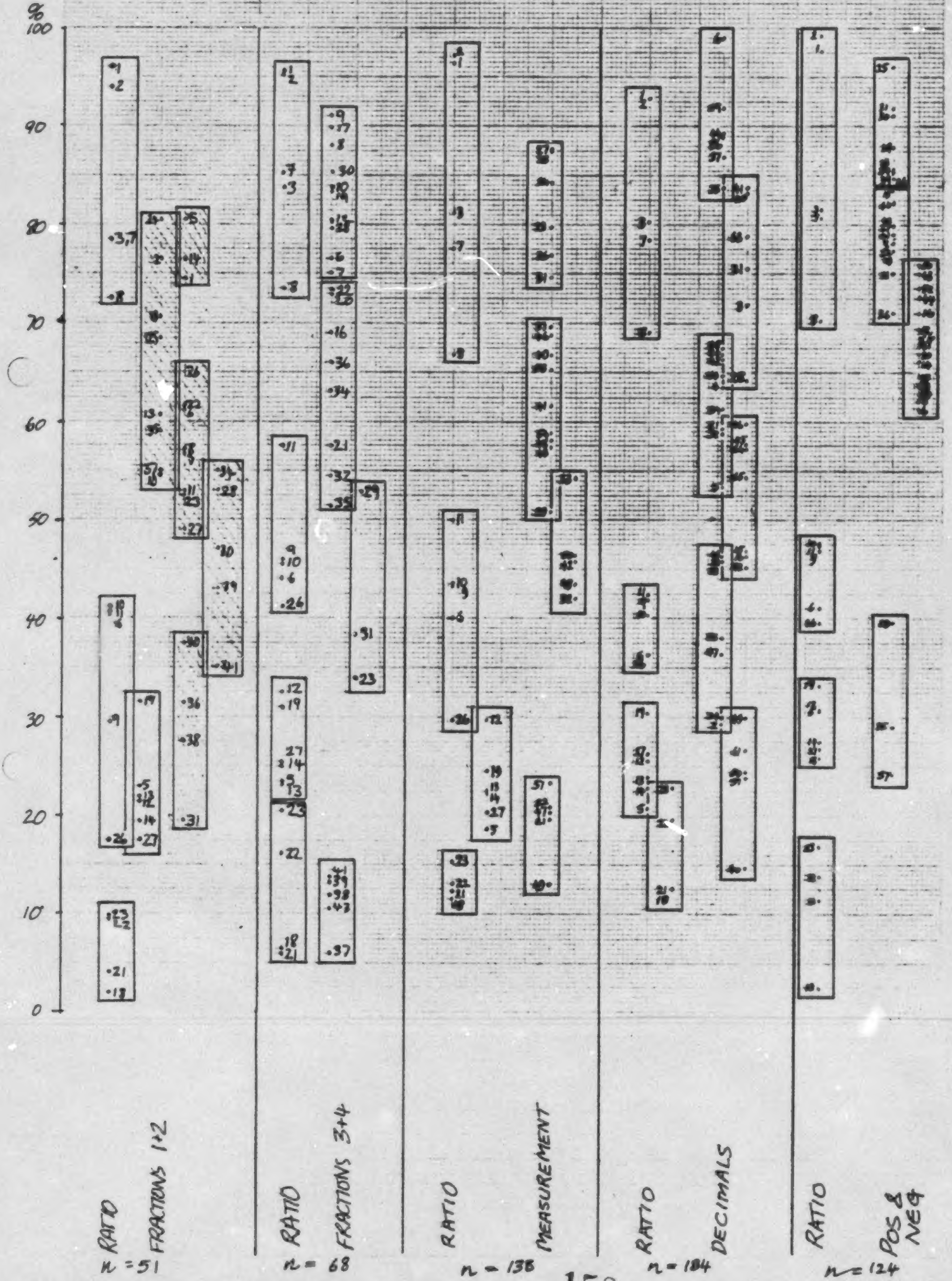


Figure 6.6 continued

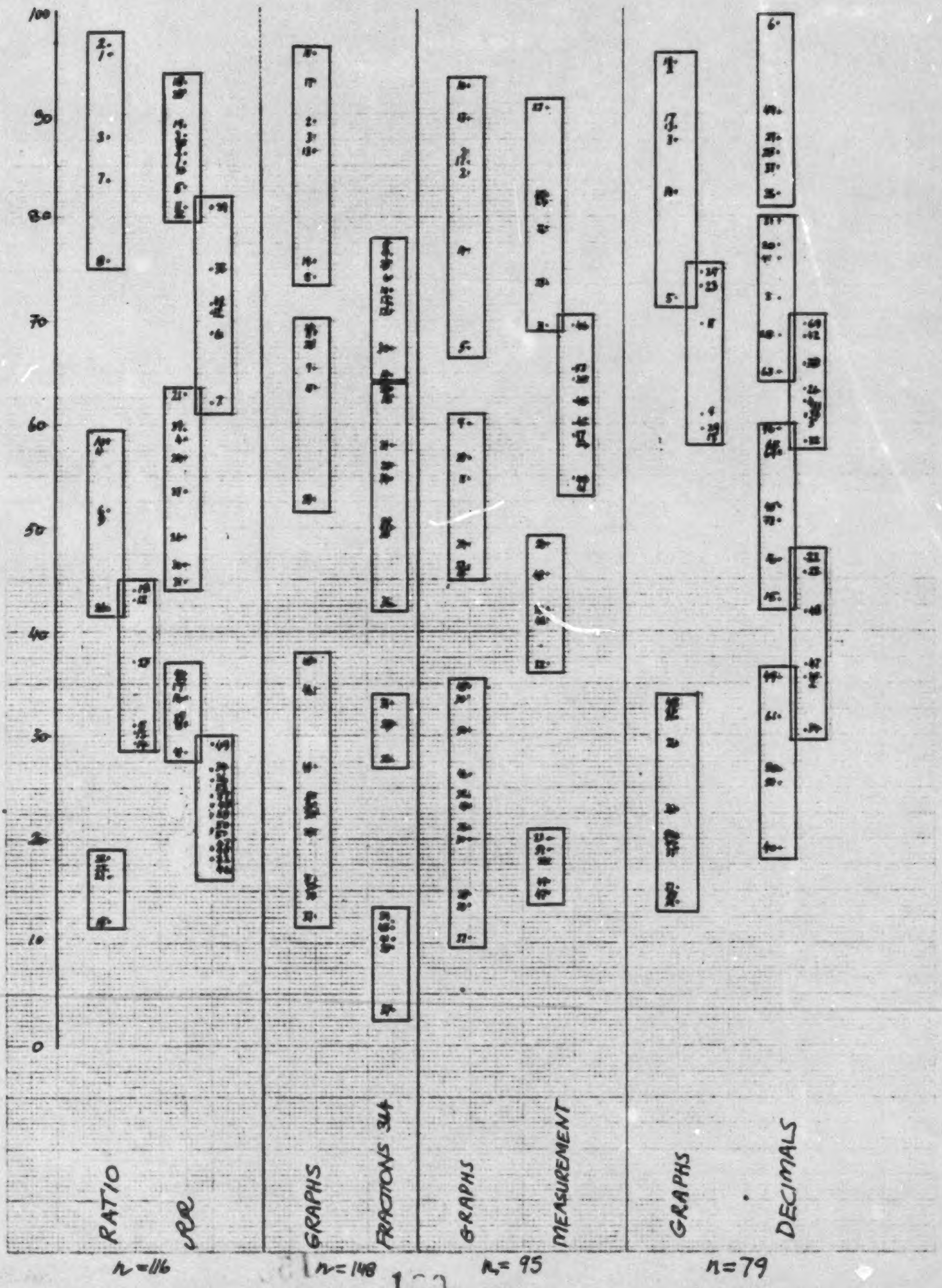


Figure 6.6 continued

ITEM FACILITIES IN SUBSAMPLES

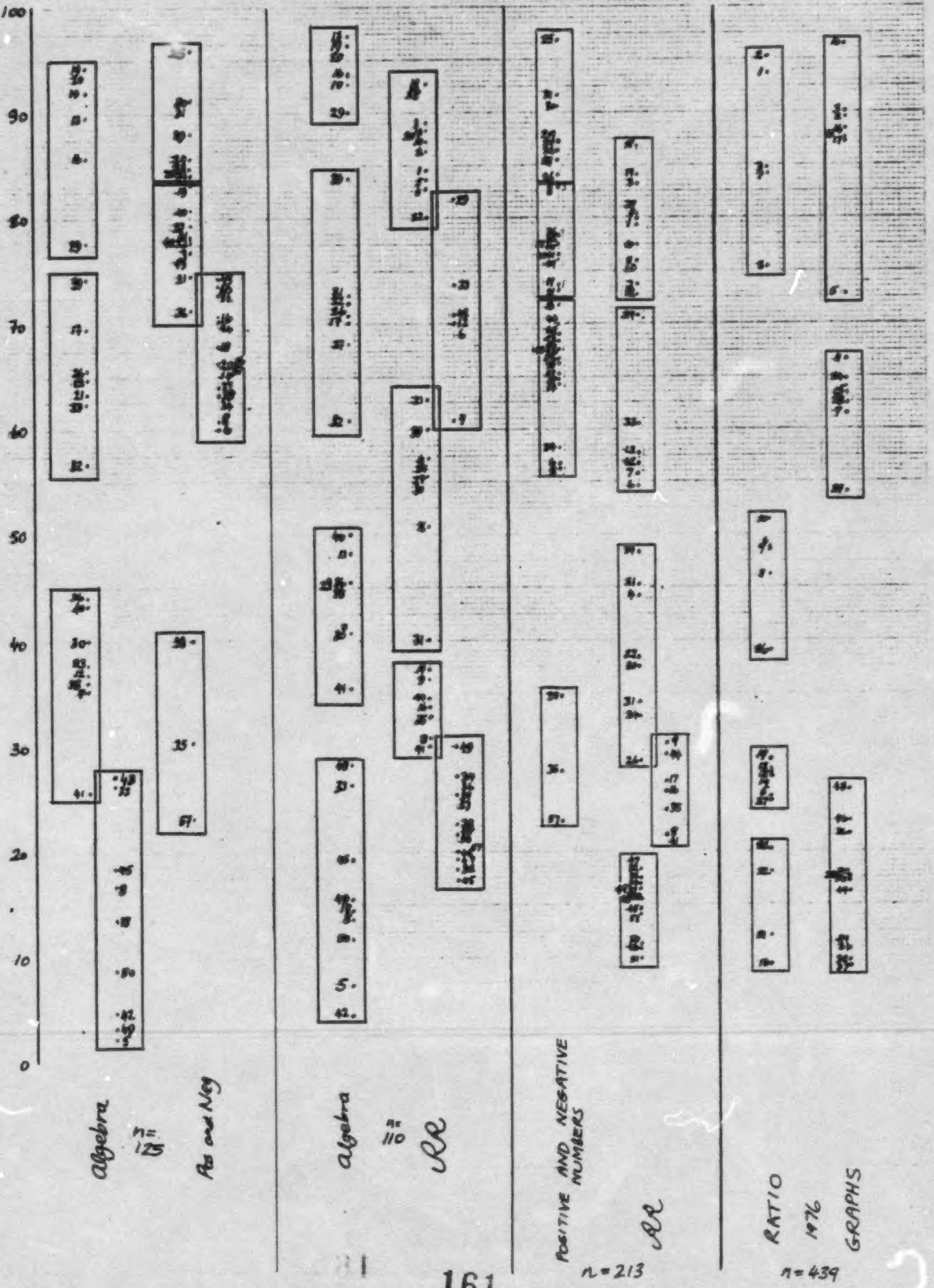
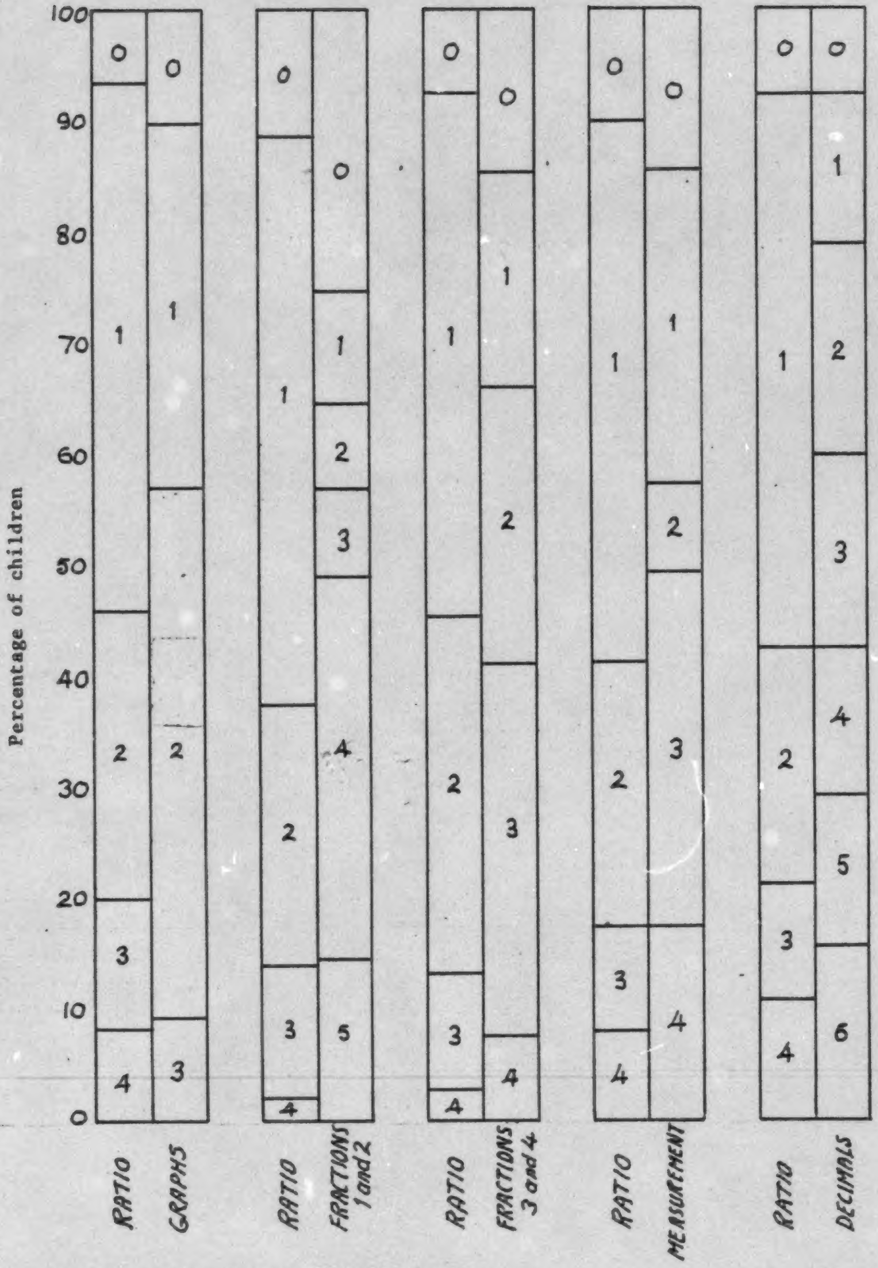


Figure 6.7



Marginal distribution in subsamples

Figure 6.7 continued

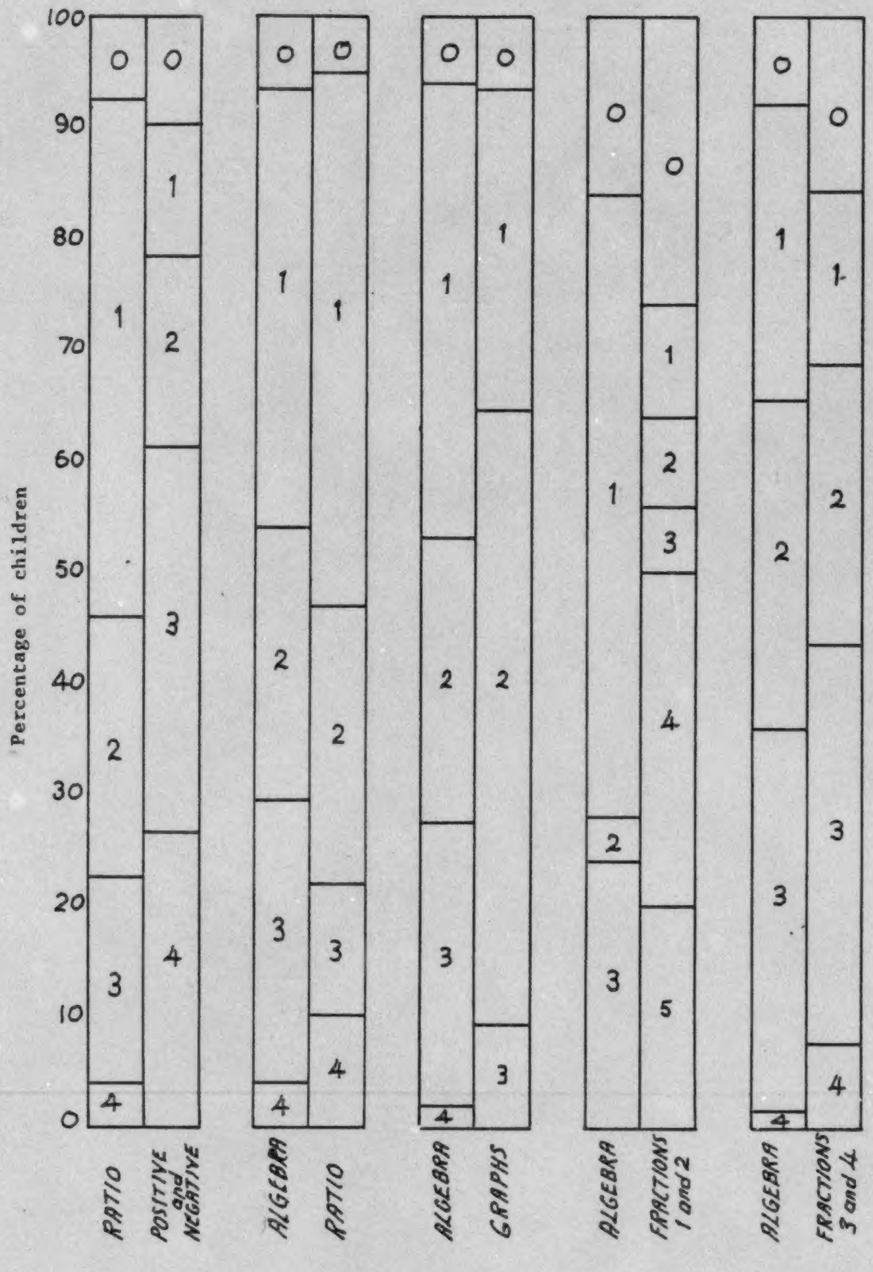
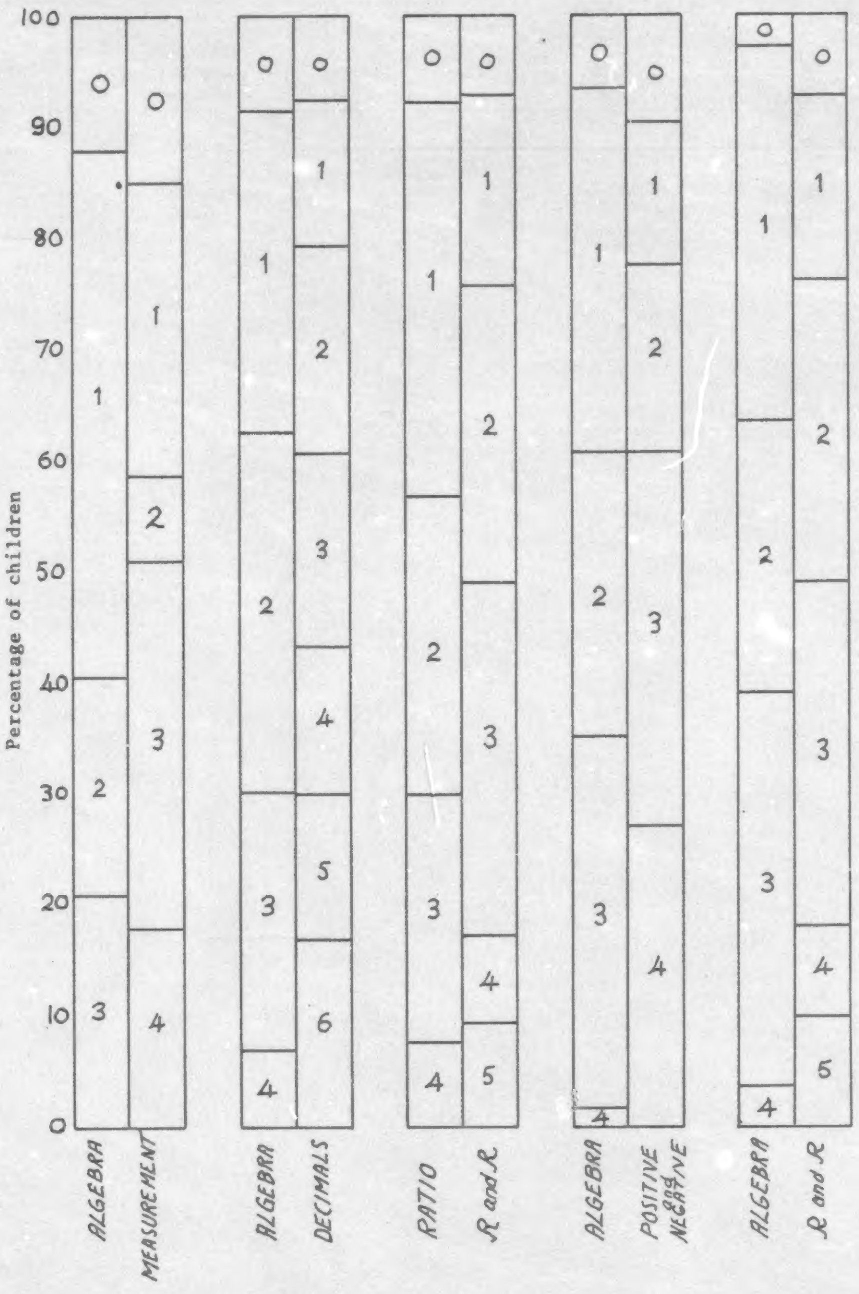
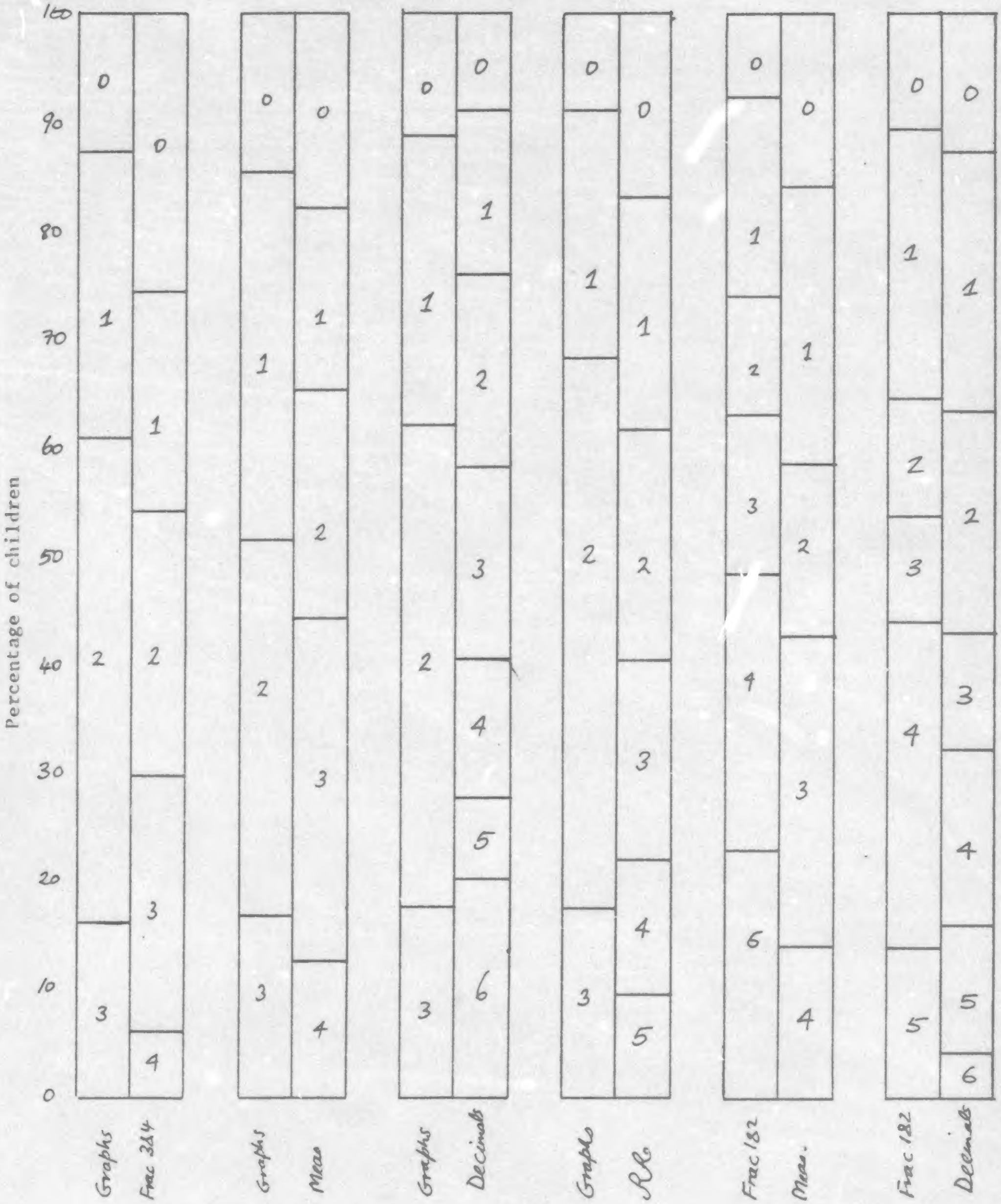


Figure 6.7 continued



10.10

Figure 6.7 continued



10.12

Figure 6.7 continued

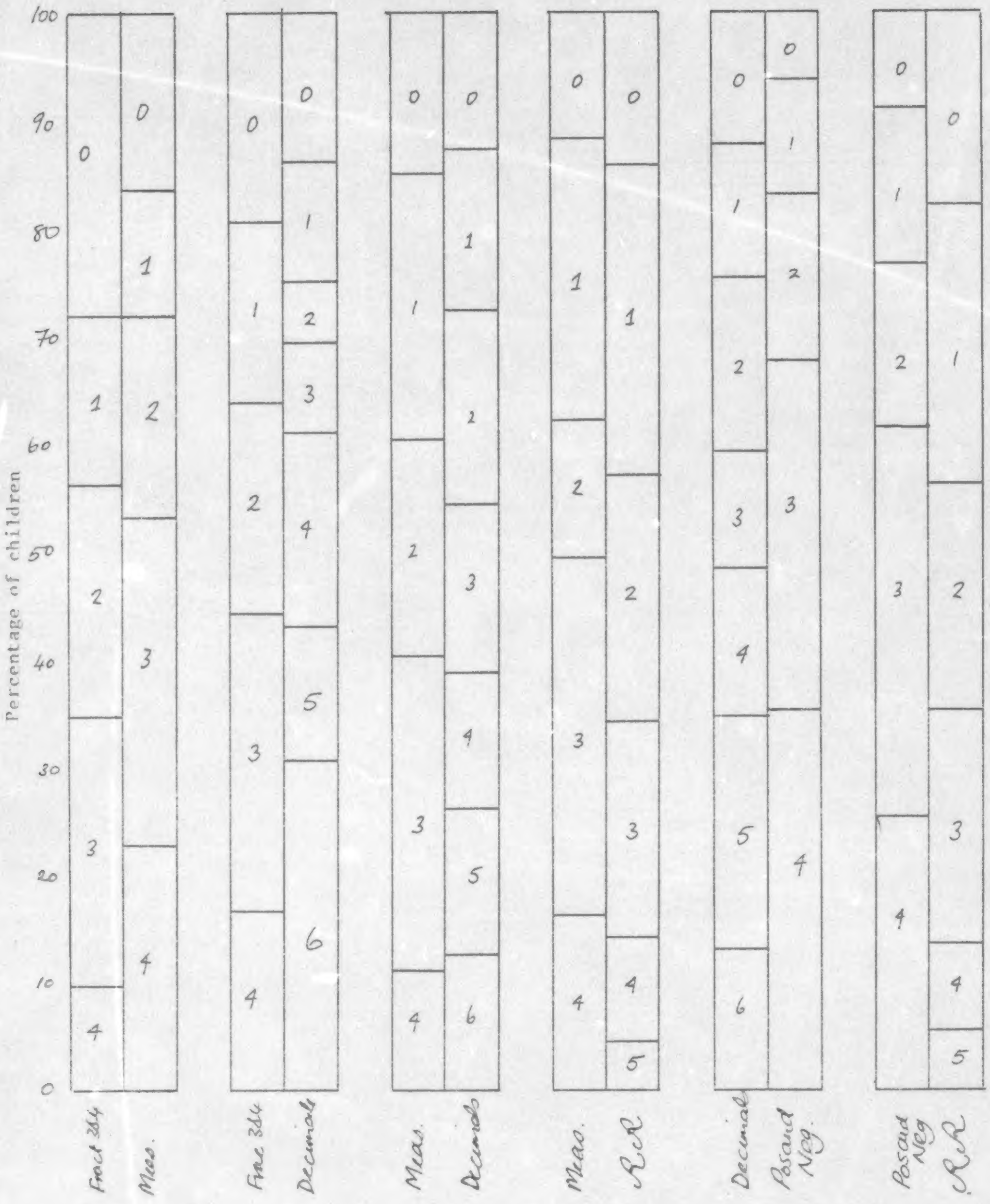
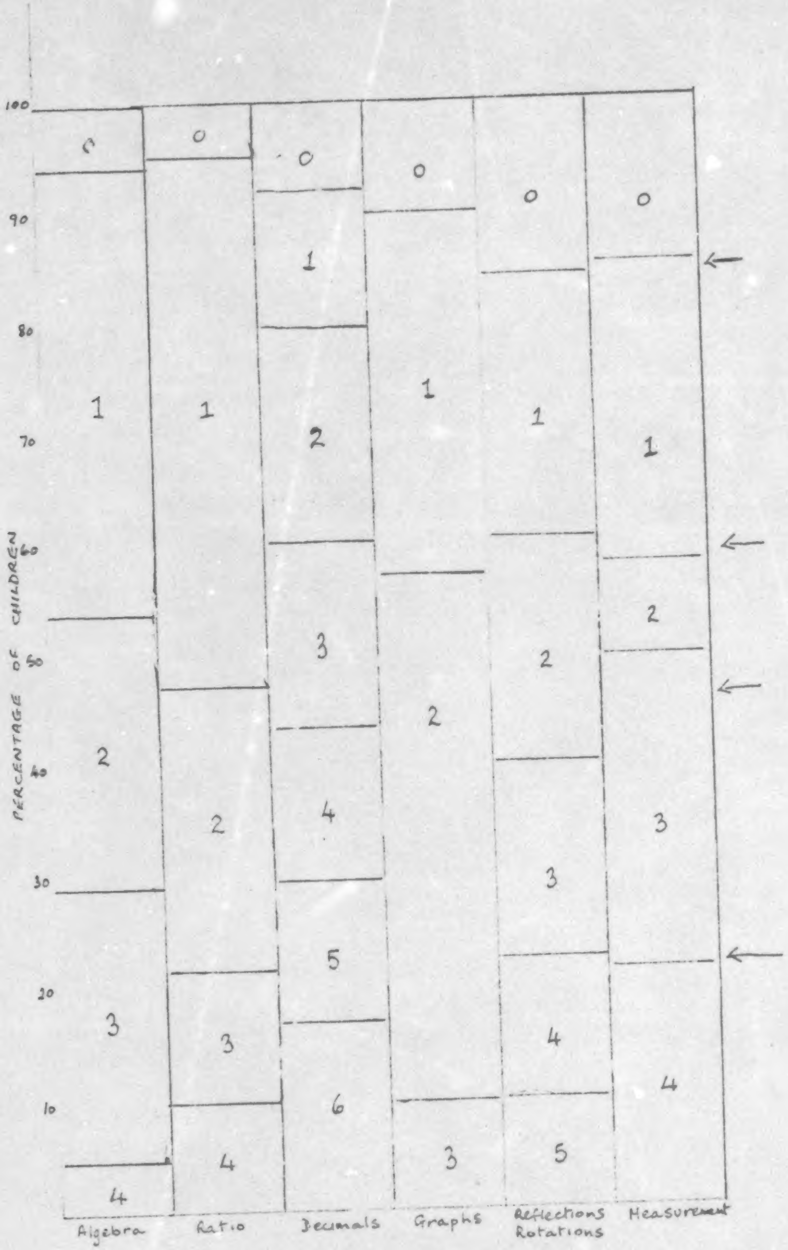


Figure 6.8



Matching Marginal Distributions of Subsamples

Figure 6.10

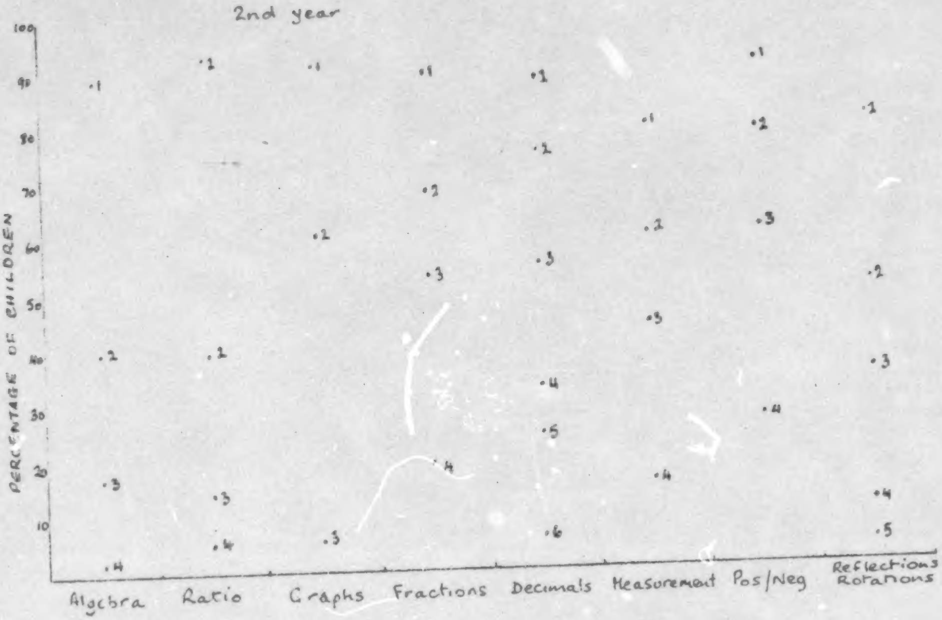


Figure 6.9

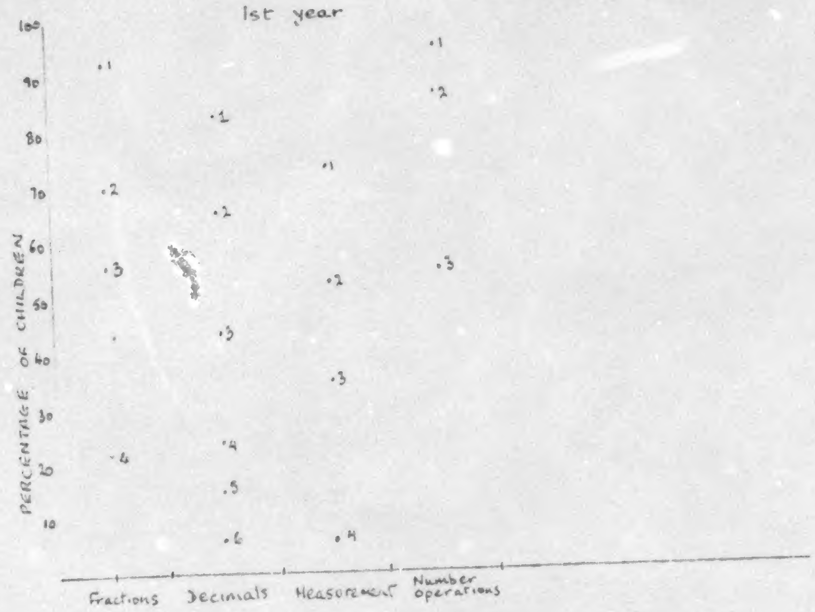


Figure 6.11

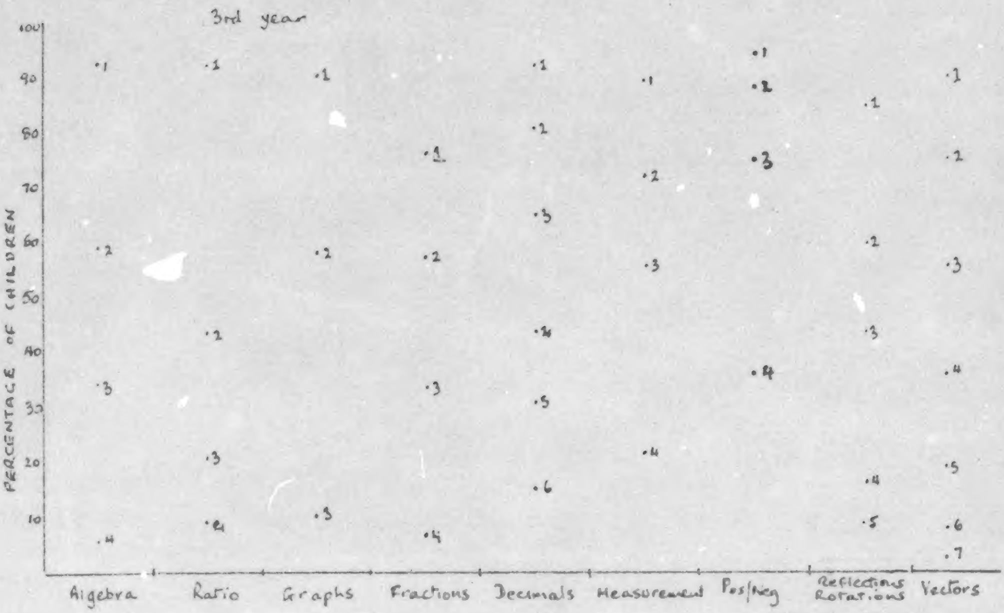
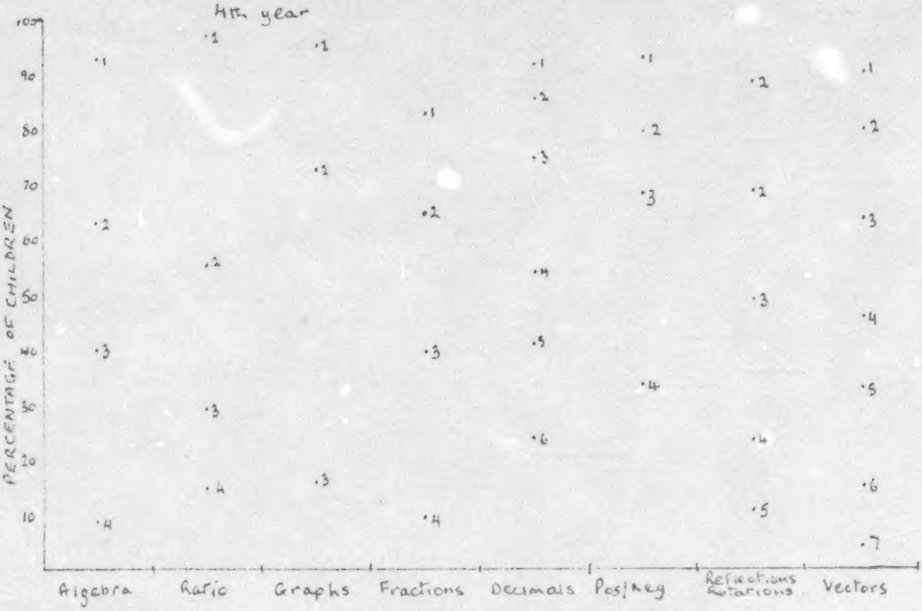


Figure 6.12



Matrices A and M have been omitted. Those items which were common to the two papers formed a common core for four levels . The joint sample performed as follows : 14yr (Levels - percent) 0-7.5, 1-10.0, 2-14.5, 3-16.0, 4-52.0 : 15yr. 0-5.5, 1-2.5, 2-14.5, 3-15.0, 4-68.0

Correlations Between Topic Hierarchies

To show that the crossmatching of topic hierarchies was valid, the subsamples of children who took two tests were subjected to further scrutiny. Besides matching facilities of items and finding an approximate correspondence between the proportions of children achieving each level of the two hierarchies, a cross-tabulation between the two topics was made. Table 6.17 illustrates this procedure for the subsample who completed Ratio and Graphs.

Table 6.17

A Comparison Between Performance on Graphs and Ratio (Age 13, 14, 15)

		Graphs Levels				
		0	1	2	3	ROW TOTAL
Ratio Levels	0	13	13	2	0	28
	1	29	105	73	2	209
	2	3	21	78	12	114
	3	0	4	37	11	52
	4	0	0	20	16	36
COLUMN TOTAL		45	143	210	41	439

Cell entries are numbers of children

A statistical analysis of the correspondence between achievement on any two tests was carried out using the γ coefficient which quantifies the probability that if a child A is better than child B on one test he is also better than child B on another test. The γ coefficients obtained varied from .85 to .59 (maximum $\gamma = 1$) and are listed in Appendix 6. These values of γ calculated on the performance of the same children on two tests support the hypothesis that there is no gross mismatch between achievement in any two mathematical areas.

In order to provide a useful guide to teachers who wished to compare levels of difficulty across topics, an approximate matching between topics was attempted by subdividing the total set of hierarchies. A teacher who has dealt with level 1 type problems in Ratio may not wish to proceed to level 2 in Ratio (particularly as there is a large facility gap between the two). An alternative might be to start teaching some ideas in Algebra and in this event it would be useful to be able to select material (problems) which are in some way comparable in terms of demand.

Figure 6.5 shows the content of each item which was included in the cross matched hierarchies (in condensed form). Considering these items and the information obtained from the attempts at cross matching the research team made the decision to work with only four 'stages' or subdivisions on the total set of hierarchies. (Note that we are now using stages to refer to subdivisions of a cross matching of levels of hierarchies.) The contents of levels or items in each stage would be the same for each year group although the position of the line drawn to delineate each stage would be at a different facility since the items themselves were solved by different percentages of children in each age group. Figures 6.13 to 6.16 indicate the levels contained in each stage (facilities are for the 3rd year sample). Table 6.18 gives the facility range for each stage for each of the age groups tested. All the items (variable numbers) which appear in each stage are given in Appendix 7.

Table 6.18

Limits of Four Stages
Hardest Items

	13 yrs.	14 yrs.	15 yrs.	
Stage 1	70	70	80	per cent
Stage 2	48	50	58	" "
Stage 3	18	22	30	" "
Stage 4	2	5	8	" "

Description of the Stages

In order to provide generalised descriptions of the contents of each stage, methods used by children to solve the items were considered, as well as the mathematical demand of the items. The four stages are described in the next sections. While the descriptions are very general, they do illustrate the considerable gap that exists between children as far as their knowledge, application and flexibility in using mathematics is concerned.

Stage 1

This includes the easiest levels of most of the topics. There were some children, probably about ten per cent, who could not achieve a 2/3 criterion pass mark on Stage 1 items (see number of children labelled level 0 in each of the topic hierarchies). It is not known what areas of knowledge these children posessed. Each of the tests assumed that numbers up to 20 were known, numbers up to 1,000 recognised and that the child had the ability to repeat a unit of measure in order to find the length or area of an object, all of which are topics covered in the Junior school curriculum.

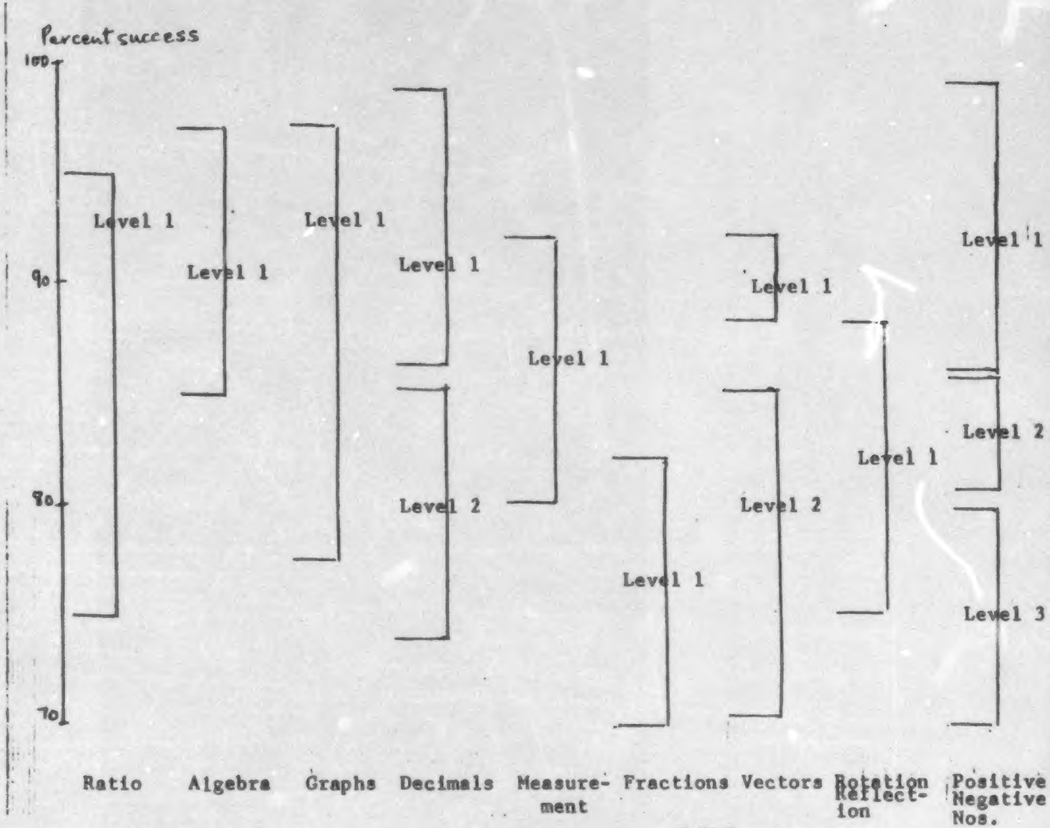
The conceptual demand of the Stage 1 items is greater than a knowledge of just whole numbers. The whole numbers must be viewed in pairs (coordinates), triples (vectors), as multipliers (ratio) or extendable (decimals). Many of the items are testing the knowledge of a new mathematical vocabulary e.g. tenths, halves, reflections. Children capable of solving Stage 1 items could be regarded as knowing 'meanings' in the mathematical language but nearly 30 per cent of our child population, even at age 15, could go no further, i.e. could not apply this language and achieve success on stage 2 items .

Many of the Stage 1 items have a visual aspect, diagrams are given or grids provided and the child usually needs to perform only one step for completion of an item. Counting is one such step and plotting a number pair is another.

Stage 2

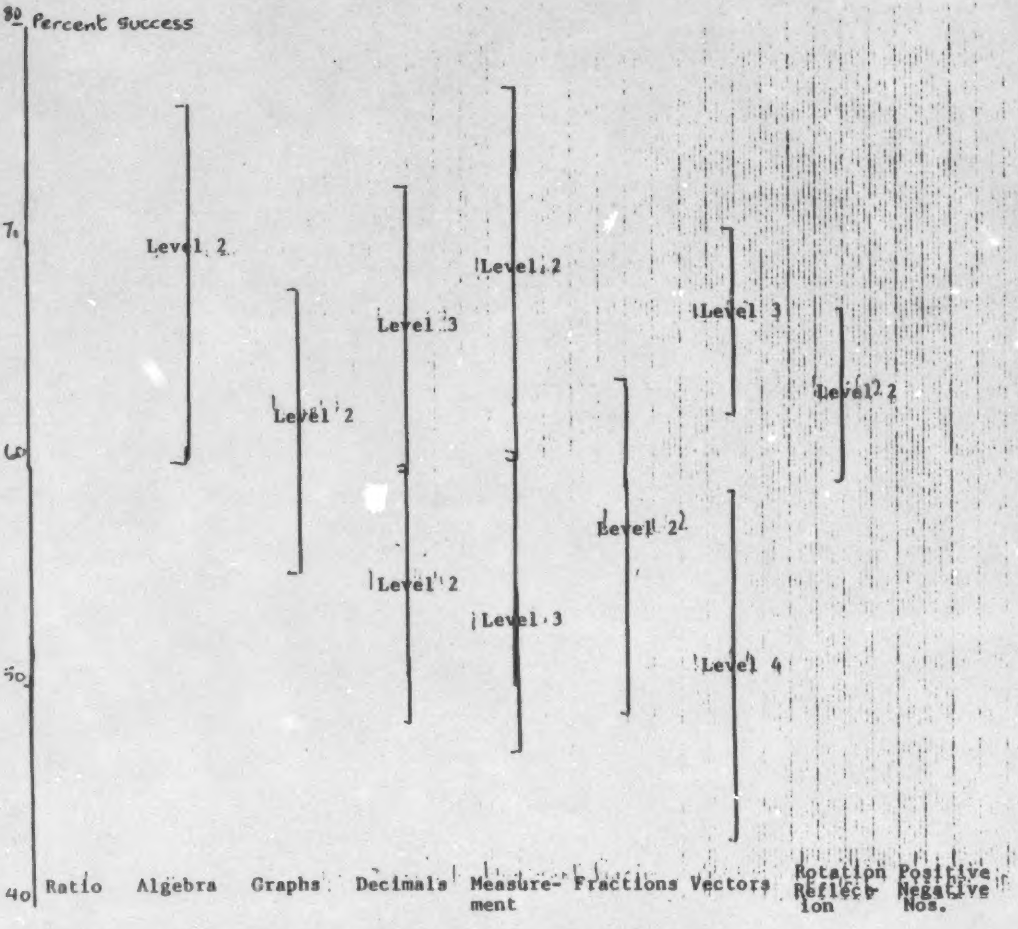
The position of the line drawn to delineate the end of Stage 2 meant that two levels of understanding in Decimals, two in Measurement, two in Matrices and two in Vectors were included, but none in Positive and Negative Numbers or Ratio. This was because there was a 20 per cent facility gap between levels 1 and 2 on Ratio and also on Positive and Negative Numbers whereas on Decimals etc. the two levels tended to be close in facility. Stage 1 items were mainly

Figure 6.13



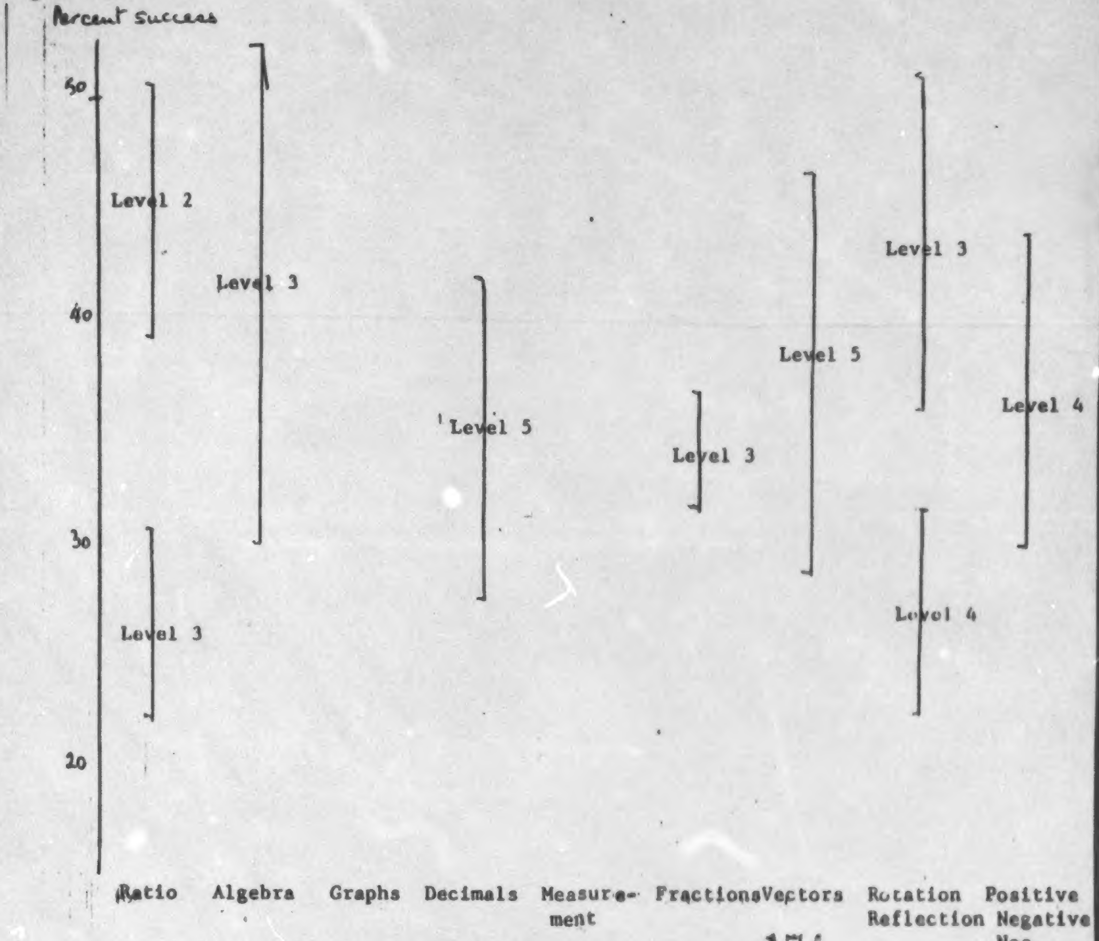
Stage 1 Levels (3rd Year Sample)

Figure 6.14



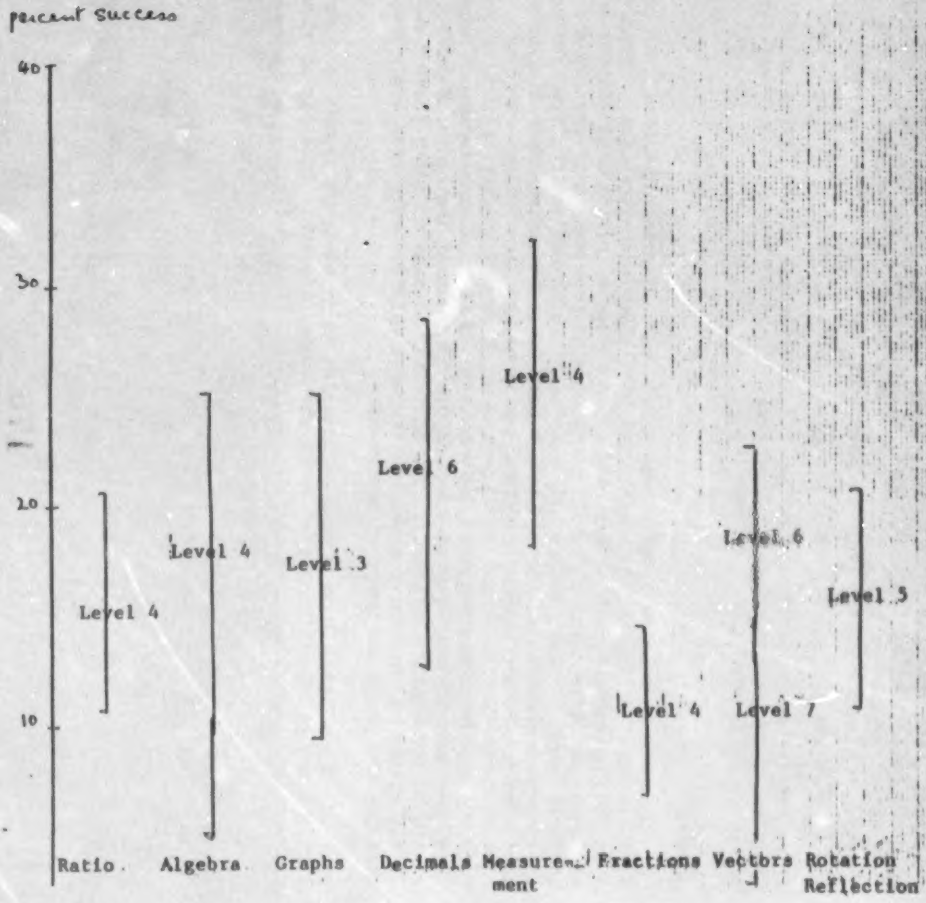
Stage 2 Levels (Facilities from 3rd Year Sample)

Figure 6.15



Stage 3 Levels (Facilities from 3rd Year Sample)

Figure 6.16



Stage 4 Levels (Facilities from 3rd Year Sample)

concerned with understanding the meaning of new conventions, whereas Stage 2 seems to be concerned with the application of these conventions and is therefore more concerned with the understanding of when they should be applied than with the pure knowledge of the language or symbols. In a sense, what was introduced in Stage 1 becomes operative in Stage 2 with an additional dimension of difficulty in the problems.

Stage 2 extends and uses what was contained in Stage 1, in some cases the vocabulary of mathematics is still being built up (e.g. Decimals and Vectors) and in other cases that vocabulary is being used. Problems may require two steps for completion or need an interpretation in the form of a graph or the application of formulae. The child can no longer simply carry out an instruction, but is now required to see what is needed for solution. Although the child is not required to invent a problem solving strategy the general question of 'what method do I use' is inherent in many of the items.

Stage 3

The main difference between the items in Stage 2 and those in Stage 3 is that Stage 3 includes an aspect of abstraction; questions are not always tied to a diagram and the child is asked to hypothesise about situations which are not shown. The harder items require the use of a strategy, not a simple interpretation, with the result that many children show an incorrect method of solution (e.g. the "addition strategy" in the Mr. Short and Mr. Tall problem from Ratio) or make errors because they focus perhaps on one aspect of the question and ignore others (e.g. in Fractions the child has to cope with $\frac{2}{7} = \frac{\Delta}{14} = \frac{10}{\square}$ and see that $\frac{10}{\square}$ is connected to the $\frac{2}{7}$ as well as to $\frac{4}{14}$). Other items require decisions to be made on the significance of the decimal point or the size of the measuring unit. The hardest Stage 3 items were successfully solved by 20-30 per cent of the sample.

Stage 4

Items in this stage involve aspects of abstract reasoning as well as the application of a breadth of mathematical knowledge to the solution of problems. Many 'child methods' such as building up an answer by addition (in Ratio) are no longer sufficient and the child is faced with problems which require a different method. Such methods may have been taught but the children who now produce them when needed are probably those who could assimilate and store them when taught and who can now decide when to use them.

All the items which tested the knowledge of the infinite nature of numbers (e.g. How many numbers between 0.41 and 0.42?) occur at the end of Stage 4 or are of comparable difficulty. The fact that so many children chose a small finite number as the answer shows how far most children were from truly understanding the range and power of mathematics.

It is not until Stage 4 that mathematics as an abstract system, and not merely a way of quantifying real world phenomena, appears. This way of viewing mathematics is generally embodied in the hardest Stage 4 items.

Summary

In this chapter each of the topic hierarchies has been described (all the items in the hierarchy levels are given in Appendix 7). The analysis which resulted in certain items being grouped together was made on the data obtained from testing large numbers of children. The methods used to cluster items have been detailed in Chapter 5. Within the limits imposed by the test papers themselves (i.e. all the levels are based on test paper items) the levels in each hierarchy are good indicators of the various degrees of complexity of the topic. The stages are very much cruder guides to overall mathematical difficulties. As stated previously an exact match between topics was impossible and the general descriptions of the contents of the stages are brief. Readers however are in a position to see exactly what each stage contains and provide their own descriptions.

CHAPTER SEVEN

STRATEGIES AND ERRORS

The interviews using the test items, gave some indication of the methods children use to solve mathematical problems in a number of different topic areas. Answers which resulted from the use of recognisably incorrect methods were coded when the written tests were marked. The codes for each test paper together with the frequency of their occurrence appear in Appendix 1. Some errors appeared to be committed by large numbers of children independent of type of school and textbooks or teaching schemes. In addition many of these regularly occurring errors were not restricted to any particular age group and some (as seen from the longitudinal survey) persisted from year to year. The study of strategies and errors is the subject of further research and the following sections are intended to be illustrative rather than comprehensive.

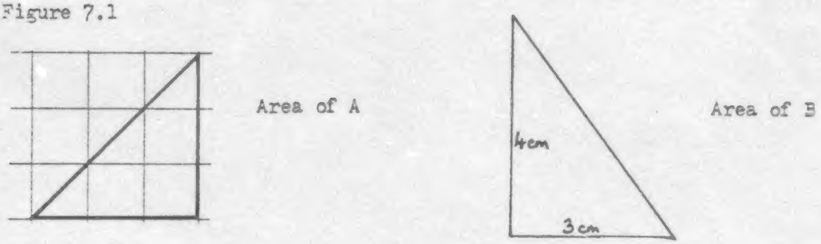
Strategies

Correct but Limited

The levels in the topic hierarchies have in many cases been described in terms of the most naive methods used to solve the problems they contain. It is recognised that the problems could equally well be solved by the use of more sophisticated methods but when an item has a high facility it seems likely that a number of children are using a naive method to solve it. Children are not necessarily consistent in their use of a method; indeed on the eel questions (Chapter Three) only two of the 29 interviewed used the same method throughout. In a number of instances teacher-taught algorithms were to a large extent replaced by 'child methods' or adapted in some way. For example, in Ratio, no child on interview used the algorithm $x/a = y/b$ and only one child made an attempt to use the unitary method (both of these methods appear in the textbooks). A 'child method' was to break the problem into sections, obtain results for these sections and add e.g. for 5:2 this method would be "take it once, take it again, take a half and add". This method although adequate for ratio problems involving n:2 is found to be deficient when the fractional amount is not "one half", since it requires the manipulation of other less common fractions. Lunzer (1966) has quoted similar results pointing to a preference for additive strategies in solving ratio problems. Those children who could solve the harder Ratio problems did so because they had a more sophisticated method available even though they also had used the additive method on easier items. The rule $x/a = y/b$ (three values known, find the fourth) was written in some form and used consistently on only 20 papers out of the 2,257 marked in the 1976 sample. Fifteen of the 20 came from the same school (out of 100 tested) and were among those who did well on the paper as a whole. (Dalrymple-Alford (1979) has reported on an experiment involving the teaching of an algorithm for solving Ratio problems - even with such instruction the majority of the children used additive methods on the posttest.)

More general indications of the methods used by children come from the items that occur in each of the stages (across topics). The majority of stage 1 items can be solved by counting or addition. The area questions in this stage are superimposed on grids and the importance of the squares is shown by the large number of children who can obtain the area of a triangle when the grid is given compared with the number who successfully find the area of a triangle when only the dimensions are given (see Figure 7.1).

Figure 7.1



Success rate

Age	12	13	14
Success rate (%)	77.5	86.9	91.4

Success rate

Age	12	13	14
Success rate (%)	31.4	38.7	47.5

Although one half is used in stage 1, no other use of fractions or decimals is required. The desire on the part of many children to avoid working within the set of non-integers is shown by the success rate on two items, one from the Decimals test and one from the Fractions test, see Table 7.1.

Table 7.1

Responses to Two Selected CSMS(M) Items

Item	Age 12	13	14	15	
1. 16 ÷ 20	7	11	25	36	(percentage giving fraction or decimal answer)
2. A piece of ribbon 17 cm long has to be cut into four equal pieces. Tick the answer you think is most accurate for the length of each piece:					
a) 4 cm, remainder 1 piece					
b) 4 cm, remainder 1 cm	37	30	26	27	(percentage selecting alternatives b and c)
c) $4\frac{1}{4}$ cm	43	54	61	61	
d) $\frac{4}{17}$ cm					

Forty per cent of the sample (age 12-15) stated that item 1 (Table 7.1) is impossible and for item 2, 27 per cent chose the remainder form of the answer. The child methods which lead to this type of response are based heavily on models which are imbedded in the whole numbers. Division was often interpreted as sharing an amount of sweets between friends (the stories for ÷ on the Number Operations paper were almost all of this type) and for this model one must have at least the same number of sweets as friends so 16 ÷ 20 is 'impossible'.

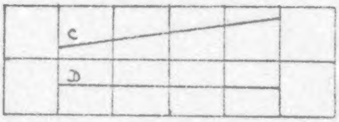
Incorrect

Naive methods are often sufficient for the successful completion of many of the items which appear on the CSMS(M) test papers. Harder items (Stage 4 problems for example) usually require a move from addition or counting and it is because they require rather more than addition that the errors first seen in the interviews occurred so frequently in the survey. For example on the Ratio paper a consistent and common error was that which we call the 'addition strategy'. Here the child looks at the difference a - b rather than a/b. For example question 4b on the Ratio paper required the construction of a figure similar to one given with a ratio of two (vertical line) to three (base), the new figure to have a base of five. The correct answer is 3.3 cm (we allowed anything between 3.1 and 3.5 to be correct), and the frequency of the response 4 cm was 47, 37 and 43 per cent for the 2nd, 3rd and 4th year subjects respectively. Children reasoned that the new base is 2 cm longer than the initial base so the new vertical line must be 2 cm longer than the initial line. This strategy was used on the harder questions and the frequency of such a response was greater than 25 per cent, and even exceeded 50 per cent on four questions of the Ratio paper. In fact in 1976, 30 per cent of the 2,257 children tested used "addition" on three or four of these questions consistently. That the mistake was not restricted to a single age group is illustrated by the fact that of 150 second year children tested in 1976 (Ratio data) and then again in 1977, those who added in 1976 were on the whole still adding in 1977. These children were not the least able, in that they did get other questions correct; they saw they needed a method for the harder questions but they opted for the wrong method. On a different level there were a number of children (varying from two per cent to 20 per cent depending on the question) who interpreted enlarging as doubling. One is tempted to conjecture that this may be due to practice (in class) on many questions involving doubling and few which require a more complicated ratio.

Errors

On the Measurement paper a number of children showed that they had many misconceptions regarding the nature of measurement. Two items which were adapted from Piaget's research on conservation showed particularly clearly the misconception that all line segments drawn between two boundary lines are equal in length (see Figure 7.2). The percentages who stated C and D (Figure 7.2) were the same length were 51, 50, and 46 for 1st, 2nd and 3rd year subjects respectively.

Figure 7.2



(i) Line C is longer ....

(ii) Line D is longer ....

(iii) C and D are the same length ....

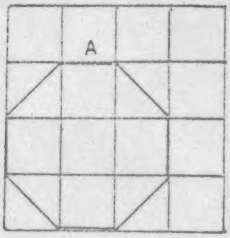
(iv) You cannot tell ....

That this was not an isolated example is illustrated by another question (see Figure 7.3). The percentages who said the answer was 8 cm were 43, 43, and 42 for 1st, 2nd, and 3rd year subjects respectively.

The essence of measurement is that the unit of measure is repeated; the number of times it is used is recorded and that number is dependent on the size of the measuring unit. The results of the two questions given in Figure 7.4 illustrate that there is some lack of such understanding.

Figure 7.3

The 8-sided figure A is drawn below on centimetre square paper.



Draw a ring round the correct answer:

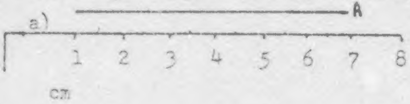
The distance all round the edge of A is:

8 cm more than 8 cm less than 8 cm you cannot tell

Figure 7.4

1. How long is each line in centimetres (cm):

a)



Length of line A: ....

2. John measures how long paths A and B are, using a walking stick. Then he measures how long paths C and D are, using a metal rod.

The answers are:

The answers are:	Path A:	13 walking sticks
	Path B:	$14\frac{1}{2}$ walking sticks
	Path C:	15 rods
	Path D:	$12\frac{1}{2}$ rods

Draw a ring round the answer you think is true in each question:

a) Path B is longer than Path A: True / False / Cannot Tell

b) Path C is longer than Path B: True / False / Cannot Tell

c) Path D is longer than Path C: True / False / Cannot Tell

For question 1 (Figure 7.4) the answer 7 may have come about through counting end points or through ignoring the fact that measuring commenced at 1. The percentages for the three year groups (1st, 2nd, 3rd) who gave the answer seven were 46, 30 and 23 respectively. Similarly, the percentages of those who said 'true' for part b, question 2 (Figure 7.4) were 51, 34 and 27 (note that the first years opted in large numbers for this answer and they would have been expected to have had the more recent experience with the ideas being tested as the topic is an important component of the primary school curriculum.

A somewhat different type of error was shown on the fractions paper when children attempted to remember a rule of operation on fractions, e.g. addition, but the version they remembered was incorrect. A popular misconception was that the rule for addition of fractions was to "add the tops and add the bottoms". For example, percentages who gave a sum of 4/15 for the problem of 1/10 + 3/5 were 16, 27, 24 and 21 for 1st through 4th year subjects respectively (more of the 1st year had the question correct than any subsequent year). Every addition of fractions (including 3/8 + 2/8) had more than 15 per cent of the children using this incorrect reasoning for the 2nd, 3rd and 4th years.

Summary

Further examples of errors that occurred in large numbers on the survey can be found by looking at the frequency of certain error codes for each test (Appendix 1). In general, the inappropriate strategies, i.e. those which were limited in generalisability or incorrect were characterised by a desire to

a) work within the set of whole numbers,
b) use counting or addition.

The errors often indicated a lack of appreciation of the nature of measurement or the essence of numbers other than whole numbers (fractions and decimals), so that inappropriate rules (not necessarily ones that had been taught) were applied.

In general the methods used by children to solve the problems were other than pure repetition of rote-learned rules. Even if the methods were limited they were part of the child's repertoire applied to an exercise rather than a blind application of a learned algorithm.

As indicated in the opening paragraph, the errors were not confined to a particular age group nor to children using a particular text book. So prevalent were some errors that the CSMS(M) research has been extended (funding for a new project) for three years, 1980-1983, to investigate further certain of the strategies and errors which occur in children's work. The new project will also attempt to identify teaching methods to be incorporated into modules designed to make children aware of errors and the limitations of particular strategies. The starting points of the new project, Strategies and Errors in Secondary Mathematics (SESM), are the data obtained from the CSMS(M) survey; in particular the evidence of the frequently occurring child methods (both incorrect and correct but limited) and the errors.

CHAPTER EIGHT

SUMMARY

The CSMS(M) Research project began with the notion of attempting to trace the development of 'levels' of understanding in secondary school mathematics. In order to do this, it was decided to divide mathematics into separate topic-areas and investigate development within each one. The areas chosen were to some extent arbitrary and non-exhaustive of the total content of the secondary curriculum. Nevertheless they covered most of the central ideas taught in secondary schools. The content areas were:

Number operations

Place value and decimals

Fractions

Ratio

Positive and negative numbers

Algebra (generalised arithmetic)

Graphs

Measurement

Rotations and Reflections

Matrices

Vectors

CSMS(M) Tests

Taking as its premise the view that the understanding of concepts is evidenced in the ability to use relationships and operations in the solution of problems, the CSMS(M) team began by identifying, with the help of syllabuses and curriculum materials, those relationships and operations which seemed to be most important in each topic area. A set of short problems which appeared to require these relationships and operations was then produced, drawing where appropriate from those used in previous research studies (e.g. Karplus 1975, Piaget, 1968, etc.). Problems requiring routine algorithms were not generally included, except occasionally for comparative purposes. Particular attention was given throughout to such issues as readability, non-ambiguity, the avoidance of 'jargon', and so on.

Preliminary forms of the written tests were used as interview instruments with approximately 300 children. About 25-35 children, drawn from the full ability range and all the year groups thought appropriate for the testing, were interviewed using each set of problems. During the course of the interviews various strategies for solving the problems, both correct and incorrect, based on the children's different conceptions and misconceptions, were noted. These findings formed the basis for modification and extension of the test material, which was then organised into class-test format and piloted with whole-class groups. Final versions of all tests appear in Appendix 1.

The Survey

Large-scale testing programmes on the finalised tests were carried out in 1976 and 1977, at the ends of the second and third year of the research programme. Schools and classes representing different areas of the country (urban and rural), different forms of organisation (e.g. middle, grammar and comprehensive) and different ability levels, as identified by the Calvert Non-Verbal Intelligence Test (NFER, 1958), were selected from a number of volunteers. The team did not however aim to control statistically for any variables othef than 'non-verbal intelligence'. The schools selected were asked to give the Calvert test to all their second year children so that the intake of the school could be judged. (It was also checked that there were no particular reasons why the intake should have changed significantly from year to year.)

On this basis a sample of at least 200 children was chosen, for each test in each year, so as not to differ significantly from the overall balance of the English population as defined by the previous standardisation of the Calvert test. In many cases the sample size per test per year was much larger than this, especially in the case of tests which were used in both 1976 and 1977. In no case was the total sample for any topic drawn from less than 12 different schools.

Each child took at least two of the mathematics tests, to enable a comparison to be made between the different levels in the different tests.

In addition a longitudinal sample of 500 second year children were selected so as to contain equal proportions of children in each ability range, as measured by the Calvert test scores. The children were tested in one or more of three mathematical areas in July of 1976, 1977 and 1978. to determine the extent to which their understanding increased from year to year.

Development of the Hierarchies

All the class tests were marked and coded for computer processing. Different codes were available for the answers which occurred frequently on the interviews and class-test trials and were thought to identify different mathematical strategies.

For each test, an output was available of the percentage giving each response to each question in each year, together with overall facilities and inter-question correlations, as measured by the ϕ-coefficient. Values of Loevinger's H_{ij} were also calculated to provide evidence of the degree of dependence of one answer on another.

A fair proportion of items were rejected at this stage of the analysis since they did not correlate sufficiently well with the remaining items. In some cases this may have been due to the fact that the item was testing a relationship which was independent of most of the relationships tested by other questions; in other cases it may be attributed to some ambiguity in the item itself.

The remaining items were clustered (see Chapter Five for a discussion of alternative methods which were tried) according to four criteria:

(i) the inter-item correlation in each cluster should be as high as possible,

(ii) each cluster should span a narrow band of facility (not more than 20 per cent and normally less), and the clusters should be sequential and not overlap in the facility-values of their items,

(iii) the items in each cluster should represent some level of demand which can be described in mathematical terms,

(iv) the clusters should be scalable i.e. in only a minimal number of cases should children attain a level, as described below, without having attained all the easier ones.

The result was a series of successive levels in each topic-area, each exemplified by a number of items. Children were judged to have attained a particular level if they successfully answered 2/3 of the items in that level. When this was done, in each case less than seven per cent of children had attained a level without attaining all the easier levels. Children could therefore reasonably be assigned to the highest level that they had attained.

A hierarchy of levels in each topic-area was therefore identified; each level being described generally in terms of mathematical demand as exemplified in the particular item. Chapter Six describes the hierarchies for each topic in detail, including:

1. the items in each level with their facilities for each year group,
2. a general description of the content of each level,
3. the percentage of children in each year group at each level.

The facility range of the items in each level is known for each year group, and the proportion of children in each year group in each level is also known.

The individual topic hierarchies were then matched with each other by 1) looking at the performance of a group of children who took two tests and calculating the values of the γ coefficient, 2) crossmatching items of the same facility (from the large sample), and 3) analysing the items from the different topic areas in terms of their mathematical demand.

This crossmatching of topic hierarchies was used to develop a model or description of four overall 'stages' in mathematics. The levels of each topic which appear in each stage and a general description of their contents was given in Figures 6.13-6.16 and Tables 6.1-6.12. An attempt was then made to abstract features common to the items in each stage. The features for each stage are described below.

Stage 1

This includes the easiest levels of most of the topics. There were some children, probably about ten per cent, who could not achieve a 2/3 criterion pass mark on Stage 1 items (see number of children labelled level 0 in each of the topic chapters). It is not known what areas of knowledge these children possessed Each of the tests assumed that numbers up to 20 were known, numbers up to 1,000 recognised and that the child had the ability to repeat a unit of measure in order to find the length or area of an object. All topics which are covered in the Junior school curriculum. Stage 1 items extend these concepts and provide an extra demand. The extra requirement might be a new notation or the use of one half as well as whole numbers. Counting and addition are the principal operations required for the successful completion of Stage 1 items and the problems usually require the coordination of two pieces of information. Many items have a diagram provided.

Stage 2

Stage 1 items were mainly concerned with understanding the meaning of new conventions, whereas Stage 2 seems to be concerned with the application of these conventions and is therefore more concerned with the understanding of when they should be applied than with the pure knowledge of the language or symbols. What was introduced in Stage 1 becomes operative in Stage 2 and there are possibly two dimensions of difficulty in the problems. In Graphs for example the child is asked to interpret a time-distance graph where as in Stage 1 he is asked to plot points with integer (or $1\frac{1}{2}$) coordinates. The problems can usually be solved by the application of a rule e.g. the area of a figure when the formula is required.

Stage 3

The main difference between the items in Stage 2 and those in Stage 3 is the appearance of the first measure of abstraction; questions are not always tied to a diagram or the child is asked to hypothesise about situations which are not shown. The harder items require the use of a strategy, not a simple interpretation, with the result that many children show an incorrect method of solution such as the "addition strategy" in the Mr. Short and Mr. Tall problem or make errors because they focus perhaps on one aspect of the question and ignore others, e.g. in Fractions the child has to cope with $\frac{2}{7} = \frac{\triangle}{14} = \frac{10}{\square}$ and see that $\frac{10}{\square}$ is connected to the $\frac{2}{7}$ as well as to $\frac{4}{14}$

Stage 4

Items in this last stage involve abstraction as well as the application of a fund of knowledge to the solution of problems. Many 'child methods' such as building up an answer by addition (in Ratio) are no longer sufficient and the child is faced with problems which require a different method. Such methods may have been taught but the children who now produce them when needed are probably those who could assimilate and store them when taught and who can now decide when to use them.

Discussion

The line delineating Stage 4 is at a facility level of 30 per cent for 15 year olds, 20 per cent for 14 year olds and 18 per cent for 13 year olds; showing a steady increase of performance with age, the percentages themselves display the fact that the vast majority of children even at age 15+ were not able to deal with abstraction and application in mathematics. In Decimals and Fractions the questions require the child to appreciate the nature of these new numbers and not be firmly fixed within the set of whole numbers. The child must be able to see that there is an answer to (16 ÷ 20) and not simply say "the question is impossible because you cannot divide a small number by a larger", as is the case when one is restricted to whole numbers. Stage 1 and 2 items in Fractions are very often of the type 'what name do we give to this', Stage 3 items require rather more in that the connection between elements is needed. Stage 4 items need a departure from concrete referents, in particular they need a recognition that multiplication or division is needed and an ability to carry out that operation. The same need for abstraction appears in the Graph questions for although a diagram is presented ($y = 2x$) the child is no longer asked simply to describe what the diagram shows in terms of (1) a number (e.g. How many children came to school by train?) or (2) a pair of numbers (coordinates), but as a relationship between sets of unknown numbers.

Certain errors and hypotheses on why they occurred were identified; Chapter Seven includes a brief discussion of some examples of errors which were made by as much as 50 per cent of the topic sample tested. These errors will be the subject of further investigation in a new SSRC project Strategies and Errors in Secondary Mathematics, SESM, based at Chelsea College. The planned research is to investigate in depth the methods children use to solve problems in mathematics and to formulate some teaching modules based on these methods in order to prevent the occurrence of the errors identified in the CSMS(M) research.

Implications

Some inferences which can be drawn from the results of the programme are as follows:

1. The majority of children can cope with only the simplest and most concrete of the levels identified. This is consistent with the findings of the science wing (Shayer, Wylam and Küchemann, 1976) which states that the majority of children do not progress beyond the concrete operational stage (in Piagetian terms) while at secondary school. This suggests that a number of mathematical topics which are generally taught across the ability range in the early years of the secondary school may be unsuitable for the majority of pupils - or at least we need to give careful consideration to the 'level' or demand (in terms of the hierarchies) implied by a given instructional approach.

2. The children in the longitudinal sample progressed on average only one level over two years; over the course of one year the majority either stayed within the same level or progressed one level. This suggests a relatively slow rate of growth in mathematical understanding (as measured by the CSMS(M) Tests).

3. In general the levels attained by children in any one year spanned the whole range in each topic; thus the range in each year was very wide compared with the average progress from year to year (see Chapter Six, Figures 6.9 -6.12). This suggests that when selecting mathematical material for presentation to children it is unwise to base this primarily on age.

Future Research

The primary goal of the CSMS(M) research was to produce hierarchies of understanding for a number of topics. The tests have been used in interviews, trial testing and then with a large sample of children - and they are now available as research tools for mathematics educators and as aids for the practising teacher. There is considerable scope for research which seeks to refine and extend the CSMS(M) work.

The test papers used in this research are appended and together with teachers' guides will be published by the National Foundation for Educational Research. Certain of them have already been used in other countries (e.g. Ratio in Greece and Canada, Fractions in Germany and Canada). Thus there is the possibility for cross-cultural studies. The results obtained from the large English sample tend to blur teaching and school effects and it is possible that where the teaching methods and materials (used in other countries) are well defined that their effects can be ascertained (e.g. Hasemann in Osnabruck obtained very different results on the Computation section of Fractions).

The hierarchies obtained in the current research are heavily dependent on the test items. Similar studies using different items but on the same topics need to be done. Particularly important relevant research would be that which attempted to provide items which 1) filled the 'gaps' in the hierarchies, 2) extended them to more complex mathematics or to pre-level 1 mathematics, and 3) which tested the descriptions assigned to each level in each hierarchy. The cut-off between levels in each hierarchy was often based on an obvious facility gap. Such a gap may simply result from the lack of items at a particular level of difficulty being included on a testpaper, alternatively there may be such an increase in cognitive demand when the items are made more complex that it is impossible to provide intermediate level items.

The mathematics investigated on each test, varied in difficulty but seldom reached that demanded in the 'O' level examination syllabus. For example in Ratio and Proportion there were no items involving the relationship between three amounts a:b:c nor were there any items dealing with Trigonometry. Each hierarchy had three or more levels; additional research could provide levels in a harder difficulty range. Level 0 has been described in each hierarchy in terms of an inability to solve level 1 items; we have no information on what children at this level can do. An extension of the research into the primary school should show the existing secondary levels but with the facility range of the easiest levels being wider and the hardest level being attained by fewer children. This may result in many more children at level 0 and therefore there would be a need for the writing of items which are pre-level 1. The process of testing younger children would probably need to involve much more verbal (oral) communication rather than written and although it could be seen as an extension of the CSMS work it would quite likely require a different format.

The descriptions of each level in each hierarchy have been based on the mathematical content of the items and the methods children use to solve those items. A planned part of the CSMS research (which was omitted because of pressure of time) was the testing of these descriptions. Such a testing might take two forms, firstly within particular topics, items could be written which matched the descriptions of each level, these would then be tried on a number of children. The analysis of the results would have to include an investigation of the association between items having the same description and also the scalability of sets of items, each of which matched a hierarchy level. The second form would start with a new topic not yet investigated by CSMS, generalising the descriptions of the stages to include this topic; writing items to fit the descriptions; testing children and then analysing the new topic hierarchy to see if there was a close correspondence between the hypothesised and resultant hierarchies.

As described in Chapter Six the Piagetian test includes five tasks which are mathematical in content and adapted from the work of Piaget. This test was administered to 500 children, but for reasons noted in earlier sections the data were not included in the analyses leading to the development of the hierarchies. Further analysis is planned and this quite likely will include the testing of other children with an adapted form of the test.

Strategies and Errors in Secondary Mathematics (SESM)

As noted previously, there is a new project which is directly linked to the work of CSMS. The Strategies and Errors in Secondary Mathematics (SESM) project has been funded by the Social Science Research Council, SSRC, for three years - 1980-1983. The project aims to investigate certain errors and strategies in mathematics which the CSMS(M) research showed to be widespread amongst secondary school children in the years two to four. During the CSMS(M) research 300 children had been interviewed on 11 different topics in mathematics and the methods they used for the solution of problems (both correct and incorrect) were recorded. The incorrect methods led to errors, and these were subsequently also noted in a wide-scale survey of 10,000 children in which written tests were used. Certain of these errors occurred on as many as half the scripts of the sample, although the children came from different schools and were taught by teachers using different teaching schemes. The aims of SESM are to study these errors more closely by

1. investigating in depth the methods children use when faced with specific problems in four topic areas - Ratio, Algebra, Measurement, Fractions and Decimals,

2. investigating the errors that are made and why.

The project also intends to

3. investigate the effects of conflict and the provision of different embodiments, on the occurrence of these errors bearing in mind which children seem to benefit from this intervention,

4. develop modules designed to circumvent these errors.

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APPENDIX ONE

Test papers, marking schemes, frequency of codes, Ø values.

Notes

1.1 Ratio

1.2 Algebra

1.3 Graphs

1.4 Fractions 1 & 2

1.5 Fractions 3 & 4

1.6 Measurement

1.7 Reflections and Rotations

1.8 Place Value and Decimals

1.9 Number Operations

1.10 Positive and Negative Numbers

1.11 Matrices

1.12 Vectors

Notes on the Marking Schemes

On each test paper the questions are subdivided and each subsection if given a Variable Number. The Ø correlation matrix is given in terms of these variable numbers.

The coding scheme for the marking of each test is given in its original form although adapted versions are published by the NFER. When the papers were being marked according to the codes it was thought advisable to use only one computer punch card for each child. Therefore to fit within the limit of the number of available columns on a punch card, certain variables were coded together on specific test papers. When this was done e.g. Decimals, the coding sheet shows which two variables were coded together by giving a Column Number for the pair (the column numbers are given in brackets). The tables which show the frequency of the codes is given in terms of column numbers where these are applicable.

Codes on each marking scheme are 1-9 and 0. Code 0 was used (unless otherwise stated) for omissions, Code 9 for an incorrect answer which had not been identified and so coded elsewhere. In some cases these '9' answers were further analysed; this analysis does not appear in this monograph.

The following appear for each topic:-

1) Test paper
2) Marking or Coding Scheme
3) Frequency of Codes (1976, 1977)
4) Ø Correlation Matrix (1976, 1977)

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CSMS

TEST R

NAME .... TODAY'S DATE ....

BOY or GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ... day ..... month .... year

---

1. Onion Soup Recipe for 8 persons

8 onions
2 pints water
4 chicken soup cubes
2 dessertspoons butter
½ pint cream

a) I am cooking onion soup for 4 people.

How much water do I need? ....(1)

How many chicken soup cubes do I need? ....(2)

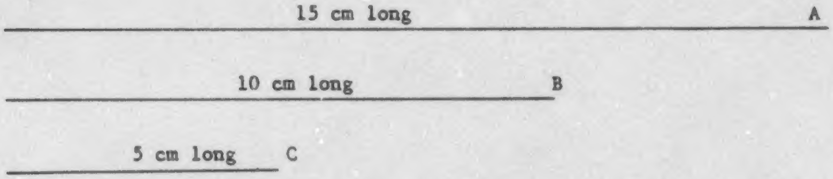
b) I am cooking onion soup for 6 people.

How much water do I need? ....(3)....

How many chicken soup cubes do I need? ....(4)....

How much cream do I need? ....(5)....

2a). There are 3 eels A, B and C in the tank at the Zoo.



The eels are fed sprats, the number depending on their length.

If C is fed two sprats, how many sprats should B and A be fed to match?

B ....(7)... A ....(8)...

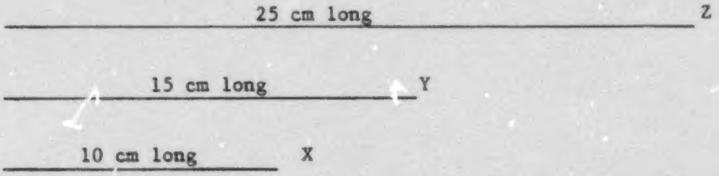
If B eats 12 sprats, how many sprats should A be fed to match?

A ...(9)....

If A gets 9 sprats, how many sprats should B get to match?

B ......(10)

2b). Three other eels, X, Y and Z are fed with fishfingers, the length of the fishfinger depending on the length of the eel.



If X has a fishfinger 2 cm long, how long should the fishfinger given to Z be? Z .... (11)

If Y has a fishfinger 9 cm long, how long should the fishfinger given to Z be? Z .... (12)

If Z has a fishfinger 10 cm long, how long should the fishfingers given to X and Y be? X ....(13).... Y ....(14)....

---

3. In an office Mr. Adams comes in to work 2 days a week.

Mr. Brown comes in to work 4 days a week.

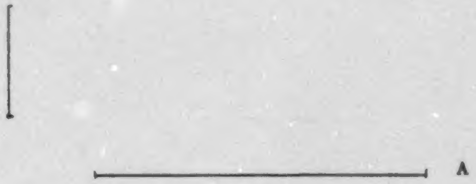
Mr. Carter comes in 6 days a week. (6)

The bill for lighting the office for these three men is 240p.

How much should each pay for it to be fair?

Mr. Adams .... Mr. Brown .... Mr. Carter ....

4a.



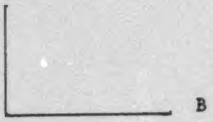
Finish drawing the diagram below so that it is the same shape but bigger than this diagram

(15)

(16)

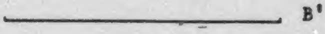
(17)

4b).



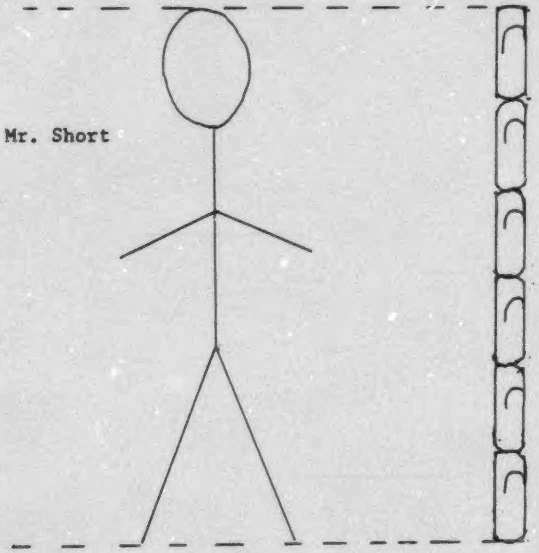
Work out how long the missing line
should be if this diagram →
is to be the same shape but bigger
than the one above

.... cm.



(18)

5.



You can see the height of
Mr. Short measured with
paper clips.

Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:

Mr. Short's height is four matchsticks

Mr. Tall's height is six matchsticks

How many paper clips are needed for Mr. Tall's height? ....(19)....

6. In a particular metal alloy there are

1 part mercury to 5 parts copper
3 parts tin to 10 parts copper
8 parts zinc to 15 parts copper

You would need how many parts mercury to how many parts tin?

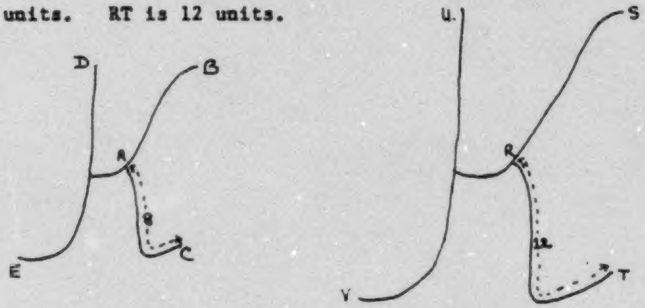
...... parts mercury to .... parts tin. (20)

You would need how many parts zinc to how many parts tin?

...... parts zinc to ...... parts tin. (21)

---

7. These 2 letters are the same shape, one is larger than the other.
AC is 8 units. RT is 12 units.



The curve AB is 9 units. How long is the curve RS? (22)

The curve UV is 18 units. How long is the curve DE? (23)

---

8. % means per cent or per 100, so 3% is 3 out of every 100.

a) 4 children out of the hundred on the school trip forgot to bring their lunch.

What percentage is this? .... (24)

b) 6% of children in a school have free dinners. There are 250 children in the school.

How many children have free dinner? .... (25)

c) The newspaper says that 24 out of 800 Avenger cars have a faulty engine.

What percentage is this? .... (26)

d) The price of a coat is £20, in the sale it is reduced by 5%; how much does it now cost? .... (27)

RATIO CODING SCHEME

Question	Variable	CODES 1	4	5	6	7	8
1	1	1					
	2	2					
	3	1½					
	4	3					
	5	3/8 (¼+⅛)					⅓
2a	7	4		6		3	12
	8	6		8		4	17
	9	18		24	14	13	17
	10	6		4½	7	8	4
2b	11	5		8	6	4	17
	12	15		18	11	10	19
	13	4		2½	6	8	
	14	6		5	8	9	4
3	6	40,80,120			60,80,100 or equal steps	40,80 120	80, 80, 80
4a	15 (vertical)	4			8		
	16 ↕ gap	2	ignored		7		1
	17 ↔ gap	3	ignored		7½		1½
4b	18	3.2-3.5			4	3	2
5	19	9		12	8	7	
6	20	2,3		2,6	1,8		1,3
	21	1,1½ 16,9 8,4½		16,6	8,8 3,13		8,3
7	22	13½		18	13	10	4½
	23	12		9	14	17	27
8	24	4					
	25	15					
	26	3					192
	27	19			5 or 4	1	16 or 15

Year 2 Ratio 1976 Frequency of Codes

Var No.	Code							
	1	4	5	6	7	8	9	0
1	94.4						5.2	0.4
2	94.6		0.1				5.1	0.1
3	84.5						14.6	0.9
4	75.0						23.9	1.1
5	24.4					20.6	50.1	4.9
6	43.8			7.7	0.2	10.1		4.9
7	85.4		0.5	0.1	2.1	0.6	10.0	1.2
8	75.9		4.4		1.5	0.4	16.6	1.2
9	45.0		10.2	8.5	0.9	9.4	23.6	2.4
10	45.6		7.7	10.1	1.2	8.0	24.2	3.0
11	47.0		5.6	13.4	3.2	1.4	27.2	2.1
12	22.7		12.5	5.5	2.1	8.4	42.1	6.6
13	22.6		11.7	6.6	3.1		46.5	9.4
14	23.7		20.0	6.6	1.2	4.4	34.4	9.6
15	70.9			2.4			20.6	6.1
16	40.0	10.7				19.0	24.1	6.1
17	40.1	9.7				17.9	26.1	6.1
18	7.9			47.6	15.0	1.2	22.5	5.7
19	28.1		2.0	51.4	1.6	0.4	14.4	2.1
20	27.0		5.1	0.5		21.4	35.1	10.9
21	11.4		1.2	0.9		19.2	52.0	15.2
22	13.7		6.9	43.6	0.7	0.4	23.7	10.9
23	15.4		8.5	32.2	1.1	2.9	27.1	12.7
24	83.7						12.0	4.2
25	37.6	0.1					53.7	8.5
26	32.0					7.4	50.0	10.6
27	20.4			5.6	1.6	40.9	23.1	8.4

Year 3 Ratio 1976 Frequency of Codes

Var No.	Code 1	4	5	6	7	8	9	0
1	94.3						5.5	0.3
2	95.0						5.0	
3	84.0						15.1	0.9
4	74.8						23.7	1.4
5	22.7					23.6	48.8	5.0
6	46.0			6.4	1.4	8.2	29.2	8.6
7	83.1		0.5		2.3	0.4	11.2	2.5
8	74.7		2.7		2.0	0.4	17.9	2.3
9	49.5		8.9	7.2	0.5	6.6	24.4	2.9
10	50.3		5.6	8.5	0.5	6.0	24.9	4.2
11	47.8		6.5	12.1	4.0	0.8	24.3	4.4
12	30.0		12.5	4.3	2.6	5.6	34.3	10.7
13	27.4		11.6	5.9	2.5		39.6	13.0
14	28.2		16.6	6.0	1.6	4.6	30.8	12.4
15	72.6			1.4			16.3	9.6
16	45.1	9.1				14.6	21.6	9.5
17	43.3	8.1				12.0	27.1	9.5
18	11.0			39.4	16.7	1.6	23.5	8.0
19	29.6		2.5	50.6	0.8		15.4	1.2
20	29.2		3.3	0.3		26.9	27.1	13.3
21	12.3		1.4	1.2		25.8	41.7	17.6
22	19.7		6.4	40.3	0.9		17.2	15.5
23	20.5		6.0	29.3	0.4	3.3	23.2	17.3
24	85.7						10.0	4.3
25	45.5						46.0	8.5
26	39.6					10.4	37.9	12.0
27	26.5			5.0	1.0	35.9	20.3	11.3

Year 4 Ratio 1976 Frequency of Codes

Var No.	Code 1	4	5	6	7	8	9	0
1	95.8						4.1	0.1
2	95.5						4.3	0.1
3	88.1						10.6	1.3
4	78.6						19.7	1.7
5	31.4					21.7	41.2	5.7
6	56.7			4.9	0.3	5.5	24.4	8.0
7	91.2		0.6		0.6		5.4	2.3
8	83.3		2.0		0.9		11.3	2.5
9	56.8		5.8	7.4	0.3	6.2	19.3	4.2
10	56.4		4.2	8.7	0.9	6.5	17.0	6.4
11	55.1		5.9	9.7	3.3	0.6	20.3	5.1
12	40.6		8.3	4.1	0.7	4.9	29.6	11.9
13	32.2		8.8	8.8	1.7		33.0	15.4
14	32.8		12.9	5.1	0.6	5.4	27.0	16.4
15	74.1			2.0			14.8	9.1
16	46.7	6.8				14.3	23.0	9.1
17	44.6	7.0				10.9	28.4	9.1
18	19.7			39.7	11.7	0.6	19.0	9.3
19	42.0		0.9	39.1	0.9	0.1	13.2	3.8
20	43.2		2.6	0.1		17.2	20.6	16.2
21	18.3		2.0	0.6		16.2	39.9	23.0
22	28.7		5.2	34.2	0.1	0.1	18.6	13.0
23	28.8		4.5	25.1	0.9	4.8	21.0	14.9
24	85.8						8.8	5.4
25	55.7						32.6	11.7
26	47.0					8.8	27.7	16.5
27	35.4		0.1	3.9	3.2	27.4	16.5	13.6

Year 2. Ratio 1977 Frequency of Codes n = 296

VAR. NO.	1	2	3	4	5	6	7	8	9	0
1	93.9	0	0	0	0	0	0	0	5.7	.3
2	93.6	0	0	0	0	0	0	0	6.1	.3
3	78.0	0	0	0	0	0	0	0	20.9	1.0
4	73.0	0	0	0	0	0	0	0	25.7	1.4
5	20.6	0	0	0	0	0	0	16.6	57.4	5.4
6	35.8	0	0	0	.7	2.4	0	10.5	32.4	18.2
7	74.7	0	0	0	.7	0	2.7	0	19.3	2.7
8	65.5	0	0	0	4.4	0	2.0	.3	24.3	3.4
9	35.8	0	0	0	8.4	7.1	1.0	8.1	34.1	5.4
10	36.5	0	0	0	7.8	11.1	2.0	7.1	29.1	6.4
11	42.2	0	0	0	7.8	10.1	4.4	1.0	30.1	4.4
12	24.3	0	0	0	8.8	6.4	2.7	5.1	43.9	8.8
13	19.9	0	0	0	11.1	5.7	1.7	.3	44.9	16.2
14	20.3	0	0	0	17.2	5.4	2.0	4.1	35.5	15.5
15	50.3	0	0	0	0	2.0	0	.3	25.3	22.0
16	27.0	0	0	8.8	0	1.0	0	13.2	28.0	22.0
17	26.4	0	0	9.1	0	1.0	0	10.8	31.1	21.6
18	5.4	0	0	0	0	47.6	9.5	2.4	17.6	17.6
19	29.4	0	0	0	5.1	49.7	.7	.3	10.5	4.4
20	24.7	0	0	.3	4.1	.7	0	16.6	30.4	23.6
21	10.1	0	0	0	1.0	4.4	0	16.6	39.2	28.7
22	11.1	0	0	0	5.7	37.5	.7	1.0	22.3	21.6
23	13.9	0	0	0	5.7	28.0	1.4	1.4	26.0	23.6
24	68.9	0	0	0	0	0	0	0	18.9	12.2
25	33.8	0	0	0	0	0	0	0	45.9	20.3
26	24.7	0	0	0	0	0	0	8.8	40.9	25.7
27	16.6	0	0	0	1.7	3.7	0	31.4	25.0	21.6

Year 3 Ratio 1977 Frequency of Codes n=257

VAR. NO.	1	2	3	4	5	6	7	8	9	0
1	93.8	0	0	0	0	0	0	0	6.2	0
2	94.2	0	0	0	0	0	0	0	5.8	0
3	84.0	0	0	0	0	0	0	0	14.0	1.9
4	74.3	0	0	0	0	0	0	0	23.7	1.9
5	24.5	0	0	0	0	0	0	20.2	45.5	9.7
6	45.9	0	0	0	1.6	5.4	.8	5.1	30.0	11.3
7	83.7	0	0	0	.4	0	3.9	.4	10.1	1.6
8	75.1	0	0	0	1.9	0	2.7	0	18.7	1.6
9	50.2	0	0	0	8.6	4.3	1.6	9.7	23.3	2.3
10	53.6	0	0	0	5.4	6.4	1.9	8.9	20.6	3.5
11	51.4	0	0	0	5.6	12.1	2.3	1.9	22.2	3.5
12	34.6	0	0	0	13.6	4.3	1.9	5.8	31.9	7.8
13	27.2	0	0	0	9.3	8.6	3.5	0	35.4	16.0
14	26.8	0	0	0	14.4	5.4	2.3	6.2	28.4	16.3
15	63.4	0	0	0	0	3.1	0	.8	12.8	19.8
16	34.6	0	0	7.4	0	.8	0	17.5	19.8	19.8
17	33.1	0	0	8.6	0	1.2	0	16.7	21.0	19.5
18	10.1	0	0	0	0	37.7	14.0	.4	24.5	13.2
19	31.1	0	0	0	2.3	48.2	0	.4	15.2	2.7
20	33.9	0	0	0	3.9	0	0	20.2	23.7	18.3
21	12.5	0	0	0	.8	.8	0	19.5	39.3	27.2
22	16.7	0	0	0	5.1	38.5	1.6	0	19.1	19.1
23	17.9	0	0	0	5.8	24.5	.8	2.3	28.8	19.8
24	83.3	0	0	0	0	0	0	0	11.3	5.4
25	48.2	0	0	0	0	0	0	0	37.4	14.4
26	40.1	0	0	0	0	0	0	4.3	34.6	21.0
27	26.1	0	0	0	0	4.7	.8	31.1	21.4	16.0

Year 4. Ratio 1977 Frequency of codes n=190

VAR. NO.	1	2	3	4	5	6	7	8	9	0
1	97.9	0	0	0	0	0	0	0	1.1	1.1
2	98.4	0	0	0	0	0	0	0	.5	1.1
3	85.8	0	0	0	0	0	0	0	13.2	1.1
4	81.1	0	0	0	0	0	0	0	17.4	1.6
5	26.8	0	0	0	0	0	0	23.2	39.5	10.5
6	43.2	0	0	0	1.1	3.2	.5	4.7	31.6	15.8
7	84.7	0	0	0	0	0	1.1	.5	12.1	1.6
8	73.7	0	0	0	4.7	0	1.1	0	18.9	1.6
9	47.9	0	0	0	6.3	5.3	1.1	10.0	25.3	4.2
10	50.5	0	0	0	6.3	5.3	1.1	7.9	23.2	5.8
11	54.7	0	0	0	7.4	10.0	2.1	2.1	19.5	4.2
12	31.6	0	0	0	8.9	4.7	1.1	13.2	28.9	11.6
13	27.9	0	0	0	9.9	4.2	2.6	0	38.9	17.4
14	28.9	0	0	0	14.7	4.2	1.1	4.2	30.5	16.3
15	63.2	0	0	0	0	3.7	0	.5	16.3	16.3
16	38.4	0	0	7.4	0	1.1	0	17.4	19.5	16.3
17	34.2	0	0	9.9	0	1.1	0	15.3	23.7	16.3
18	11.1	0	0	0	0	43.2	10.5	.5	17.9	16.8
19	35.8	0	0	0	1.1	41.6	0	.5	13.7	7.4
20	30.5	0	0	0	2.6	0	0	21.1	24.7	21.1
21	13.2	0	0	0	1.6	.5	0	21.6	34.7	28.4
22	19.5	0	0	0	4.2	41.6	0	0	19.5	15.3
23	22.6	0	0	0	7.4	25.8	1.1	3.2	24.2	15.8
24	79.5	0	0	0	0	0	0	0	13.2	7.4
25	57.9	0	0	0	0	0	0	0	31.6	10.5
26	45.8	0	0	0	0	0	0	5.8	33.2	15.3
27	37.9	0	0	0	.5	2.1	1.6	24.2	16.9	16.8

RATIO: PHI CORRELATION MATRIX, N OF CASES = 2257 (1976)

FAC	ITEM																											
95.0	2																											
94.8	1	697																										
86.4	7	164	184																									
85.4	3	368	438	157																								
85.0	24	184	185	205	185																							
77.8	8	113	132	712	135	199																						
76.0	4	311	273	184	335	194	193																					
72.4	15	110	153	219	155	219	187	201																				
50.5	10	116	113	272	168	200	308	237	210																			
50.2	9	106	120	316	153	212	344	260	237	617																		
49.8	11	129	146	245	157	196	261	234	219	320	334																	
48.5	6	148	172	246	224	238	235	295	304	350	356	323																
45.8	25	128	120	191	190	306	179	226	236	271	263	251	330															
43.8	16	82	106	163	108	150	154	171	442	250	236	209	266	226														
42.6	17	85	97	164	124	142	163	179	440	244	220	218	271	249	667													
39.2	26	116	131	192	202	288	223	288	271	333	344	345	460	389	269	298												
32.9	19	90	105	171	136	153	204	242	201	341	341	288	368	282	236	240	385											
32.7	20	89	83	172	162	184	204	223	214	293	300	292	372	267	230	221	402	326										
30.7	12	72	95	208	149	195	258	254	227	441	409	404	356	293	242	253	377	379	345									
28.0	14	78	93	158	128	148	207	190	207	321	329	299	344	272	229	237	352	332	345	467								
27.2	13	93	98	167	133	164	199	202	218	345	339	336	358	284	236	233	375	353	351	488	823							
27.0	27	111	98	157	163	210	210	248	192	302	319	296	351	373	223	218	421	366	349	367	371	384						
26.0	5	121	125	158	198	177	190	275	225	317	303	297	365	309	253	264	361	355	351	382	348	356	363					
21.2	23	78	78	140	134	178	181	202	203	318	296	291	348	283	273	276	369	365	358	397	364	379	367	369				
20.3	22	85	89	146	133	202	182	229	222	343	331	313	375	310	277	292	428	419	398	396	371	391	408	431	620			
13.8	21	67	70	128	114	146	161	182	183	267	259	268	301	249	232	225	350	344	499	336	331	357	381	376	374	423		
12.5	18	37.	46	88	80	73	128	134	161	214	219	217	240	207	218	217	255	313	263	287	303	301	302	270	356	390	322	
		2	1	7	3	24	8	4	15	10	9	11	6	25	16	17	26	19	20	12	14	13	27	5	23	22	21	18

Algebra 1

Name .... *School* .... *Class* ......

Date .... *Date of Birth* ....
day *month* *year*

Boy or Girl ....

Trial Item 1

What number does $a + 4$ stand for if $a = 2$ ....

What number does $4a$ stand for if $a = 2$ ....

Trial Item 2

Fill in the gaps:
(work down the page)

$x \longrightarrow 3x$	$x \longrightarrow x+3$	$x \longrightarrow 7x$	$x \longrightarrow x+8$
$2 \longrightarrow 6$	$5 \longrightarrow 8$	$2 \longrightarrow .$	$3 \longrightarrow .$
$5 \longrightarrow .$	$4 \longrightarrow .$		
	$n \longrightarrow .$		

Now check your answers against the answers on the back page.

---

1. Fill in the gaps:

$x \longrightarrow x + 2$	$x \longrightarrow 4x$
$6 \longrightarrow .$ Var.1	$3 \longrightarrow .$ Var.3
$r \longrightarrow .$ Var.2	

---

2. Write down the smallest and the largest of these: smallest largest

$n + 1$, $n + 4$, $n - 3$, n, $n - 7$. .... Var.4 ....

---

3. Which is the larger, $2n$ or $n + 2$? Var.5 ....

Explain: ....

---

4. 4 added to n can be written as $n + 4$. Add 4 onto each of these:

8	$n + 5$	$3n$
......	Var.6	Var.7 ......

n multiplied by 4 can be written as $4n$. Multiply each of these by 4:

8	$n + 5$	$3n$
......	Var.8	Var.9 ......

---

5. If $a + b = 43$ $a + b + 2 =$ Var.10 ......

If $n - 246 = 762$ $n - 247 =$ Var.11 ......

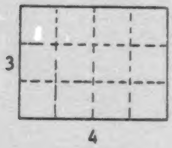
If $e + f = 8$ $e + f + g =$ Var.12 ......

6. What can you say about a if $a + 5 = 8$ ......Var.13.........

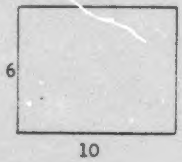
What can you say about b if $b + 2$ is equal to $2b$ ......Var.14......

---

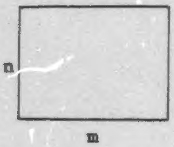
7. What are the areas of these shapes?



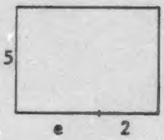
A = ...Var.15.



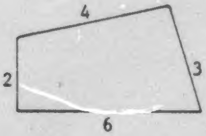
A = ...Var.16..

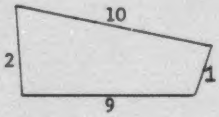


A = ...Var.17..



A = .Var.18....

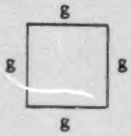




8. The perimeter of this shape is equal to $6 + 3 + 4 + 2$, which equals 15.

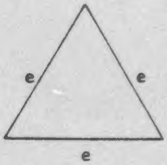
Work out the perimeter of this shape. p = ...Var.19...

9.

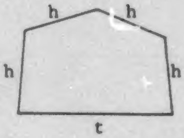


This square has sides of length g.
So, for its perimeter, we can write $p = 4g$.

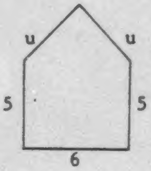
What can we write for the perimeter of each of these shapes?



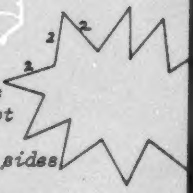
p = ...Var.20...



p = ...Var.21...



p = ...Var.22...



Part of this figure is not drawn. There are n sides altogether, all of length 2.

p = ...Var.23...

---

11. What can you say about u if $u = v + 3$ and $v = 1$ ....Var.26.......

What can you say about m if $m = 3n + 1$ and $n = 4$ ....Var.27..........

12. If John has J marbles and Peter has P marbles, what could you write for the number of marbles they have altogether? ....Var.28..........

---

13. a + 3a can be written more simply as 4a.

Write these more simply, where possible:

2a + 5a = .Var.29........

2a + 5b = .Var.30........

(a + b) + a = .Var.31.......

2a + 5b + a = .Var.32........

(a − b) + b = .Var.33........

3a − (b + a) = ..Var.34.......

a + 4 + a − 4 = ..Var.35.......

3a − b + a = ..Var.36.......

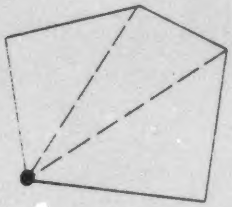
(a + b) + (a − b) = ..Var.37.......

---

14. What can you say about r if r = s + t

and r + s + t = 30 ....Var.38..........

15.



In a shape like this
you can work out the number of diagonals by
taking away 3 from the number of sides.

So, a shape with 5 sides has 2 diagonals;

a shape with 57 sides has ...Var.39.... diagonals;

a shape with k sides has ...Var.40.... diagonals.

---

16. What can you say about c if c + d = 10

and c is less than d ....Var.41.....

---

17. Mary's basic wage is £20 per week.
She is also paid another £2 for each hour of overtime that she works.

If h stands for the number of hours of overtime that she works, and
if W stands for her total wage (in £'s)
write down an equation connecting W and h: ....Var.42......

What would Mary's total wage be if she worked 4 hours of overtime? ....Var.43..........

18. When are the following true -always, never, or sometimes?
Underline the correct answer:

$A + B + C = C + A + B$ Always. Never. Sometimes, when ...Var.44.......

$L + M + N = L + P + N$ Always. Never. Sometimes, when Var.45

---

19. $a = b + 3$. What happens to a if b is increased by 2? Var.46

$f = 3g + 1$. What happens to f if g is increased by 2? Var.47

---

20. Cakes cost c pence each and buns cost b pence each.
If I buy 4 cakes and 3 buns,
what does

$4c + 3b$ stand for? Var.48

---

21. If this equation ⟶ $(x + 1)^3 + x = 349$
is true when $x = 6$,

then
what value of x
will make this equation ⟶ $(5x + 1)^3 + 5x = 349$
true?

Var.49

$x =$ ....

---

22. Blue pencils cost 5 pence each and red pencils cost 6 pence each.
I buy some blue and some red pencils and altogether it costs me 90 pence.

If b is the number of blue pencils bought, and
if r is the number of red pencils bought,
what can you write down about b and r? ....Var.50............

Trial Item
ANSWERS ⟶

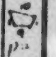
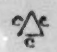
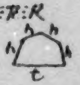
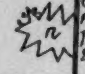
What number does $a + 4$ stand for if $a = 2$ ...6...

What number does $4a$ stand for if $a = 2$ ...8...

$x \to 3x$	$x \to x+3$	$x \to 7x$	$x \to x+8$
$2 \to 6$	$3 \to 8$	$2 \to 14$	$3 \to 11$
$5 \to 15$	$4 \to 7$		
	$n \to n+3$		

ALGEBRA CODING SCHEME

	1			2	3+	4+ CODES				5+			6		7+ AREA			
	x → x+2 b →	r →	x+4 → x 3 →	SMALLEST/ LARGEST	WHICH IS LARGER? 2n OR n+2	ADD 4 ONTO n+5	3n	MULTIPLY BY 4 n+5	3n	a+b=43 a+b+2=	n−246=762 n−247=	e+f=8 e+f+g=	a+5=8	b+2 IS EQUAL TO 2b	3 □ 4	6 □ 10	n □ m	5 □ e+2
1 CORRECT	8	r+2	12	n−7, n+4	"DEPENDS" + AWARE OF AT LEAST TWO OF: n=2 n=1 n=0 n NEGATIVE ("n>2" ONLY)	n+9	3n+4	4(n+5) 4n+20	12n	45	761	8+g	3	2	12	60	nm n×m	5(e+2) 5e+10
2 CORRECT WEAK					"DEPENDS" + ONLY ONE OF: n=2 n=1 n=0 n=NEG												MISUNDERSTOOD BUT CONSISTENT ← WITH	
3 AMBIGUOUS [INADEQUATE PUNCTUATION, +, ×; NOT NECESSARILY WRONG]		r2 2r x+2		OTHER WAY ROUND n+4; n−7 MIXED: [illegible]	"DEPENDS" OR "USUALLY 2:1" BUT NONE OF: n=2, 1, 0, NEG	n+5+4 n+54 n9	3n+4 IFF "n+514" 3n4 n×3+4	4n+5 n+5×4 4n5 n54 n+54	3n×4 3n4 43n			8g		"b+2 =2b"			n,m	5e+2 e+2×5 5e2 e25 MISUNDERSTOOD
4 AMBIGUOUS ASSOCIATION [WRONG PUNCTUATION +, ×, BUT NOT WRONG OPERATION ON NUMBERS]		r×2			2n	n×9 n+5×4	3n×4		12+n			8×g					n+m	5e×2
5 ASSOCIATION WITH LETTER [EXPLICIT WRONG OPERATION ON NOS]						n+20	7n 12n	n+20 n+9	7n									e+10 10e 7e
6 ASSOCIATION LETTER IGNORED		+2 2				9	7 12	20 9	7 12			9						10 7
7 SPECIFIC/ CONSISTENT [ALPHABET CODE]		t 20 m 21 2t				23 22 24 19 w w+9 [illegible]	3r 3w	4s	3r			15 ONLY						35 (50)
8 SPECIFIC/ OBVIOUS (* ANY "OTHER" NUMERICAL ANS)	12	*	7	*	n+2 "SAME"	*	*	*	*	*	763	12	*	b=1 b=0	*	*	12	25 20 30
9 OTHERS		INCLUDE 8		ONE OR MORE WRONG														
0 BLANK																		

8	9a PERIMETER				10 CABBAGES AND TURNIPS 8c+6t	Nog VEG	11 h=13 V=1	n=3 n+4	12a MARBLES
 p=									
22 7.5 21 20	3c c3	4h+t 4h+1t	2u+16 u^2+16 2^u+16	2n 2^n $\frac{n}{2}$	EXPLICIT "COST OF C CABBAGES AND t TURNIPS"	c+t	4	13	J+P
	LONG c+c+c	h+h+h+h+t	u+u+16 2u+2.5+6	n2 n^2 n^y	LITERAL "8p×(C)CABBAGES PLUS 6p×(t)TURNIPS"				
	eee e,e,e (indexes)	hhht 4ht	uu556 2u16	n,2 t^2 2^n $\frac{n}{2}$	IMPLICIT "(TOTAL)COST" "COST OF CABBAGES AND TURNIPS"	c,t ct	V= n-3	$n=\frac{n-4}{3}$	J and P J,P JP J_c+P_c n_J+n_P J∧P
	c×e×e	4h×t	2u×16	n+2.	"THE NUMBER of cabbages + turnips" (GENERAL)	c×t			J×P
		5ht	32u 18u 2u+150 2u+60	32n 32+n 17n 6n					
					1 CAB, 1 TURNIP 8p+6p 14p	2			
	15 125			28 p	100 8 CABS 6 TURNIPS AND 100	100			26 27 25 Z
*	*	*	*	32+2 34 6 16 17	8 CAB, 6 TURNIPS (14 CABBAGES AND TURNIPS)	14	2	*	*
			SIMPLE ARITH ERROR	include 32+2n d×n nn	INCLUDE 14ct 14c+t 14c,t ch 8×c+6×c				J=P

13.

$$\begin{array}{ll} 2a+5a = 7a & \\ 2a+5b = 2a+5b & 3a-(b+a) = 2a-b \\ (a+b)+a = 2a+b & a+4+a-4 = 2a \\ 2a+5b+a = 3a+5b & 3a-b+a = 4a-b \\ (a-b)+b = a & (a+b)+(a-b) = 2a \end{array}$$

ALGEBRA Contd.

2a+5a	2a+5b	(a+b)+a	2a+5b+a	(a−b)+b	3a−(b+a)	a+4+a−4	3a−b+a	(a+b)+(a−b)
7a	2a+5b OR "IMPOSSIBLE" OR 'BLANK' IF AT MOST ONE OTHER BLANK.	2a+b	3a+5b	a	2a−b	2a	4a−b 4^a-b	2a 2^a
								2a+b−b OR a+a
BRACKET EXPANDED: (EVEN IF WRONG) SIMPLIFIED BUT INADEQUATE PUNCTUATION 'BLANK' IF >1 OTHER BLANK		a^2+ab a^2+b 2a+ab 2a,b 7a b a^2b	3a5b $2a^2+5b$	$ab-b^2$ ab−2b b^2a	$-3ab-3a^2$ $3a^2-b$ (2ab) 3ab−4a	a^2	4ab $3a^2-b$	a^2-b^2 a^2
STRAIGHT FORWARD		2a×b	WRONGS	(OR × a−2b a+b	INSTEAD 4a−b	OF +) 2a+8 0 a	2a−b 3a−b	a 2a+2b 2a−2b 2a+b 0
ASSOCIATIONS (ab) ABSORPTIONS (a) REPLACEMENTS (c) 8a	7ab 12a	3ab 4a 2c	8ab	3ab ab b	5ab 2b		5ab 2ab 2a	
*	*	*	*	*	*	*	*	*
		INCLUDE ITEM WRITTEN DOWN UNCHANGED (with or without brackets) OR NOT COMPLETELY SIMPLIFIED ab+a aba		ab+b a−kb		2a+4−4		
	'BLANK' IF ALL OTHERS BLANK (other than 2a+5a=7a)							

ALGEBRA Contd.

14	15+		16+	17→		18		19		20	21	22		
$r=s+t$ $r+s+t=30$	SIDES→ DIAGONALS 57→	k→	$c+d=10$ c LESS THAN d	WAGES	4 HOURS	$A+B+C$ $=C+A+B$	$L+M+N$ $=L+P+N$	$a=b+3$ $\Delta b=2$ $\Delta a=?$	$f=3g+1$ $\Delta g=2$ $\Delta f=?$	CAKES AND BUNS $4c+3b$	$x=\frac{6}{5}$	BLUE AND RED PENCILS	MACHINE	▨
$r=15$	54	$k-3$	$c<5$ $(c>5)$	$W=20+2h$ $w=20+2\times h$	28	ALWAYS OR ALW/SOME WHEN "GENERAL" "POSITIVE"	SOMETIMES (ALWAYS) WHEN $M=P$	$\Delta a=2$	$\Delta f=6$	EXPLICIT "TOTAL COST" "COST OF 4 CKS AND 3 BUNS"	$\frac{6}{5}$ 1·2	$5b+6r=90$	×5 +50	1 CORRECT
		WRONG BUT CONSISTENT WITH ($k-2$ ONLY)	$c\leq5$ > $c\leq4$ $c<4$ $c<6$ SYSTEMATIC LIST (4 VALUES) 1–4 1234 01234(5)	TWO NUMERICAL PAIRS OF VALUES (CORRECT) $W=28$ $h=4$ $W=30, h=5$ etc OR 28,4 30,5 etc				STATIC- $a+?=b+5$ $a+2=b+3$ $a=b+1$ $a+2.$	"CORRECT" $f+6$ = ANYTHING $f=3(g+2)+1$ $(f=3g+7)$ $f=3g-5$ $f+6.$	LITERAL "4 CAKES AT c PENCE AND 3 BUNS AT b PENCE"		TWO NUMERICAL PAIRS OF VALUES (CORRECT) FROM 6,10 12,5 18,0 0,15 OR $b\leq18$, $r\leq15$		2 CORRECT WEAK [SEVERAL CORRECT NUMERICAL ANSWERS]
RE-WRITE $r=30-s-t$ $r=30-r$ "$r<30$"		$-3k$ $(-2k)$ $3-k$	$C=10-d$	"$W+2h$ = wages" $W+2h$ [illegible] $2h$ EXPRESSIONS INVOLVING "n_w, n_h" (CORRECT). [illegible] eg $W=20+h2$		ALWAYS/ WHEN "SPECIFIC", eg $A=1$ $B=2$ $C=3$ SOMETIMES etc	SOMETIMES ALWAYS (= MAYBE) SOMETIMES WHEN M = NUMBER	"NO CHANGE" (NOT "NOTHING") AMBIGUOUS eg	PUNCTUATION $f: 3g+2+1$ $f: [illegible]$	IMPLICIT "COST" "AMOUNT"	$1<x<2$ $x=5/6$ (NOT $x=1$ $x=2$)	TWO CORRECT PAIRS BUT CATEGORY [illegible] $b+r$ = NUMBER OF PENCILS EXPRESSIONS INVOLVING n_b, n_r (CORRECT)	"NO"	3 AMBIGUOUS
		$k3$ $(k2)$ $3k$ $(2k)$ $k+3$ $(k+2)$ $k\times3$ $(k\times2)$	UNSYSTEMATIC 2134, etc OR LESS THAN 4 VALUES (MORE THAN ONE)	$W=20+h$ $h\rightarrow w$ $20+h$ $w=w+h$ w = wages + h TOTAL WAGES $=W+h$				"INCREASES" OR "INCREASES BY ...." eg "f increases by 2" $f+2$		"NUMBER OF CAKES + BUNS" (GENERAL)		$b+r$ = amount of money $b+r=90$ or $=90$ $b+r\leq90$		4 LETTER AS OBJECT GENERAL
				$W+h.$ wh $w\cap h$				STATIC-INCORRECT $a=b+5$ $a=2b+3$ $a=3b+3$ ALL "OTHER" EXPRESSIONS in b or g	$f=3g+3$ $=3g-1$ $f=5g+\ldots$ $f=6g\ldots$			$b+r$		5 LETTER AS OBJECT, GENERAL / ASSOCIAT.
		-3 (-2) 3 (2)	$C=4$ 3 2 1 0	ONE PAIR OF VALUES (CORRECT) $W=28$ $h=4$ etc OR 28,4 etc				$a=2$	$f=6$	(1 bun, 1 cake) $4p+3p$ $7p$		ONE PAIR OF VALUES (CORRECT) $b=6$, $r=10$ etc [EXCEPT "6, 10"]		6 ONE CORRECT NUMERICAL ANSWER
		$k\rightarrow h$ (9) i (h) 8 (7) 9 (8)		(NUMBERS CORRECT) $2h=24W$ $4h=28W$, etc OR $24=W+2h$ $28=W+4h$ etc				$(a\rightarrow c)$ etc	$(f\rightarrow l)$ etc	$25p$ (NOT $\times18p$)		$6b+10r$ etc (NUMBERS CORRECT) $b=30$ $r=60$ etc (6 BLUE + 10 RED)	CONSISTENT 10 NOT 5 NOT ANY 5 NUMBER	7 LETTER AS OBJECT, SPECIFIC/ CONSISTENT
10	*	*	* (ALL ARITH ERRORS OTHER THAN $C=5$)	$W=20$, $h=2$ (IMPLIES $W=20$ $h=2$) OR $20W+2h$ £22 OR $W=10h$, etc $22=W+h$	*			$a=5$	$f=7$	4 CAKES, 3 BUNS 7 cakes	30	$5b+6r$ $b=5$, $r=6$ $b<r$ $b+r=11$ $b+1=r$ etc	5 10 OR 10 5	8 LETTER AS OBJECT, SPECIFIC/ OBVIOUS
		k	INCLUDE $c<d$	INCLUDE ALL ARITH ERRORS eg $W=28$ $h=8$ $W+h>20$ $28=W+8h$ $2h=4W$ $28W+8h$ [illegible]		NEVER	NEVER	"CHANGES" $a\neq b+3$ $a<$ EXPRESSION	$f\neq3g+1$ $f<$ EXPRESSION	INCLUDE $7cb$ $7c,b$ $7c+b$ etc $4\times c+3\times b$		($b+r$ = PENCILS bought) INCLUDE ALL ARITHMETICAL ERRORS eg $b=10$ $r=6$	INCLUDE 5 NOT 50 5 NOT 10	9 OTHERS
														0 BLANK

TEST 1 YEAR 2 NUMBER OF CHILDREN = 1128 ALGEBRA 1976

VAR	1	2	3	4	5	6	7	8	9	0
1	83.2	0	0	0	0	0	0	5.2	5.8	5.8
2	64.7	0	3.9	.3	0	7.2	2.5	5.4	5.7	10.4
3	74.4	0	.1	0	0	0	0	7.9	9.8	7.8
4	66.3	0	1.6	0	0	0	0	3.6	26.1	2.4
5	2.1	1.9	1.5	62.1	0	0	.1	24.1	1.0	7.3
6	61.3	0	5.9	.2	1.2	20.0	.4	2.8	4.0	4.2
7	21.7	0	3.9	.2	40.8	17.3	0	5.3	5.2	5.6
8	7.6	0	11.5	.1	38.9	16.4	0	6.7	8.5	10.2
9	39.2	0	6.2	.5	11.3	16.0	0	6.9	8.5	11.3
10	92.1	0	0	0	0	0	0	3.6	.5	3.7
11	58.9	0	.1	0	0	0	0	22.3	9.7	9.0
12	24.7	0	5.7	0	0	10.2	4.1	28.7	13.7	12.9
13	85.5	0	.1	0	0	0	0	1.6	1.6	11.3
14	29.1	0	2.7	0	0	0	0	10.7	27.0	30.6
15	80.8	0	.4	0	0	0	0	12.1	.4	6.4
16	78.9	0	.4	0	0	0	0	13.8	.4	6.5
17	51.3	3.0	1.0	4.8	.1	0	.3	6.4	13.9	19.2
18	7.0	.1	14.5	3.6	27.2	9.5	1.1	6.0	12.1	18.9
19	95.3	.1	0	0	0	0	0	1.7	0	2.9
20	90.3	.3	.2	0	0	0	.9	2.2	1.9	4.3
21	57.4	.4	22.3	1.0	3.5	0	0	2.7	7.4	5.1
22	27.7	25.9	20.4	.8	2.0	0	0	4.3	11.0	8.0
23	9.3	14.5	.8	.4	3.3	.1	2.7	25.4	20.3	23.1
24	0	1.2	2.5	1.5	0	5.0	20.1	48.6	11.9	9.3
25	2.2	0	.4	0	0	6.1	9.1	64.7	7.9	9.5
26	49.3	.1	.4	0	0	0	0	15.2	13.6	21.5
27	43.5	.1	0	0	0	0	0	22.3	7.0	27.0
28	47.3	0	17.8	1.7	0	0	5.7	6.7	6.2	14.6
29	77.4	0	.1	.7	9.5	0	0	4.7	.4	7.2
30	29.3	0	6.6	1.1	45.3	.1	0	5.5	.7	11.3
31	37.7	0	9.0	1.1	18.7	0	0	7.4	4.7	21.5
32	40.2	0	3.2	1.3	26.4	0	0	7.0	2.0	19.9
33	15.2	0	3.4	25.4	19.7	0	0	6.8	4.3	25.3
34	10.6	0	3.1	19.9	21.9	0	0	7.6	5.0	31.8
35	29.6	.4	2.6	16.3	11.2	0	0	9.4	5.4	25.2
36	26.5	0	1.7	6.4	23.5	0	0	8.4	2.5	31.0
37	11.4	1.3	2.8	28.5	9.9	0	0	6.9	5.0	34.0
38	27.4	0	2.2	0	0	0	0	24.1	17.7	28.5
39	62.6	0	0	0	0	0	0	22.6	0	14.8
40	33.2	1.0	2.0	.8	.1	2.6	14.2	10.5	13.7	22.1
41	7.1	13.9	1.2	1.2	0	42.6	0	2.0	9.1	23.0
42	2.0	0	1.1	5.5	12.5	.2	.8	13.6	21.1	43.3
43	65.0	0	0	0	0	0	0	15.6	3.6	15.8
44	58.4	0	14.3	0	0	0	0	0	10.5	16.8
45	10.5	0	16.2	0	0	0	0	0	55.5	17.7
46	9.4	10.6	.9	4.9	22.1	.7	1.9	8.1	15.1	26.4
47	3.0	1.4	.5	9.7	29.4	2.2	1.3	1.7	14.3	36.4
48	3.5	3.8	6.3	2.3	0	2.1	1.2	35.5	19.0	26.4
49	4.3	.2	1.1	0	0	0	0	1.7	33.5	59.3
50	2.2	.2	.6	10.6	2.0	3.3	5.6	4.4	27.0	44.1
51	1.9	0	1.0	0	0	0	24.8	30.4	2.3	39.6
52	0	0	0	0	0	0	0	0	0	100.0
53	0	0	0	0	0	0	0	0	0	100.0
54	0	0	0	0	0	0	0	0	0	100.0
55	0	0	0	0	0	0	0	0	0	100.0
56	0	0	0	0	0	0	0	0	0	100.0
57	0	0	0	0	0	0	0	0	0	100.0
58	0	0	0	0	0	0	0	0	0	100.0

TEST 1 YEAR 3 NUMBER OF CHILDREN = 961 ALGEBRA 1976

VAR	1	2	3	4	5	6	7	8	9	0
1	87.9	.1	0	0	0	0	0	3.9	5.0	3.1
2	75.0	0	1.0	.2	0	8.1	1.0	4.2	4.3	6.1
3	83.7	0	0	0	.1	.2	0	4.4	7.7	4.0
4	72.4	0	1.8	0	0	0	0	4.6	19.5	1.8
5	3.4	2.7	2.4	70.7	0	0	.1	15.7	.7	4.3
6	67.7	0	3.5	0	.7	20.0	0	2.2	3.1	2.7
7	36.0	0	3.2	.2	31.4	16.0	0	4.8	5.4	2.9
8	16.8	.1	19.1	.2	31.1	15.2	.1	4.7	6.3	6.3
9	44.5	0	5.4	.3	13.8	16.2	0	6.8	7.1	5.8
10	96.9	0	0	0	0	0	0	1.8	0	1.4
11	73.9	0	0	0	0	0	0	13.4	7.5	5.2
12	40.8	0	3.2	0	0	6.3	2.0	26.0	11.4	10.2
13	92.3	0	0	0	0	0	0	1.1	1.7	4.9
14	41.1	.1	1.6	0	0	0	0	11.6	23.5	22.2
15	91.4	0	0	0	0	0	0	5.4	.7	2.5
16	88.6	.1	0	0	0	0	0	8.1	.6	2.6
17	67.1	1.0	.4	5.3	0	0	0	5.2	9.5	11.4
18	11.8	.2	17.5	2.9	28.0	12.6	.4	5.0	8.6	13.0
19	97.4	0	0	0	0	0	0	1.1	.2	1.2
20	93.8	.2	0	0	0	0	.1	2.1	1.5	2.4
21	67.6	.3	20.0	1.4	.9	0	0	2.1	4.9	2.8
22	36.9	27.0	16.1	1.7	3.3	.1	0	2.8	7.4	4.7
23	23.0	15.1	.7	.6	4.8	0	.4	17.5	15.6	22.3
24	.2	.9	4.0	2.7	0	5.3	23.3	52.1	6.9	4.6
25	3.6	0	.4	0	0	5.2	6.7	72.7	5.6	5.7
26	61.0	0	.5	.1	0	.1	0	14.2	10.0	14.2
27	61.8	0	.1	0	0	0	0	14.3	4.8	19.0
28	63.3	0	14.0	1.4	0	0	2.2	2.9	5.3	10.9
29	86.4	0	.3	.4	7.5	0	0	1.9	.4	3.1
30	44.7	0	9.6	.2	34.2	0	0	4.2	.5	6.6
31	53.3	0	9.5	.7	14.6	0	0	5.1	3.7	13.1
32	59.7	0	2.4	.8	20.3	0	0	4.6	.8	11.3
33	22.8	.2	4.1	27.9	14.5	0	0	4.8	5.4	20.4
34	14.8	0	4.2	24.1	19.4	0	0	4.3	6.6	26.7
35	43.6	.8	2.4	13.3	11.2	0	0	6.0	6.9	15.7
36	46.7	0	1.4	5.1	20.2	0	0	4.6	2.0	20.1
37	17.5	1.7	4.9	28.6	7.6	0	0	2.8	5.4	31.5
38	35.2	0	5.7	0	0	0	0	21.3	15.5	22.3
39	74.6	.1	0	0	0	0	0	16.2	.2	8.8
40	51.1	.9	4.0	.9	0	.9	8.2	8.8	10.3	14.8
41	11.3	18.5	4.0	1.1	0	39.3	0	1.2	8.3	16.1
42	5.3	.1	2.7	12.9	13.9	.1	2.2	11.2	19.1	32.4
43	76.6	0	0	0	0	0	0	11.0	3.1	9.3
44	71.9	0	10.9	0	0	0	0	0	6.8	10.4
45	24.5	0	13.7	0	0	0	0	.1	50.7	11.0
46	21.0	9.5	2.4	4.7	20.2	.4	1.1	8.1	14.7	17.9
47	6.5	2.2	1.6	18.0	26.8	2.5	.2	2.1	15.3	24.9
48	10.7	2.9	8.0	2.8	0	1.8	.6	38.9	15.6	18.6
49	11.8	0	2.4	0	0	0	0	3.6	35.3	46.9
50	10.0	.7	1.1	17.0	.8	2.4	6.2	3.9	22.6	35.3
51	3.3	0	2.3	0	0	[illegible]	21.2	35.1	1.1	36.9
52	0	0	0	0	0	0	0	0	0	100.0
53	0	0	0	0	0	0	0	0	0	100.0
54	0	0	0	0	0	0	0	0	0	100.0
55	0	0	0	0	0	0	0	0	0	100.0
56	0	0	0	0	0	0	0	0	0	100.0
57	0	0	0	0	0	0	0	0	0	100.0
58	0	0	0	0	0	0	0	0	0	100.0

TEST 1 YEAR 4 NUMBER OF CHILDREN = 731 . ALGEBRA 1976

VAR	1	2	3	4	5	6	7	8	9	0
1	82.1	0	0	0	0	.1	0	4.4	5.7	7.7
2	68.4	0	3.7	.3	0	5.5	.1	2.9	6.2	13.0
3	78.4	0	0	0	0	.3	0	2.9	9.8	8.6
4	71.7	0	2.3	0	0	0	0	2.5	20.2	3.3
5	5.9	3.7	2.2	66.3	0	0	0	15.2	1.6	5.1
6	68.9	0	6.4	0	2.1	13.7	.1	1.9	3.3	3.6
7	41.0	0	4.1	0	30.4	12.0	0	2.9	4.9	4.7
8	24.5	0	17.6	0	29.4	11.1	0	3.1	5.2	9.0
9	50.2	0	8.1	.3	12.6	10.8	0	2.9	6.6	8.6
10	94.5	0	0	0	0	0	0	2.2	0	3.3
11	73.9	0	0	0	0	0	0	10.9	6.8	8.3
12	49.8	0	7.1	.1	.1	5.5	1.1	15.7	7.8	12.7
13	93.0	0	.1	0	0	0	0	1.1	1.0	4.8
14	40.2	0	2.6	0	0	0	0	12.9	20.7	23.7
15	90.3	0	0	0	0	0	0	5.3	.3	4.1
16	89.9	0	0	0	0	0	0	6.2	.3	3.7
17	74.7	1.1	.4	2.6	0	0	0	2.6	6.2	12.4
18	16.1	0	18.6	1.5	30.5	8.8	.1	5.2	7.0	12.2
19	95.6	.1	0	0	0	0	0	.8	0	3.4
20	92.9	0	.3	0	0	0	.3	.8	1.1	4.7
21	71.8	1.0	12.9	3.4	1.5	0	0	1.0	3.0	5.5
22	44.7	21.9	12.3	2.9	2.2	0	0	1.0	7.1	7.9
23	23.0	17.5	.5	1.0	3.8	.1	.3	16.7	15.3	21.8
24	1.5	1.1	4.8	1.8	0	1.6	36.8	38.2	7.9	6.3
25	5.3	0	.4	.1	0	3.0	7.7	72.4	3.7	7.4
26	69.9	0	.4	0	0	0	0	7.7	4.9	17.1
27	67.4	0	.1	0	0	0	0	8.6	3.3	20.5
28	63.5	0	15.6	2.9	0	0	2.5	2.5	2.5	10.7
29	87.4	0	.1	.3	6.0	0	0	1.6	.3	4.2
30	50.9	0	5.5	1.1	34.2	0	0	2.2	.3	5.9
31	54.7	0	18.2	1.1	8.3	0	0	3.0	2.6	12.0
32	65.9	0	3.6	.8	17.4	0	0	2.6	.1	9.6
33	32.0	.3	6.4	23.4	14.5	0	0	2.9	4.7	15.9
34	19.6	0	5.9	27.9	16.1	0	0	1.8	5.5	23.3
35	51.7	.7	4.0	12.2	8.8	0	0	3.0	3.7	16.0
36	55.7	0	1.9	6.4	13.5	0	0	1.4	1.2	19.8
37	25.2	1.1	9.3	23.7	7.7	0	0	1.4	6.6	25.2
38	31.3	0	7.9	0	0	0	0	20.8	16.0	23.9
39	72.0	0	0	0	0	.1	0	16.4	.5	10.9
40	52.9	1.4	3.3	.4	0	1.1	7.8	4.7	10.0	18.5
41	15.5	19.3	3.7	.8	0	35.0	0	.8	8.9	16.0
42	7.9	0	3.0	18.6	12.4	0	1.2	10.5	15.5	30.8
43	75.9	.1	0	0	0	0	0	10.7	2.5	10.8
44	73.2	0	11.9	.1	0	0	0	0	4.1	10.7
45	27.2	0	11.5	0	0	0	0	0	50.1	11.2
46	25.9	7.4	1.9	6.2	17.9	.7	.7	9.3	11.1	19.0
47	13.0	3.1	1.5	16.7	23.3	1.8	.3	4.0	13.1	23.3
48	17.0	4.1	9.3	2.2	0	1.8	.5	25.9	21.1	18.2
49	15.7	0	4.2	0	0	0	0	2.1	30.2	47.7
50	12.2	.7	1.0	17.1	2.5	2.9	4.5	4.4	22.7	32.1
51	4.9	0	1.9	0	0	0	23.5	27.6	2.2	39.8
52	0	0	0	0	0	0	0	0	0	100.0
53	0	0	0	0	0	0	0	0	0	100.0
54	0	0	0	0	0	0	0	0	0	100.0
55	0	0	0	0	0	0	0	0	0	100.0
56	0	0	0	0	0	0	0	0	0	100.0
57	0	0	0	0	0	0	0	0	0	100.0
58	0	0	0	0	0	0	0	0	0	100.0

TEST 1 YEAR 2 NUMBEP OF CHILDREN = 292 ALGEBRA 1977

VAR	1	2	3	4	5	6	7	8	9	0
1	55.5	0	0	0	0	0	0	8.6	15.8	20.2
2	55.5	0	1.7	0	0	1.4	.7	5.5	9.6	25.7
3	53.8	0	0	0	0	0	0	4.8	18.8	22.6
4	61.3	0	.7	0	0	0	0	3.8	28.4	5.8
5	.7	1.7	1.0	59.2	0	0	0	26.4	0	11.0
6	61.3	0	4.8	0	2.4	11.3	0	3.4	6.5	10.3
7	22.9	0	4.8	0	38.0	9.6	0	5.8	8.9	9.9
8	4.5	0	14.0	0	39.7	8.6	0	4.5	13.0	15.8
9	33.9	0	5.5	.3	17.5	9.2	0	5.1	11.3	17.1
10	86.6	0	0	0	0	0	0	3.8	.7	8.9
11	52.4	0	0	0	0	0	0	26.4	8.2	13.0
12	17.5	0	2.1	0	0	7.5	5.5	33.9	17.1	16.4
13	82.9	0	0	0	0	0	0	1.4	3.1	12.7
14	26.0	0	3.8	0	0	0	0	11.3	30.1	28.8
15	83.2	0	0	0	0	0	0	12.7	.3	3.8
16	80.1	0	0	0	0	0	0	15.4	.3	4.1
17	46.6	1.0	.3	3.8	0	0	0	8.6	19.2	20.5
18	7.5	0	11.6	4.5	19.5	14.0	.7	11.0	13.0	18.2
19	90.8	0	0	0	0	0	0	2.1	0	7.2
20	84.2	.3	.7	0	0	0	.7	3.1	2.1	8.9
21	52.1	1.0	24.0	.7	2.1	0	0	3.8	6.2	10.3
22	29.8	21.6	21.2	0	2.4	0	0	3.4	8.2	13.4
23	11.0	11.3	0	0	6.8	0	2.1	34.6	7.2	27.1
24	0	0	0	0	0	0	0	0	0	100.0
25	0	0	0	0	0	0	0	0	0	100.0
26	47.6	0	0	0	0	0	0	13.7	12.7	26.0
27	38.7	0	0	0	0	0	0	22.9	7.5	30.8
28	49.7	0	12.7	2.4	0	0	8.2	3.8	3.8	19.5
29	73.6	0	0	.3	7.9	0	0	6.2	.7	11.3
30	27.7	0	4.1	0	39.7	0	0	5.5	2.4	20.5
31	34.6	0	8.9	1.0	19.5	0	0	6.8	8.2	20.9
32	39.4	0	4.5	0	26.4	0	0	6.2	3.4	20.2
33	18.8	0	.3	22.6	16.4	0	0	6.8	9.2	25.7
34	12.3	0	1.7	17.5	20.9	0	0	7.5	7.9	32.2
35	32.9	.3	0	14.7	7.9	0	0	13.0	4.5	26.7
36	26.4	0	.3	2.7	23.6	0	0	9.9	3.4	33.6
37	9.6	1.0	2.4	28.8	8.2	0	0	6.2	8.2	35.6
38	25.7	0	2.1	0	0	0	0	18.5	19.5	34.2
39	59.6	0	0	0	0	0	0	22.6	.3	17.5
40	29.8	1.0	1.4	.3	0	.3	20.9	9.9	8.9	27.4
41	6.5	11.0	2.1	1.0	0	39.0	0	.3	13.4	26.7
42	2.1	.3	1.4	11.0	12.7	0	.3	16.8	14.0	41.4
43	58.9	0	0	0	0	0	0	16.8	4.8	19.5
44	62.0	0	8.6	0	0	0	0	0	11.3	18.2
45	9.6	0	14.4	0	0	0	0	0	57.2	18.8
46	8.2	9.6	.3	5.8	15.8	.3	2.4	8.9	19.5	29.1
47	2.4	0	0	9.2	24.3	3.8	1.4	2.1	19.5	37.3
48	8.2	1.4	7.2	1.7	0	3.4	1.0	28.8	18.5	29.8
49	1.7	.3	.7	0	0	0	0	2.7	34.6	59.9
50	2.1	0	.3	14.4	.7	3.4	4.8	9.2	20.9	44.2
51	0	0	0	0	0	0	0	0	0	100.0
52	0	0	0	0	0	0	0	0	0	100.0
53	0	0	0	0	0	0	0	0	0	100.0
54	0	0	0	0	0	0	0	0	0	100.0
55	0	0	0	0	0	0	0	0	0	100.0
56	0	0	0	0	0	0	0	0	0	100.0
57	0	0	0	0	0	0	0	0	0	100.0
58	0	0	0	0	0	0	0	0	0	100.0

TEST 1 YEAR 3 NUMBER OF CHILDREN = 246 ALGEBRA 1977

VAR	1	2	3	4	5	6	7	8	9	0
1	75.2	0	0	0	0	0	0	4.5	7.7	12.6
2	72.4	0	2.4	0	0	2.4	.4	4.5	4.5	13.4
3	71.5	0	0	0	0	0	0	3.3	12.6	12.6
4	71.1	0	.8	0	0	0	0	2.8	22.4	2.8
5	3.7	1.6	3.3	67.1	0	0	0	17.9	.4	6.1
6	67.5	0	6.9	.8	1.6	11.8	0	.8	4.5	6.1
7	38.6	0	6.1	0	33.3	9.8	0	1.2	5.7	5.3
8	16.3	0	26.4	0	33.3	10.2	0	1.6	3.3	8.9
9	43.1	0	7.7	.4	20.7	9.8	0	2.8	5.7	9.8
10	92.3	0	0	0	0	0	0	3.7	.4	3.7
11	68.7	0	0	0	0	0	0	15.4	7.7	8.1
12	38.6	0	4.1	0	0	5.3	3.3	26.8	10.6	11.4
13	93.9	0	0	0	0	0	0	.4	1.2	4.5
14	39.4	0	4.1	0	0	0	0	13.4	19.9	23.2
15	92.7	0	0	0	0	0	0	5.7	0	1.6
16	89.8	0	0	0	0	0	0	7.3	0	2.8
17	69.1	.8	.8	4.1	0	0	0	6.1	6.1	13.0
18	11.4	0	15.4	1.2	36.2	6.5	.4	7.7	7.7	13.4
19	95.9	0	0	0	0	0	0	.8	0	3.3
20	91.9	1.2	0	0	0	0	.4	1.6	.8	4.1
21	72.0	.8	15.4	.4	.8	0	0	1.2	4.5	4.9
22	39.8	26.8	14.6	0	3.7	0	0	2.0	6.1	6.9
23	19.5	15.9	0	0	4.1	0	.8	36.6	3.7	19.5
24	0	0	0	0	0	0	0	0	0	100.0
25	.4	0	0	0	0	0	0	0	0	99.6
26	67.5	0	0	0	0	0	0	8.1	9.8	14.6
27	62.2	0	0	.4	0	0	0	13.8	4.5	19.1
28	58.1	0	15.9	3.7	0	0	3.7	2.8	2.4	13.4
29	88.6	0	0	0	7.3	0	0	.4	0	3.7
30	43.9	0	6.1	0	37.4	0	0	2.0	0	10.6
31	55.7	0	12.2	.4	13.4	0	0	1.6	4.1	12.6
32	62.2	0	3.3	.4	19.9	0	0	2.0	1.2	11.0
33	23.2	0	3.3	29.7	16.7	0	0	1.6	7.3	18.3
34	20.3	0	1.6	20.3	21.5	0	0	2.0	7.3	26.8
35	45.9	.8	2.8	17.1	7.7	0	0	4.9	4.9	15.9
36	48.0	0	.4	3.7	19.9	0	0	2.0	3.7	22.4
37	19.1	2.8	8.1	23.2	7.7	0	0	2.0	9.3	27.6
38	31.7	0	5.3	0	0	0	0	27.2	11.4	24.4
39	76.8	0	0	0	0	0	0	13.4	.4	9.3
40	44.3	.8	4.1	0	0	2.0	17.9	2.4	9.3	19.1
41	8.9	19.5	2.4	1.6	.4	43.1	0	0	8.9	15.0
42	4.9	0	1.6	15.9	14.2	0	.4	15.9	13.0	34.1
43	74.8	0	0	0	0	0	0	11.4	2.0	11.8
44	70.3	0	10.2	0	0	0	0	.4	9.3	9.8
45	15.9	0	13.8	0	0	0	0	.4	58.9	11.0
46	16.3	13.0	.4	6.5	18.3	.4	1.2	11.0	15.4	17.5
47	5.3	1.6	0	17.9	28.5	2.0	1.2	3.7	13.8	26.0
48	15.0	.4	8.9	2.8	0	4.1	.8	28.5	21.1	18.3
49	8.5	0	.4	0	0	0	0	3.7	28.0	59.3
50	4.5	.4	0	19.1	1.2	5.7	8.1	8.1	16.7	36.2
51	0	0	0	0	0	0	0	0	0	100.0
52	0	0	0	0	0	0	0	0	0	100.0
53	0	0	0	0	0	0	0	0	0	100.0
54	0	0	0	0	0	0	0	0	0	100.0
55	0	0	0	0	0	0	0	0	0	100.0
56	0	0	0	0	0	0	0	0	0	100.0
57	0	0	0	0	0	0	0	0	0	100.0
58	0	0	0	0	0	0	0	0	0	100.0

TEST 1 YEAR 4 NUMBER OF CHILDREN = 192 ALGEBRA 1977

VAR	1	2	3	4	5	6	7	8	9	0
1	74.5	0	0	0	0	0	0	4.2	10.9	10.4
2	76.0	0	1.0	.5	0	1.6	0	3.6	5.2	12.0
3	73.4	0	0	0	0	0	0	1.6	13.5	11.5
4	75.5	0	.5	0	0	0	0	6.3	16.1	1.6
5	3.6	3.1	3.1	76.0	0	0	0	8.9	0	5.2
6	69.3	0	6.3	.5	2.6	13.0	0	2.1	3.6	2.6
7	42.7	0	7.3	0	28.1	12.5	0	4.7	2.6	2.1
8	22.9	0	17.7	0	27.6	9.4	0	7.3	7.3	7.8
9	50.5	0	6.3	0	13.5	12.0	0	5.2	5.7	6.8
10	94.3	0	0	0	0	0	0	4.2	0	1.6
11	70.3	0	0	0	0	0	0	15.6	8.9	5.2
12	54.7	0	3.6	0	0	2.1	.5	18.8	12.5	7.8
13	92.7	0	.5	0	0	0	0	2.1	2.1	2.6
14	40.6	0	2.1	0	0	0	0	16.7	22.4	18.2
15	90.1	0	0	0	0	0	0	6.8	0	3.1
16	92.2	0	0	0	0	0	0	4.7	0	3.1
17	82.3	.5	0	4.2	0	0	0	.5	4.7	7.8
18	18.8	0	17.2	2.6	31.3	9.4	1.0	5.7	3.6	10.4
19	95.3	0	0	0	0	0	0	2.1	0	2.6
20	94.8	0	0	0	0	0	.5	1.6	.5	2.6
21	69.3	1.0	21.4	.5	.5	0	0	2.1	2.1	3.1
22	43.2	25.5	13.0	.5	5.2	0	0	2.6	4.2	5.7
23	31.8	17.7	.5	1.0	6.3	0	.5	18.2	6.3	17.7
24	0	0	0	0	0	0	0	0	0	100.0
25	0	0	0	0	0	0	0	0	0	100.0
26	76.0	0	0	0	0	0	0	7.3	6.8	9.9
27	71.4	0	0	0	0	0	0	13.0	4.2	11.5
28	62.5	0	22.4	1.0	0	0	3.1	.5	1.6	8.9
29	89.6	0	0	1.0	6.3	0	0	1.0	.5	1.6
30	47.9	0	4.2	0	40.6	0	0	1.0	0	6.3
31	56.3	0	9.4	.5	12.5	0	0	1.0	10.4	9.9
32	71.4	0	3.1	.5	19.3	0	0	.5	0	5.2
33	27.6	0	2.1	30.2	14.6	0	0	.5	11.5	13.5
34	26.0	0	3.6	22.4	17.7	0	0	1.6	13.5	15.1
35	51.0	0	6.8	21.4	7.3	0	0	1.0	3.6	8.9
36	57.3	0	1.0	8.9	17.7	0	0	.5	3.6	10.9
37	17.2	2.6	7.8	24.0	9.4	0	0	1.0	18.2	19.8
38	28.6	0	14.6	0	0	0	0	18.8	16.1	21.9
39	80.2	0	0	0	0	0	0	11.5	0	8.3
40	56.8	1.0	3.6	2.1	0	.5	12.0	3.6	8.9	11.5
41	15.1	21.4	6.3	1.6	0	31.8	0	1.6	8.3	14.1
42	7.3	0	5.7	17.7	19.3	.5	0	13.0	11.5	25.0
43	75.0	0	0	0	0	0	0	10.9	3.6	10.4
44	73.4	0	10.9	0	0	0	0	0	6.8	8.9
45	26.0	0	9.9	0	0	0	0	0	55.7	8.3
46	30.7	9.9	0	6.3	16.1	1.0	0	6.8	10.4	18.8
47	6.8	1.6	0	24.5	24.5	1.0	0	4.2	14.6	22.9
48	24.5	3.6	4.7	7.3	0	1.6	1.0	25.5	13.5	18.2
49	14.1	0	2.1	0	0	0	0	4.2	28.1	51.6
50	12.5	1.0	1.0	17.7	0	3.6	8.3	6.3	13.0	36.5
51	0	0	0	0	0	0	0	0	0	100.0
52	0	0	0	0	0	0	0	0	0	100.0
53	0	0	0	0	0	0	0	0	0	100.0
54	0	0	0	0	0	0	0	0	0	100.0
55	0	0	0	0	0	0	0	0	0	100.0
56	0	0	0	0	0	0	0	0	0	100.0
57	0	0	0	0	0	0	0	0	0	100.0
58	0	0	0	0	0	0	0	0	0	100.0

PHI CORRELATION MATRIX ALGEBRA 1976 n = 2923

96.3	19																														
94.6	10	215																													
92.5	20	425	184																												
90.1	13	267	279	[illegible]																											
87.3	15	212	157	286	[illegible]																										
85.6	16	242	189	328	[illegible]	196																									
85.[illegible]	1	171	[illegible]	221	228	200	256																								
83.5	29	263	214	313	307	254	298	215																							
79.3	3	153	232	185	253	241	241	479	200																						
72.7	43	191	218	241	257	[illegible]	233	176	267	182																					
7[illegible].4	4	137	234	252	292	279	315	239	247	207	222																				
7[illegible].2	2	149	286	242	[illegible]	278	245	437	223	379	209	280																			
69.9	39	236	214	307	[illegible]	269	313	261	283	260	295	299	285																		
68.7	11	155	254	223	215	245	278	216	224	193	261	313	276	303																	
67.9	44	212	190	214	292	243	278	221	281	199	313	314	270	350	301																
66.4	6	174	176	214	194	170	195	264	205	191	173	277	293	233	253	223															
66.3	21	[illegible]	163	364	277	249	281	267	280	234	241	286	296	279	301	282	242														
65.8	17	189	199	286	[illegible]	338	395	253	318	266	259	360	385	360	344	369	323	344													
61.5	22	[illegible]	163	337	[illegible]	243	267	219	278	248	240	300	312	276	288	290	242	777	342												
59.9	26	[illegible]	222	224	294	268	299	235	284	263	281	318	305	354	336	342	240	313	419	329											
57.9	28	146	161	219	273	240	270	210	235	192	215	285	298	311	306	311	194	384	375	356	342										
57.2	27	198	231	[illegible]	329	310	347	266	304	289	319	379	361	395	385	413	305	348	492	378	742	389									
54.9	32	181	195	266	296	276	392	251	376	249	281	353	348	350	346	398	286	434	476	426	428	352	502								
48.4	31	157	162	235	261	231	243	224	320	221	235	331	282	277	294	341	257	392	378	396	351	321	416	689							
47.0	40	163	195	233	265	262	306	307	271	298	310	379	434	498	366	408	334	375	522	398	461	422	541	478	407						
44.6	9	[illegible]	114	165	151	149	170	122	171	142	147	180	183	178	183	140	419	150	223	168	154	149	211	178	144	198					
42.6	36	143	151	232	236	245	276	205	292	238	251	325	301	325	324	361	270	338	448	351	418	336	476	627	508	493	189				
42.1	35	135	149	184	199	198	226	194	266	198	219	267	245	247	271	293	279	292	331	295	332	252	383	448	386	384	194	435			
41.8	30	141	142	195	231	223	241	203	308	212	243	291	293	276	292	330	245	346	385	342	361	304	419	650	542	428	145	524	425		
38.3	12	121	179	180	232	234	263	229	243	243	231	310	361	300	325	333	302	356	462	363	386	380	455	452	369	530	182	462	389	404	
37.6	14	111	145	166	233	191	214	210	261	229	231	256	270	281	313	270	231	293	316	286	358	306	406	325	280	398	112	313	314	308	359
37.4	38	140	151	176	215	224	261	234	228	246	264	310	283	323	320	336	295	297	361	330	412	304	479	388	347	474	164	405	365	381	429
34.7	23	120	133	184	205	232	254	218	214	234	234	285	323	304	310	345	300	305	423	312	383	336	444	400	339	524	175	434	345	373	469
33.2	7	92	146	158	194	194	225	179	230	226	205	304	303	272	314	301	419	326	379	335	347	316	423	428	350	485	95	446	410	409	487
29.1	41	118	136	147	189	165	213	177	198	180	231	280	271	285	261	308	238	278	331	312	333	281	399	366	316	419	130	373	324	350	394
23.7	33	92	121	127	157	176	196	165	169	183	194	260	239	254	245	271	256	232	296	259	280	242	330	349	362	377	170	380	430	373	387
22.2	48	87	109	135	141	146	166	154	166	181	214	198	245	210	216	249	192	243	279	258	272	264	328	310	273	363	141	350	305	293	370
21.1	45	93	116	127	165	154	179	158	193	190	221	224	261	263	246	314	230	236	300	264	317	284	374	359	301	420	122	377	328	354	407
19.7	37	83	49	120	138	150	161	134	180	155	154	219	198	192	212	239	197	200	244	211	238	217	278	339	367	307	195	378	434	364	310
18.8	46	90	88	126	127	146	165	132	154	165	160	204	213	206	198	233	171	206	263	223	270	213	289	289	232	319	125	268	249	269	291
16.4	8	77	102	115	131	154	168	136	164	173	163	218	235	216	231	225	250	233	297	235	268	237	324	321	264	355	307	364	353	338	396
15.7	34	79	99	108	136	136	144	143	168	155	162	211	196	176	232	211	209	221	257	218	243	224	286	333	336	301	170	331	346	348	336
12.4	18	68	81	99	117	131	145	140	130	159	165	185	199	165	191	192	206	231	260	240	244	228	277	280	256	314	167	322	325	313	362
10.8	49	62	83	94	115	126	140	133	136	156	176	182	212	183	204	214	194	195	244	212	240	210	279	273	228	337	138	313	301	302	356
9.5	50	57	77	87	107	120	126	129	134	142	167	156	185	179	176	199	173	181	226	198	217	186	259	244	218	316	169	312	292	283	334
7.5	47	55	68	[illegible]	[illegible]	108	116	107	108	132	127	135	162	155	157	167	157	172	185	184	192	179	221	196	161	254	122	240	235	231	253
7.1	5	54	66	73	82	105	113	100	108	121	130	158	162	149	166	164	162	185	196	204	198	195	217	223	239	277	160	247	261	258	295
5.6	42	40	58	63	81	93	[illegible]	98	108	114	133	139	149	154	149	165	158	171	176	184	185	175	202	200	211	248	150	251	241	240	280
4.1	25	31	40	45	51	68	73	62	54	80	84	92	112	120	109	116	99	103	130	128	127	152	147	149	158	188	71	167	158	174	191
1.7	24	25	-3	16	34	33	[illegible]	32	36	47	56	49	50	56	42	67	47	76	71	86	79	62	85	86	92	112	38	108	99	94	94
FACILITY VAR		19	10	20	13	15	16	1	29	3	43	4	2	39	11	44	6	21	17	22	26	28	27	32	31	40	9	36	35	30	12

P̄HI CORRELATION MATRIX ALGEBRA 1976 contd

37.6	14																				
37.4	38	394																			
34.7	23	384	419																		
33.2	7	367	415	463																	
29.1	41	322	431	379	[illegible]																
23.7	33	312	356	424	422	445															
22.2	48	349	344	391	422	332	351														
21.1	45	343	396	421	421	412	377	372													
19.7	37	279	383	426	345	389	412	312	314												
18.8	46	246	296	333	346	274	278	303	291	268											
16.4	8	265	353	375	649	422	394	308	341	373	314										
15.7	34	273	424	415	364	313	426	306	332	370	297	353									
12.4	18	298	324	368	362	316	400	355	374	321	275	438	374								
10.8	49	319	321	384	373	346	341	406	404	330	297	422	336	435							
9.5	50	251	368	361	345	293	368	399	331	343	306	429	315	426	434						
7.5	47	239	262	291	278	262	292	324	267	258	566	317	264	351	374	365					
7.1	5	281	293	323	331	243	376	356	330	322	266	396	374	437	422	401	378				
5.6	42	263	273	317	[illegible]	267	365	354	352	363	235	376	359	429	443	532	359	441			
4.1	25	184	185	[illegible]	212	159	241	279	222	235	160	253	238	235	277	367	249	299	405		
1.7	24	74	119	117	[illegible]	98	127	143	115	116	60	114	61	120	160	166	84	130	175	295	
FACILITY VAR.		14	38	23	7	41	33	48	45	37	46	8	34	18	49	50	47	5	42	25	24

CSMS

Graphs (Final draft)

DO NOT WRITE IN THIS MARGIN

NAME ....

CLASS .... BOY or GIRL ....

SCHOOL .... DATE OF BIRTH ....

---

1. What do you understand by a graph? Put a tick by the answer you prefer.

   a) A graph has points plotted on squared paper.

   b) A graph is a diagram to show how children come to school.

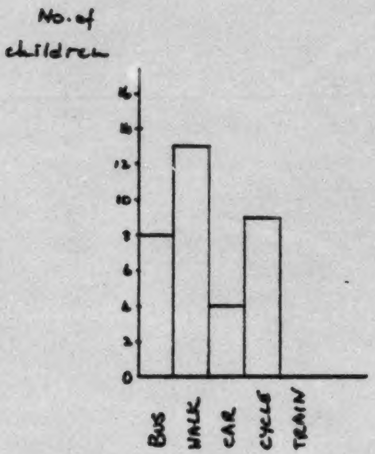
   c) A graph shows the relation between two quantities. (1)

   d) A graph is two axes with numbers on them.

   e) A graph is a way of showing numerical results clearly. 1.1

---

2. This is a diagram to show how class 1.B travels to school. For example, 8 children travel by bus, and 13 walk.



(2)

How many children come by car? 2.1

How many children cycle to school? 2.2

The other 5 children come by train. Enter this on the diagram. 2.3

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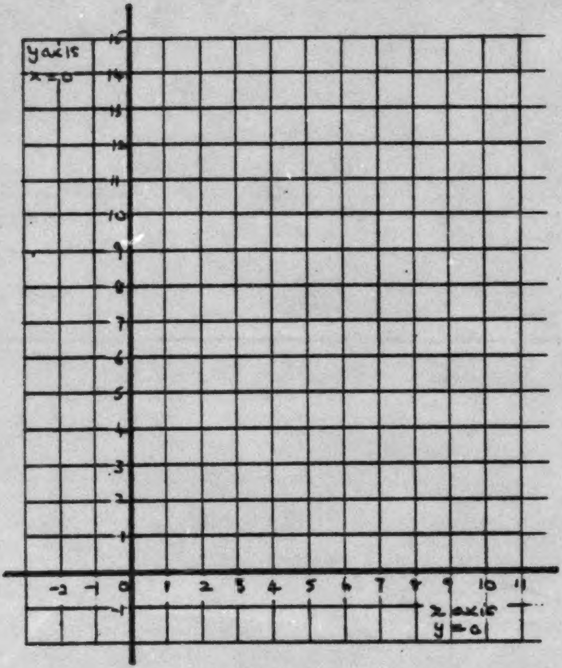
MARGIN

3.


y axis
x = 0
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
-1
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
x axis
y = 0

Plot the points (2,5), (3,7), (5,11).

These points lie on a straight line. Draw the line.

Find some other points on the line, and write them down

.... 3.1 (3)

The point (4.6, 10.2) also lies on the line. Mark its position approximately. 3.2 (4)

Plot the point (1½, 4). 3.3 (5)

How many points do you think lie on the line altogether?.........

.... 3.4 (6)

Are there any points on the line between the points (2,5) and (3,7)?

.... 3.5

If so, how many? 3.6 (7)

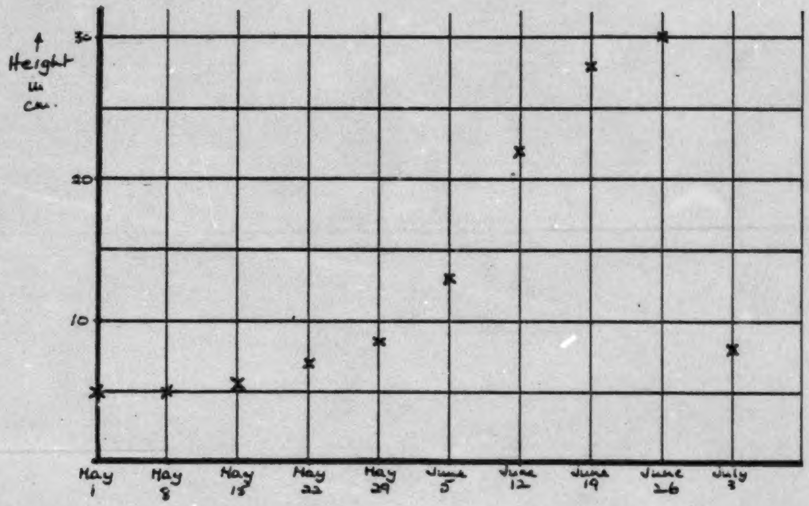
The point (6, □) lies on the line. What number does □ stand for?

.... 3.7 (8)

The point (△, 15) lies on the line. What number does △ stand for?

.... 3.8 (9)

4. Jane planted some flowers in her garden, and measured one particular one every week. This is a graph of its growth:



Did it grow all the time at the same rate? 4.1 (10)

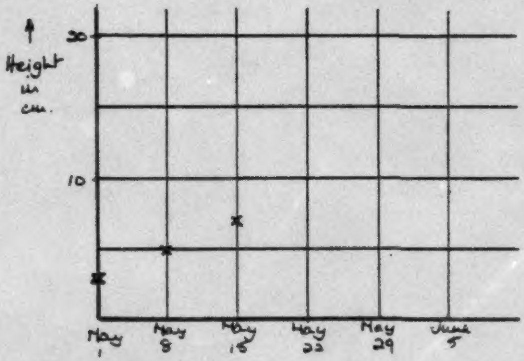
During which week did it grow fastest? The week ending 4.2 (11)

Estimate the height of the flower half way through the week ending on June 5 .... 4.3 (12)

What do you think happened in the week ending July 3? ....

.... 4.4 (13)

This plant did grow at the same rate all the time for 5 weeks. Complete the graph:



(14)
4.5

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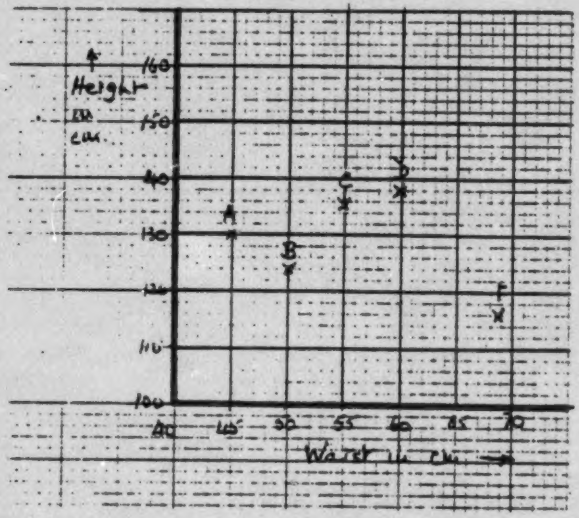
5. Ann drew a diagram to show the height and waist measurement of herself (A), Brian (B) Cheryl (C), David (D) and Frank (F).



What is Brian's height? 5.1 (15)

What is Brian's waist? 5.2 (16)

Mark in George (G) whose height is 150 cm
and waist is 70 cm 5.3 (17)

What can you say about Frank's appearance? ....
.... 5.4 (18)

Should we join up the points on the diagram? 5.5 (19)

Why do you think this? ....
.... 5.6

What can you say about the height of a child whose waist is 65 cm ?
.... 5.7 (20)

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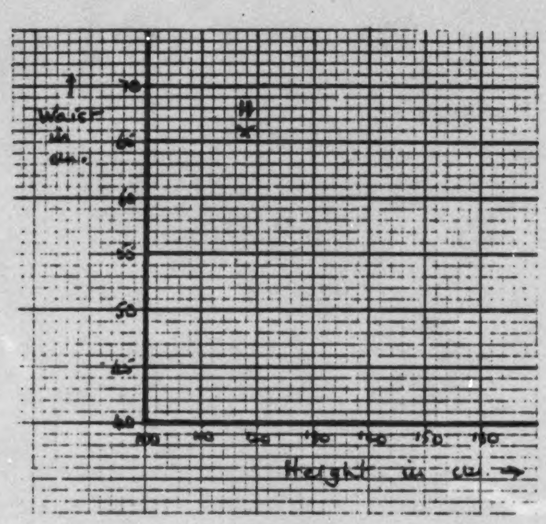
5. (Contd)

Helen drew her diagram like this, and marked on it her height and waist at the point (H).



Mark in John (J) whose height is 140 cm and waist 60 cm. 5.8

What can you say about Helen's appearance?...........

.... 5.9 (21)

DO NOT
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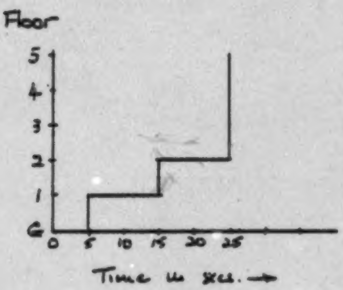
6. A lift can travel from the ground floor to the fifth floor of a building. It takes approximately 5 seconds to travel from one floor to the next, and stops for 10 seconds at each floor.

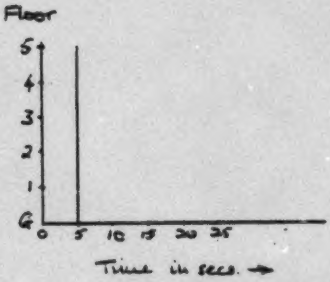
Which of the following graphs represents the journey of the lift when it starts from the ground floor, G, and stops at the 1st., 2nd., and 5th. floors only?

Put a tick by the graph you choose.

(a)

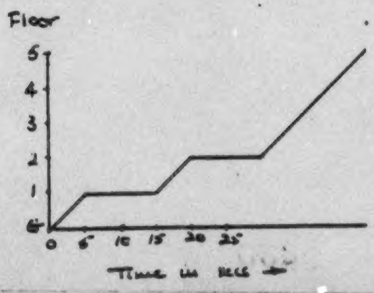


(b)

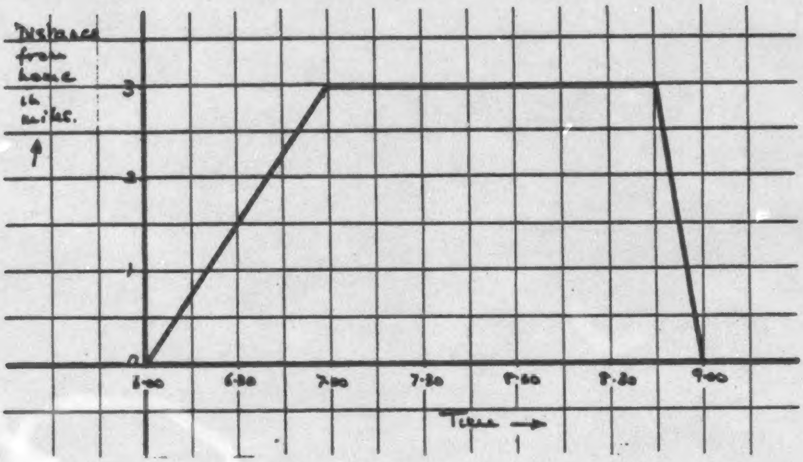


(22)

(c)



7. John leaves home to go to a disco in Cambridge 3 miles away. He walks one way and takes the bus the other way. This is a graph of his journey:



At what time did he get to the disco? 7.1 (23)

What do you think he was doing between 7.00 and 8.45? ....

.... 7.2 (24)

How was he travelling between 8.45 and 9.00? ....

.... 7.3 (25)

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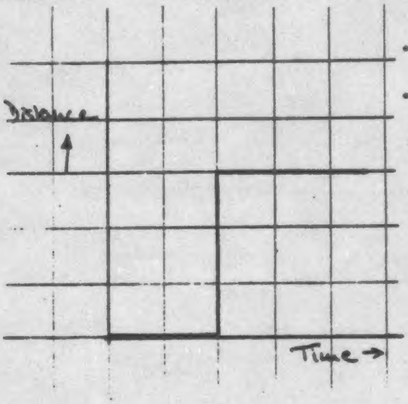
7(ii). Which of the graphs below represent journeys?

Describe what happens in each case.

(a)



....

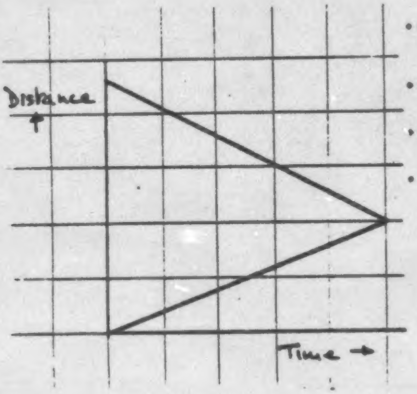
....

....

....

7.4 (26)

(b)



....

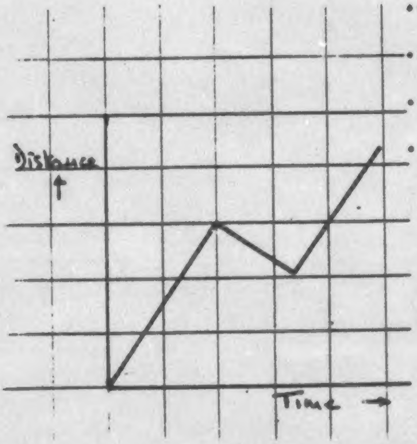
....

....

....

7.5 (27)

(c)



....

....

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....

7.6 (28)

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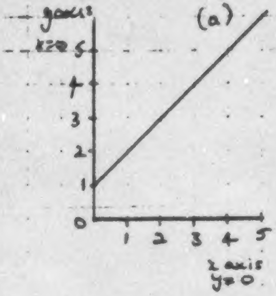
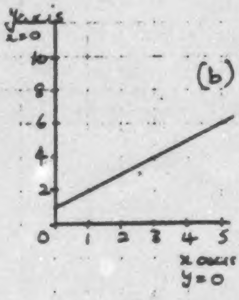
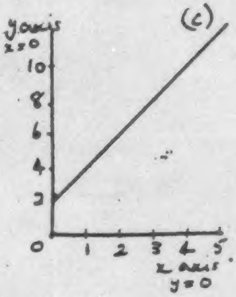
8. Two of the lines (a), (b) and (c) represent the same information.



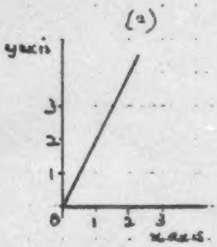


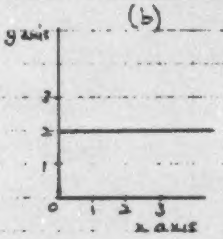


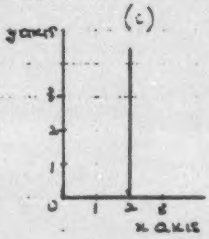
Put a ring round the letters of the two lines that represent the same information: a b c 8.1 (29)

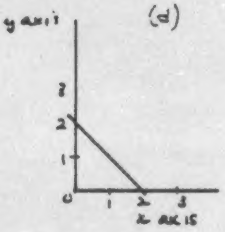
Which of the graphs below represents the line y=2x?

Put a tick by the graph you choose.









What are the equations of the other lines?

(a) 8.2 (30)

(b) 8.3 (31)

(c) 8.4 (32)

(d) 8.5 (33)

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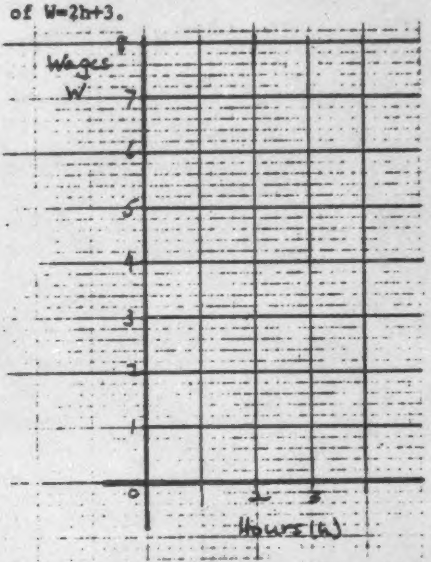
9. A boy's wages, £W, in a supermarket are made up as follows: basic wage £3, plus £2 an hour for every extra hour, h, that he works. So W=3+2h, which can be written as W=2h+3.

When h=1, what is W? 9.1 (34)

When h=2, what is W? 9.2 (35)

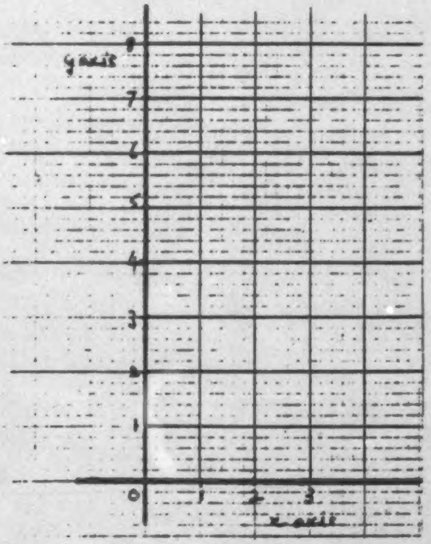
Plot this information on the grid below, and draw the graph of W=2h+3.



9.3 (36)

9.4 (37)

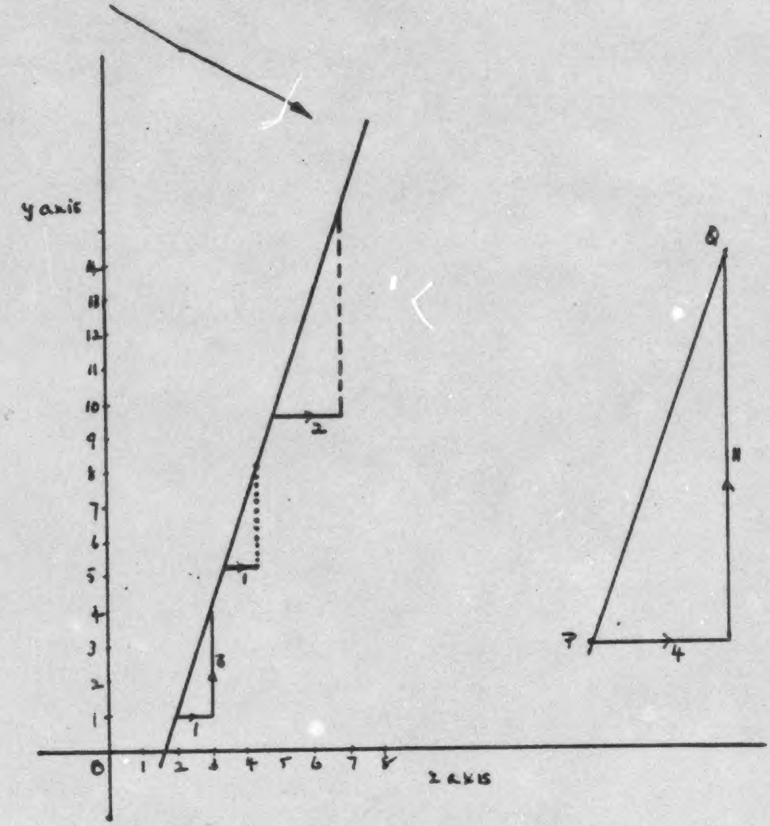
Plot the graph of y=2x+3, on the grid below:



9.5 (38)

9.6 (39)

10. Here is the graph of $y=3x-5$.



How many units long is the line of dots on the graph? 10.1 (40)

How many units long is the line of dashes? 10.2 (41)

Is the line joining PQ parallel to the first line? 10.3

Why do you think this? .... (42)

.... 10.4

11.

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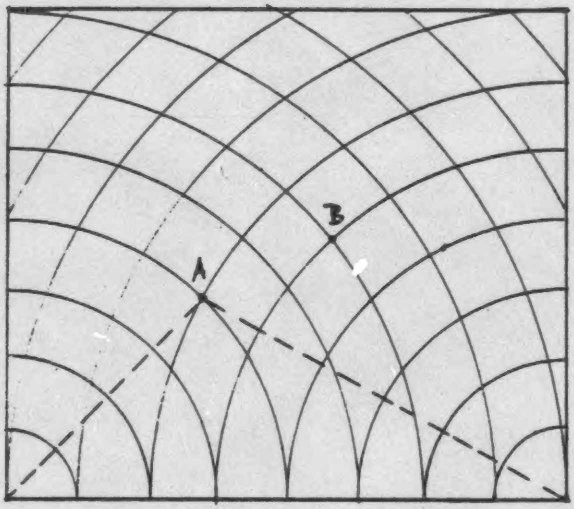
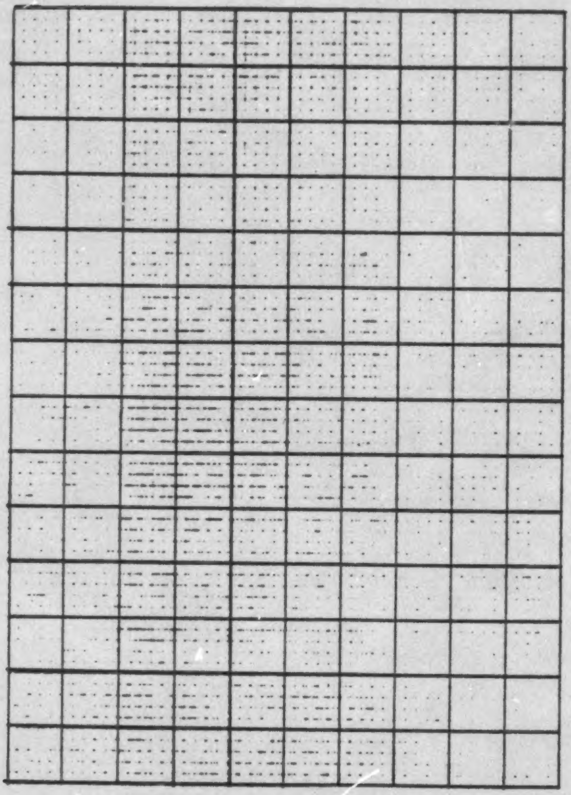


The co-ordinates of the point A are given as (4,6) in this system. What are the co-ordinates of the point B in this system? 11.1

(43)

Plot the point (7,6). 11.2

12. **The points (20,15), (-14,3) and (5,-12) have to be plotted on this piece of graph paper. Draw two axes, and mark a suitable scale on each. You need not actually plot the points.**



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12.

(44)

(45)

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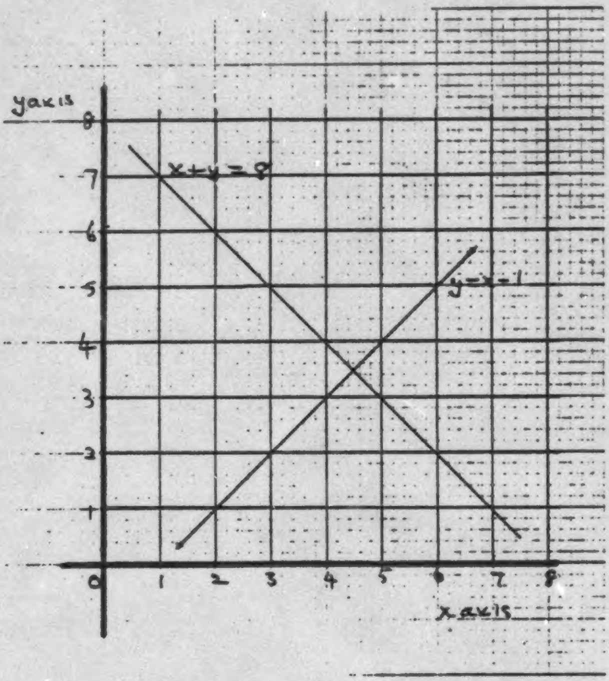
13. Here are the graphs of $y=x-1$ and $x+y=8$.



Find an x and y that make $y=x-1$ true. x=......... 13.1 (46)

y= 13.2

How many pairs of values of x and y are there that make $y=x-1$ true? 13.3 (47)

Find an x and y that make $x+y=8$ true. x= 13.4 (48)

y= 13.5

Find an x and y that make both $y=x-1$ and $x+y=8$ true at the same time. x= 13.6 (49)

y= 13.7

How many pairs of values of x and y are there that make both $y=x-1$ and $x+y=8$ true at the same time? 13.8 (50)

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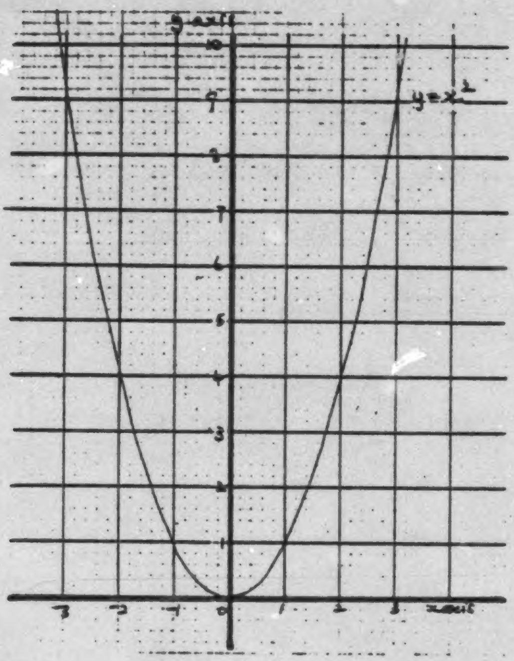
14.



This is the graph of $y=x^2$

What is the approximate value of y when x=3? 14.1 (51)

What is the approximate value of x when y=5? 14.2 (52)

What is the approximate value of 2.5^2? 14.3 (53)

What is the approximate value of $\sqrt{3}$? 14.4 (54)

Draw the straight line that would enable you to find the values of x that make $x^2=x+3$ true. What is its equation? 14.5 (55)

What are the approximate values of x that make $x^2=x+3$ true? 14.6 (56)

MARKING SCHEME GRAPHS

The marking scheme provides codes for the correct answer, and for a variety of incorrect answers. All wrong answers for which a code is not provided should be coded 9.

ALL QUESTIONS OMITTED : CODE 0

ALL 'WRONG' ANSWERS, EXCEPT WHERE INDICATED : CODE 9

Question	Code 1 Answer	Other codes and comments
PAGE 1		
1.1	c	a – 7 Between 1 and 3 answers b – 9 ticked, code 'best' d – 8 e – 2 More than 3 — 9
2.1 2.2 2.3	4 9 All correct Plot 5	2 or less correct — 9
PAGE 2		
3.1	Plot (2,5)(3,7)(5,11) rough accuracy acceptable	1 or more (x,y) reversal — 2
OMIT 3.2		
3.3	Plot (4.6,10.2) rough accuracy acceptable	(x,y) reversal — 2 (4,6) and (10,2) plotted–8
3.4	Plot (1½,4) rough accuracy acceptable	(x,y) reversal — 2
3.5	'Infinite'	'Hundreds', 'lots' etc. — 2 Finite number > 1 — 5 1 — 7
3.6 3.7	'Infinite'	'Hundreds','lots', etc. — 2 Finite number > 1 — 5 1 — 7 No — 8
3.8	13($12 \leq x \leq 14$)	
3.9	7($6 \leq x \leq 8$)	
PAGE 3		
4.1	NO	
4.2	June 12th	OK if 'June 5th to June 12th'
4.3	10($8 \leq x \leq 13$)	2 or 2½ (scale ignored) — 8
4.4	'died', 'wilted', etc. OK if discontinuity implied	'got smaller' — 9
4.5	points in straight line rough accuracy acceptable	

Question	Code 1 Answer	Other codes and comments
PAGE 4 5.1	124	$120 \leq x \leq 130$ 8
5.2	50	
5.3	Plot (70,150)	
5.4	Short and(fat or plump)	height 116, waist 68 — 2 plump — 2
5.5 5.6	No + good reason 'No relationship' 'all different people'	Not on line or curve — 5 No + 'no E' — 8 No - 'only shows one thing, only shows height' — 9 Yes +'looks neater' { 'shows pattern' 'helps you to compare children' } — 7 'Could draw a 'line of best fit' 2
5.7	Nothing/You can't say	Could/might/ought to/ should be quite tall, or probably quite tall, about x provided $138 \leq x \leq 150$ — 3 {is/would be quite tall is about x, $(138 \leq x \leq 150)$} — 7 {is/would be quite small is about x, $(138 \leq x \leq 150)$ or could/might/ought to/ should be/probably/quite small} 8 No such child — 9 is fat — 5
PAGE 5 OMIT 5.8 5.9	short and fat Height 118, waist 66	Tall and thin — 8
PAGE 6 6	c	a – 8 b – 9
PAGE 7 7.1	7.00	
7.2	'Dancing' 'Disco'	
7.3	'By bus' 'fast'	

Question	Code 1 Answer	Other codes and comments
PAGE 8 7.4	'At rest, then a long way in no time, then at rest'	Stopped walked, stopped (no reference to time) — 9 'Not a journey' or other correct but partial answer — 2 Spatial answer e.g. 'Upwards then along', North then East', Left then right' — 8
7.5	'Not a journey as goes backwards in time' (or 'Journeys by 2 people')	'not a journey' or correct but partial answer — 2 Spatial answers e.g. 'Right then left' etc. — 8
7.6	'Forward then back a little way, then forward' 'Distance travelled gets smaller-impossible'	'Is a journey' or other correct but partial answer — 2 Spatial answers e.g. 'Right then left', 'Climbing a mountain' etc. — 8 Fast, slow, fast — 3
PAGE 9 8.1	a and b	a and c — 8
8.2	y = 2x (or tick against (a))	any wrong equation — 8
8.3	y = 2	x = 2 — 7 any other equation — 8
8.4	x = 2	y = 2 — 7 any other equation — 8
8.5	x + y = 2	xy = 2 — 7 any other equation — 8
PAGE 10 9.1	5	4 — 8
9.2	7	5 — 8
9.3	Plot (1,5)(2,7) (Rough accuracy O.K.)	Plot (2,3) — 7 Plot (1,4)(2,5) — 8
9.4	Draw correct line W = 2h + 3	Line through 2 points 'wrongly' plotted — 8 Any other line — 7
9.5	Plot (1,5)(2,7) (Rough accuracy OK) (OK if line drawn without plotting points)	Plot (2,3) — 7 Plot (1,4)(2,5) — 8
9.6	Draw correct line y = 2x + 3	Line through 2 points wrongly plotted — 8 any other line — 7

Question	Code 1 Answer	Other codes and comments
PAGE 11 10.1	3 (only)	
10.2	6(only)	
10.3	No + reason involving 11 and 4	Yes, because same distance apart — 4 Yes, it looks as though it is — 5 No, because it is larger — 7 No, one is straight } — 8 No, one is a triangle } No, measures distance between — 2
PAGE 12 11.1 11.2	Both (6,5); and (7,6) plotted correctly (above B)	One answer only correct — 2
PAGE 13 12.1	Sensible scale on both axes e.g. large square ≡ 5 or ten units	At least 1 axis with odd scale e.g. large square ≡ 3, 4, 6, 7 units — 4 Scale marked in spaces $\cdots\underset{5}{\vdash\!-}\underset{10}{\vdash\!-}\cdots$ — 5 Scale starting at 1, not 0 — 7 Scale different on either side of origin — 8
12.2	Negative numbers allowed for on both axes	Negative numbers allowed for but axes along edge of page — 2 Negative numbers on 1 axis only — 7 Negative numbers ignored — 8
PAGE 14 13.1 13.2	Any pair with y=x-1 e.g. (2,1),(3,2) etc.	x, y reversed — 2
13.3	Infinite	'Hundreds', 'lots' etc. — 2 5 — 4 Finite number — 5 1 — 7 0 — 8
13.4 13.5	Any pair with x+y=8 e.g. (4,4) (5,3) etc.	
13.6 13.7	(4.5, 3.5)	(3.5, 4.5) — 2 $4.2 \leq x \leq 4.8$ or $3.2 \leq y \leq 3.8$ — 3
13.8	1 '2' OK if above answer OK (interpreted as one pair being two points)	

Question	Code 1 Answer	Other codes and comments
PAGE 15		
14.1	9	$8 \leq y \leq 10$ — 2 6 — 8
14.2	$2.2 \leq x \leq 2.4$	$2.1 \leq x < 2.2$, $2.4 < x < 2.5$ — 2 2.5 — 8
14.3	$6.0 \leq x \leq 6.4$ except 6.25	6.25 — 2 $5.5 \leq x < 6.0$, $6.4 < x \leq 7$ — 3 5 — 8
14.4	$1.6 \leq x \leq 1.8$	1.5 — 8 9 — 7
14.5	$y = x + 3$ (equation or graph)	
14.6	$2.2 \leq x \leq 2.4$ and $-1.4 \leq x \leq -1.3$	$x = 1.4$ approx — 3 One only of solutions-2

Year 2 Graphs **Frequency of Codes** 1976

Var No.	Code 1	2	3	4	5	6	7	8	9	0
1	12.3	63.5				1.8	2.8	15.9	3.6	
2	89.5								10.5	
3	94.9	3.1				1.0			1.0	
4	36.0	1.5				23.7		31.4	7.5	
5	79.2	4.1				13.6			3.1	
6	6.4	12.6	0.3		63.8	15.9	0.8	0.3		
7	4.1	6.4			27.5	8.2	36.8	15.4	1.5	
8	49.1					12.6			38.3	
9	69.4					16.2			14.4	
10	96.9					1.3			1.8	
11	67.1					4.9			28.0	
12	65.3					4.6		2.6	27.2	0.3
13	85.6	0.5				4.1			9.5	0.3
14	86.9					2.6			10.3	0.3
15	61.2					0.5		33.7	4.4	0.3
16	93.3					0.8		0.3	5.4	0.3
17	92.0					4.9			2.8	0.3
18	82.5	5.4				2.6		0.3	9.0	0.3
19	54.8	1.5				11.8	21.6		10.0	0.3
20	11.6		9.8		12.9	14.7	17.7	27.2	5.9	0.3
21	59.4	2.8				3.6		15.4	18.5	0.3
22	60.9					2.6		31.9	4.4	0.3
23	64.3					3.1			32.1	0.5
24	67.1	0.5				4.9			27.0	0.5
25	77.1	0.8				4.6			17.0	0.5
26	9.5	5.4				44.5		16.2	23.7	0.8
27	11.1	1.5				48.8		16.2	21.1	1.3
28	14.7	8.5				44.5		12.1	18.8	1.5
29	56.3					4.6		30.8	6.4	1.8
30	18.8	0.3				33.7		33.4	11.8	2.1
31	16.7					32.6	1.8	38.6	8.0	2.3
32	15.2					32.6	0.8	42.2	6.7	2.6
33	5.9					45.5	0.3	37.3	8.5	2.6
34	43.7					9.3		13.4	30.6	3.1
35	41.1					10.0		10.5	34.7	3.6
36	19.8	2.3				24.4	16.5	3.9	28.3	4.9
37	7.5	0.3		0.3		69.4	5.4	6.2	4.4	6.7
38	17.7			1.0		35.7	15.7	1.5	21.6	6.7
39	10.5					61.4	11.1	5.4	3.1	8.5
40	28.8					13.4			48.6	9.3
41	26.5					14.4			49.6	9.5
42	3.1	2.3		10.5	26.2	20.6	1.3	5.4	20.3	10.3
43	19.5	13.4				7.5			45.0	14.7
44	32.9	2.3		10.8	0.3	11.8	1.0	1.5	10.0	29.3
45	27.2	1.8				11.8	2.8	18.5	8.2	29.6
46	15.7	3.1				13.6			13.1	54.5
47	1.8	1.8		1.8	17.0	17.2	3.1	0.3	0.5	56.6
48	15.7	0.8				15.2			10.3	58.1
49	9.0					22.9			8.5	59.6
50	8.2					22.9			8.5	60.4
51	20.6	0.3				4.1			12.9	62.2
52	10.3	1.3				5.4		3.9	14.9	64.3
53	1.8	4.6	0.3			4.1		5.1	13.6	70.4
54	2.8					1.5		5.4	13.6	76.6
55	0.3					2.3			6.9	90.5
56						1.3			6.9	91.8

Year 3 Graphs Frequency of Codes 1976

Var No.	Code									
	1	2	3	4	5	6	7	8	9	0
1	10.1	71.2				2.4	4.7	9.6	1.9	0.2
2	90.6								9.3	0.2
3	91.2	5.2				1.2			2.2	0.2
4	27.9	0.3				22.1		41.8	7.7	0.2
5	76.9	4.5				15.3		0.2	2.9	0.2
6	6.2	5.1			69.2	16.7	2.0		0.7	0.2
7	4.0	3.0			22.1	10.4	41.8	17.7	0.8	0.2
8	41.8					13.3			44.6	0.3
9	61.6					17.0			21.0	0.3
10	96.8					0.5			2.5	0.2
11	67.3	0.3				2.2			30.0	0.2
12	65.7					5.2		1.3	27.6	0.2
13	88.2	0.7				4.4			6.6	0.2
14	88.4					3.4			8.1	0.2
15	63.6					1.2		31.5	3.5	0.2
16	86.7					1.2			11.8	0.3
17	89.4					6.1			4.2	0.3
18	77.6	6.2				2.5		0.3	13.0	0.3
19	46.1	1.2			0.2	12.5	29.0	0.7	9.9	0.5
20	10.3	0.2	7.4		10.4	14.5	18.9	29.8	7.9	0.7
21	55.6	3.9				2.0		20.2	17.7	0.7
22	57.9					3.0		32.7	5.4	1.0
23	64.8					2.0			32.0	1.2
24	65.2	0.7				3.7			29.3	1.2
25	75.9	1.3				2.4		0.2	19.0	1.2
26	8.4	7.4				41.8		9.3	30.3	2.9
27	9.3	4.9				44.6		6.7	31.5	3.0
28	17.2	7.1			0.2	40.2	0.2	9.6	22.6	3.0
29	55.2					6.4		31.0	4.2	3.2
30	18.2	0.2				37.7		23.7	16.2	4.0
31	10.3	0.5				36.0	3.4	34.8	10.9	4.0
32	9.8	0.5				37.4	3.0	34.0	11.3	4.0
33	3.9					48.8		30.8	12.5	4.0
34	36.5					10.3		12.5	36.2	4.5
35	35.4					11.4		10.6	37.9	4.7
36	24.4	4.0				20.9	15.8	6.1	23.7	5.1
37	12.6	0.3				64.5	8.1	5.9	3.2	5.4
38	17.5	2.4				38.0	15.0	2.0	19.7	5.4
39	12.0	0.2				63.1	12.8	3.7	2.4	5.9
40	34.7						13.5		44.9	6.9
41	31.5	0.5				13.6			47.3	7.1
42	3.2	1.2		8.9	26.1	22.1	1.2	5.9	23.6	7.9
43	31.0	12.6				6.2			39.6	10.6
44	49.8	0.8		7.9	0.5	9.9	0.7	0.8	6.4	23.1
45	35.7	0.5				10.3	1.9	23.6	5.1	23.1
46	22.6	2.4				12.3			15.8	47.0
47	4.7	1.5		6.6	15.2	18.7	2.5	0.2	0.7	50.0
48	23.9	0.3				14.6			9.9	51.2
49	16.5	0.2	0.2			17.7		0.5	12.5	52.5
50	16.7					19.0		0.2	10.8	53.4
51	26.6	0.2				1.5		0.2	15.7	55.9
52	12.1	3.4				1.9		5.9	18.5	58.2
53	6.6	4.9	2.7			4.0		5.4	10.1	66.3
54	6.9	0 .2				2.4		4.2	15.3	71.0
55	0.7					4.4			10.1	84.8
56	0.3	1.0				1.2			11.8	85.7

Year 4 Graphs Frequency of Codes 1976

Var No.	Code 1	2	3	4	5	6	7	8	9	0
1	12.8	71.7				1.2	4.1	9.0	1.2	
2	94.2								5.8	
3	89.1	6.5				1.0	0.2		3.1	
4	34.6	1.2				20.1		35.1	9.0	
5	80.1	7.5				10.2			2.2	
6	19.6	9.7			51.6	18.2	0.2		0.7	
7	11.1	7.7			21.3	11.1	34.4	13.1	1.2	
8	51.3					13.3			35.4	
9	68.8					15.5			15.7	
10	99.3					0.2			0.5	
11	79.7	0.2				0.7			19.4	
12	74.1					1.2		1.5	23.2	
13	89.6					5.6			4.8	
14	93.7					1.0			5.3	
15	73.6					0.7		23.5	2.2	
16	91.3					1.2			7.5	
17	95.2					4.1			0.7	
18	88.1	2.7				1.0			8.2	
19	54.2	3.4				13.1	23.7		5.6	
20	19.9		12.6		8.7	15.3	11.9	24.7	7.0	
21	69.0	6.3				1.9		13.1	9.7	
22	67.8					1.7		26.6	3.9	
23	75.8					2.2			22.0	
24	74.3	0.2				5.3			20.1	
25	86.7	0.7				2.9			9.7	
26	15.0	7.7				40.2		12.6	24.2	0.2
27	15.7	1.7				41.9		14.3	26.2	0.2
28	25.2	11.4				36.8		11.4	15.0	0.2
29	64.2					2.4		28.8	4.4	0.2
30	26.9	0.2				32.9		30.5	8.7	0.7
31	14.3					33.2	2.4	42.6	6.8	0.7
32	14.0					32.9	2.2	41.6	8.5	0.7
33	5.1					41.2		45.3	7.7	0.7
34	60.3					5.1		8.7	25.2	0.7
35	61.3					4.8		7.5	25.7	0.7
36	35.8	2.2				13.8	18.6	2.7	25.9	1.0
37	19.9	0.2				56.2	8.7	8.7	4.6	1.7
38	23.0	0.5				29.3	21.8	1.2	22.5	1.7
39	16.2					60.8	14.0	5.1	2.2	1.7
40	36.1					9.2			53.0	1.7
41	36.8					10.4			51.1	1.7
42	9.2	2.7		14.0	28.8	13.6	1.7	1.7	25.9	2.4
43	38.0	12.1				7.7			38.5	3.6
44	61.3	0.7		13.3	0.2	7.3	0.2	1.7	6.1	9.2
45	65.4					7.3	4.6	9.0	4.6	9.2
46	35.1	7.5				22.5			13.6	21.3
47	9.7	4.4		5.6	19.6	30.8	4.8	0.2	1.2	23.7
48	36.8	1.7				28.1			9.0	24.5
49	24.9	0.2	1.0			37.8		0.2	10.7	25.2
50	28.3					38.0			8.5	25.2
51	56.2	1.0				1.7			13.6	27.6
52	31.0	8.2				2.4		9.0	20.1	29.3
53	9.0	8.2	3.4			4.4		14.3	17.2	43.6
54	14.3					3.1		6.1	25.4	51.1
55	1.5					6.1			14.0	78.5
56		1.5				0.2			15.3	83.1

TEST 4 YEAR 2 NUMBER OF CHILDREN = 170 GRAPHS 1977

VAR	1	2	3	4	5	6	7	8	9	0
1	12.9	61.2	0	0	0	2.9	7.6	4.1	10.0	1.2
2	87.1	0	0	0	0	0	0	0	11.8	1.2
3	87.6	7.1	0	0	0	1.8	0	0	2.4	1.2
4	20.6	1.2	0	0	0	20.6	0	41.8	14.7	1.2
5	70.0	4.7	0	0	0	13.5	0	0	10.6	1.2
6	12.4	5.3	0	0	62.4	14.1	2.4	0	2.4	1.2
7	10.0	1.8	0	0	15.9	7.1	34.7	25.3	4.1	1.2
8	35.3	0	0	0	0	8.8	0	0	54.7	1.2
9	64.7	0	0	0	0	11.8	0	0	22.4	1.2
10	93.5	0	0	0	0	2.4	0	0	2.9	1.2
11	59.4	0	0	0	0	1.2	0	0	37.6	1.8
12	62.4	0	0	0	0	7.1	0	1.2	28.2	1.2
13	90.0	0	0	0	0	2.4	0	0	6.5	1.2
14	78.2	0	0	0	0	2.9	0	0	17.6	1.2
15	46.5	0	0	0	0	.6	0	48.2	3.5	1.2
16	78.2	0	0	0	0	2.9	0	0	17.6	1.2
17	88.2	0	0	0	0	5.3	0	0	5.3	1.2
18	77.1	0	0	0	0	2.9	0	.6	18.2	1.2
19	39.4	.6	0	.6	3.5	9.4	28.8	.6	15.3	1.8
20	12.4	.6	9.4	0	11.8	10.0	14.7	22.4	17.1	1.8
21	57.6	0	0	0	0	2.9	0	18.2	19.4	1.8
22	54.1	0	0	0	0	.6	0	34.1	8.8	2.4
23	53.5	0	0	0	0	2.9	0	0	40.6	2.9
24	52.9	0	0	0	0	4.7	0	0	39.4	2.9
25	73.5	0	0	0	0	3.5	0	0	19.4	3.5
26	4.1	5.3	0	0	0	32.4	0	13.5	48.2	3.5
27	4.7	4.7	0	0	0	39.4	0	7.6	44.1	3.5
28	9.4	15.9	0	0	0	33.5	0	11.8	28.8	3.5
29	50.6	0	0	0	0	4.1	0	35.3	6.5	3.5
30	31.8	0	0	0	0	29.4	0	21.2	13.5	4.1
31	15.9	0	0	0	0	31.8	7.6	30.0	10.6	4.1
32	17.1	0	0	0	0	34.1	5.9	29.4	9.4	4.1
33	7.6	0	0	0	0	39.4	0	35.9	12.9	4.1
34	35.3	0	0	0	0	10.0	0	14.1	35.9	4.7
35	34.1	0	0	0	0	10.6	0	11.8	38.8	4.7
36	14.7	1.2	0	0	0	21.8	18.2	6.5	32.9	4.7
37	5.9	0	0	0	0	67.1	8.8	9.4	4.1	4.7
38	12.4	0	0	0	0	32.4	14.1	2.9	32.9	5.3
39	9.4	0	0	0	0	59.4	15.9	7.1	2.9	5.3
40	45.3	0	0	.6	0	7.1	0	0	41.8	5.3
41	27.1	0	0	0	0	7.1	0	0	60.0	5.9
42	1.2	1.8	0	11.2	8.2	8.8	2.9	5.3	54.1	6.5
43	17.1	11.8	0	0	0	2.4	0	0	61.2	7.6
44	43.5	0	0	15.3	2.4	14.1	1.2	1.2	6.5	15.9
45	35.9	1.2	0	.6	0	14.1	5.9	21.2	4.7	16.5
46	25.3	3.5	0	0	0	11.3	.6	0	28.8	30.6
47	10.0	1.8	0	6.5	24.1	15.9	5.3	1.2	2.4	32.9
48	33.5	0	0	0	0	14.7	0	0	14.7	37.1
49	18.2	1.2	0	0	0	21.2	0	0	20.0	39.4
50	22.9	0	0	0	0	18.2	0	0	18.8	40.0
51	28.2	0	0	0	0	1.8	0	.6	25.9	43.5
52	11.2	1.8	.6	0	0	2.4	0	7.1	30.6	46.5
53	4.1	5.3	2.4	0	0	5.3	0	10.0	21.2	51.8
54	2.4	0	0	0	0	7.6	0	4.1	26.5	59.4
55	.6	0	0	0	0	4.1	0	0	24.1	71.2
56	0	0	0	0	0	.6	0	0	24.7	74.7

```
TEST  4  YEAR 3   NUMBER OF CHILDREN =  161   GRAPHS  1977

 VAR     1       2       3       4       5       6       7       8       9       0

   1    8.7    72.0      0       0       0      .6     8.1     7.5     3.1      0
   2   99.7      0       0       0       0       0       0       0     9.3      0
   3   82.6    13.7      0       0       0     1.9       0       0     1.9      0
   4   31.7     1.2      0       0       0    16.8       0    31.1    19.3      0
   5   67.7     9.3      0       0       0    16.6       0       0     4.3      0
   6    6.2     9.3      0       0    64.0    16.8      .6       0     3.1      0
   7    3.1     6.2      0       0    20.1     9.9    31.1    21.1     2.5      0
   8   34.8       0      0       0       0    19.9       0       0    45.3      0
   9   59.0       0      0       0       0    24.2       0       0    16.8      0
  10   95.0       0      0       0       0      .6       0       0     3.7     .6
  11   64.6       0      0       0       0       0       0       0    34.8     .6
  12   62.1       0      0       0       0     5.0       0     1.9    30.4     .6
  13   87.0       0      0       0       0     1.9       0       0    10.6     .6
  14   75.2       0      0      .6       0      .6       0       0    23.0     .6
  15   65.2       0      0       0       0       0       0    29.8     3.1    1.9
  16   87.0       0      0       0       0     1.2       0       0     9.9    1.9
  17   89.4       0      0       0       0     6.2       0       0     2.5    1.9
  18   78.9       0      0       0       0     1.2       0       0    18.0    1.9
  19   40.4       0      0       0     5.0     5.6    29.6       0    17.4    1.9
  20   17.4       0   11.2       0    12.4    14.9    13.7    18.6     9.9    1.9
  21   64.0       0      0       0       0      .6       0    14.3    19.3    1.9
  22   52.2       0      0       0       0     1.2       0    37.9     6.8    1.9
  23   69.6       0      0       0       0       0       0       0    28.6    1.9
  24   70.8       0      0       0       0      .6       0       0    26.7    1.9
  25   79.5       0      0       0       0       0       0       0    18.6    1.9
  26    9.3     3.7      0       0       0    34.2       0     9.9    46.0    1.9
  27   13.0      .6      0       0       0    42.9       0     6.2    37.9    1.9
  28   27.3     3.7      0       0       0    38.5       0     7.5    21.1    2.5
  29   50.9       0      0       0       0     1.9       0    36.0     7.5    3.7
  30   21.7       0      0       0       0    32.9       0    25.5    14.3    5.6
  31   11.8       0      0       0       0    31.1      .6    38.5    12.4    5.6
  32   10.6       0      0       0       0    37.9     1.2    35.4     9.3    5.6
  33    2.5       0      0       0       0    44.7       0    40.4     6.8    5.6
  34   38.5       0      0       0       0     9.9       0    13.0    32.9    5.6
  35   34.2       0      0      .6       0     9.3       0     9.3    41.0    5.6
  36   21.1       0      0       0       0    18.0    18.0     1.9    34.8    6.2
  37   11.2       0      0       0       0    64.6    10.6     6.2     1.2    6.2
  38   13.7       0      0       0       0    25.5    25.5      .6    28.6    6.2
  39   10.6       0      0       0       0    67.7     9.9     5.0      .6    6.2
  40   49.7       0      0       0       0     9.9       0       0    34.2    6.2
  41   29.2       0      0       0       0     9.3       0       0    54.7    6.8
  42    4.3       0      0     5.6    16.8     9.3     1.2     3.1    52.8    6.8
  43   28.6    11.8      0       0       0     6.8       0       0    44.7    8.1
  44   49.7       0      0     9.9     2.5    11.2     1.2      .6    10.6   14.3
  45   39.8       0      0       0       0    11.2     5.6    20.5     8.7   14.3
  46   26.1     6.2      0       0       0    19.3       0       0    20.5   28.0
  47    4.3     1.9      0     7.5    25.5    25.5     2.5       0     1.9   31.1
  48   31.1       0      0       0       0    20.5       0       0    16.1   32.3
  49   18.6     2.5      0       0       0    27.3       0       0    18.6   32.9
  50   18.6       0      0       0       0    29.2       0       0    19.3   32.9
  51   40.4       0      0       0       0     1.9       0       0    21.7   36.0
  52   20.5     2.5      0       0       0     3.1       0     6.8    28.0   39.1
  53    4.3     8.7    2.5       0       0     6.8       0     8.7    16.8   52.2
  54    6.2       0      0       0       0     5.0       0     4.3    21.7   62.7
  55     .6       0      0       0       0     6.2       0       0    14.9   78.3
  56      0     1.9      0       0       0      .6       0       0    17.4   80.1
```

```
TEST  4  YEAR 4   NUMBER OF CHILDREN =  171      GRAPHS  1977

 VAR     1       2        3        4        5       6       7       8       9       0

  1    14.6    65.5       0        0        0      .6     5.2     6.4     3.5     1.2
  2    92.4       0       0        0        0       0       0       0     6.4     1.2
  3    93.0     3.5       0        0        0     1.8       0       0       0     1.8
  4    46.2       0       0        0        0    15.2       0    29.2     8.2     1.2
  5    78.9     4.7       0        0        0    11.1       0       0     4.1     1.2
  6    17.0    10.5       0        0     50.9    15.8     3.5       0     1.2     1.2
  7    14.6     5.3       0        0     21.1    12.3    31.6    13.5      .6     1.2
  8    50.3       0       0        0        0    11.1       0       0    36.8     1.8
  9    71.3       0       0        0        0    13.5       0       0      .6     1.8
 10    96.5       0       0        0        0      .6      .6       0    19.3     1.8
 11    77.8       0       0        0        0     1.2       0       0    22.2     1.8
 12    73.7       0       0        0        0     2.3       0       0     5.3     1.8
 13    83.3       0       0        0        0     4.1      .6       0    10.5     1.8
 14    86.0       0       0        0        0     1.8       0       0     2.9     1.8
 15    63.2       0       0        0        0      .6       0    31.6     2.9     1.8
 16    85.9       0       0        0        0      .6       0       0     8.8     1.8
 17    93.6       0       0        0        0     3.5      .6       0      .6     1.8
 18    84.2       0       0        0        0     2.9      .6       0    10.5     1.8
 19    48.0       0       0        0      2.3     7.6    25.1      .6    14.6     1.8
 20    22.2       0    13.5        0      7.6     8.8    10.5    26.9     8.8     1.8
 21    67.8       0       0       .6        0     4.1       0    13.5    12.3     1.8
 22    60.8       0       0        0        0     2.3      .6    32.2     1.8     2.3
 23    69.6       0       0        0        0     1.2       0       0    25.7     3.5
 24    70.2       0       0        0        0     2.9       0       0    22.8     4.1
 25    80.1       0       0        0        0     2.3       0       0    13.5     4.1
 26    14.6     4.1       0        0        0    31.6       0     5.8    42.1     4.7
 27    15.8     3.5       0        0        0    32.2       0     4.7    41.5     5.3
 28    24.6    14.6       0        0        0    29.8       0     7.6    19.9     4.7
 29    62.6       0       0        0        0     2.9       0    20.5     8.2     5.8
 30    40.4       0       0        0        0    22.8       0    20.5     9.4     7.0
 31    23.4       0       0        0        0    25.1     5.3    33.3     5.8     7.0
 32    23.4       0       0        0        0    22.2     5.8    35.7     5.8     7.0
 33    11.1       0       0        0        0    34.5       0    40.4     7.0     7.0
 34    49.7       0       0        0        0     5.3       0     9.9    27.5     7.6
 35    49.7       0       0        0        0     8.2       0     8.8    25.1     8.2
 36    36.8     1.2       0        0        0    14.0    10.5     4.7    24.6     8.2
 37    25.7       0       0        0        0    53.2     5.3     5.3     2.3     8.2
 38    30.4      .6       0        0        0    24.6    16.4     1.2    18.7     8.2
 39    22.2       0       0        0        0    54.4     8.2     5.3     1.8     8.2
 40    62.0       0       0        0        0     5.3       0       0    24.0     8.8
 41    38.0       0       0       .6        0     7.0       0       0    45.0     9.4
 42     8.8       0       0      7.6     15.2     5.8     1.8     4.7    46.8     9.4
 43    32.2    13.5       0        0        0     4.7       0       0    36.3    13.5
 44    51.5       0       0     13.5      1.2     8.8      .6     1.8     2.3    20.5
 45    57.3      .6       0        0        0     8.8      .6    10.5     1.8    20.5
 46    44.4     2.9       0        0        0    12.9       0       0     8.2    31.6
 47     7.6     4.1       0     14.6     19.3    18.7     3.5       0       0    32.2
 48    44.4       0       0        0        0    15.2       0       0     7.0    33.3
 49    32.2      .6     1.8        0        0    19.9       0       0    11.1    34.5
 50    29.8       0       0        0        0    22.8       0       0    11.7    35.7
 51    49.7      .6       0        0        0     2.3       0       0     9.9    37.4
 52    31.6     7.6       0        0        0     3.5       0     4.7    14.0    38.6
 53     8.2    15.2     5.8        0        0     4.1       0     8.8    12.9    45.0
 54    21.1       0       0        0        0     2.9       0     5.8    20.5    49.7
 55     2.3       0       0        0        0     4.7       0       0    16.4    76.6
 56     1.8      .6       0        0        0     1.2       0       0    17.0    79.5
```

PHI CORRELATION MATRIX, N OF CASES = 1396

GRAPHS 1976

```
  FAC ITEM
 97.6   10
 91.8   17  156
 91.6    3   36 136
 91.3    2  133 270 127
 89.9   16   39 186 138  91
 89.5   14  158 188 133 194 102
 87.9   13  155 129  70 143  43 231
 82.1   18  168 290 121 188 159 163 176
 79.4   25   92 158  63 158 100 138 104 187
 78.5    5  132 251 502 198 119 163 147 183 100
 70.9   11  164 229  96 155 109 234 216 206 201 168
 68.4   24  162 213 111 184 130 185 206 253 512 195 263
 68.1   12   81 143  69 127  66 157 183 136  88 131 278 192
 67.9   23  150 232 118 186 146 181 168 227 474 200 249 800 200
 65.9    9  160 199 289 197 135 183 191 203 228 318 288 288 168 253
 65.9   15   92 210 153 175 340 164 173 192 217 193 298 294 197 259 269
 61.7   22  104 189 101 123  48 110 141 150 207 208 247 253 142 240 238 188
 60.6   21   53 193 105 137 104 160 150 246 181 192 171 208 132 205 193 186 157
 58.2   29  110 208 141 172 101 194 157 160 200 196 233 295 147 288 290 259 176 210
 51.9   19   86 136  39  84  70  85 176 169  92 149 166 162  49  98 155 110 101 100 123
 48.5   44   78 168 117 161  58 134 149 135 153 218 147 178 130 178 232 150 178 154 204 123
 46.6    8  181 163 158 119 122 150 148 155 198 216 193 224 153 205 554 196 146 168 236 121 167
 45.6   34   88 178 110 107  96 124 163 175 198 152 243 244 154 252 275 208 161 160 213 135 205 228
 44.6   35   85 178 125 117 104 127 160 178 189 164 251 253 142 249 317 223 171 166 217 125 192 267 894
 42.1   45  115 169 132 164  98 191 192 171 222 244 287 280 217 266 328 255 230 215 308 193 600 275 370 357
 33.7   51  192 134 117 106  57 154 129 135 163 180 229 249 166 249 266 234 231 168 287 120 306 245 307 308 463
 33.5   40   43  89  99 132  96  58  72  77  90 126 116 129  98 129 189 106 115  83 127  79 174  82 101  81 191 195
 32.2    4   88 143 186 108 118 125 119 165 217 281 261 304 202 286 317 210 189 181 220 157 175 247 247 251 328 246 161
 31.7   41   77 113  72 127  33 142  87  72 140 146 134 161  86 151 171 135  96 101 180  92 193 123 159 160 220 238 326 161
 29.9   43   62 114  78  73  47  79  93 100 142 124 176 228 118 207 192 139 195 132 220  61 249 177 135 135 264 217 190 193 134
 26.5   36   52 119  82 110  98  99 113 128 160 132 241 226 153 211 253 205 162 148 236  99 212 261 574 584 342 300 103 260 177 193
 25.4   48   81  84 105  68  97 135 136 156 203 145 203 262 139 218 274 222 196 174 218 161 305 255 291 299 418 485 178 288 214 244
 24.4   46   78  83 123  56  62 112 138 130 177 162 201 227 141 211 256 182 200 160 213 139 277 228 285 282 367 453 184 266 234 256
 20.9   30   46 101  53  58  73  95 115  79 126 123 170 197  57 183 187 176 159  90 164 110 167 182 268 253 253 311  98 245 138 133
 19.2   38   65  92 121  53 103 119 130 123 166 157 192 237 135 198 243 181 148 177 232 110 218 200 361 363 320 330 128 279 180 214
 18.8   28   52 130  86 102  45 110  94 119 190 149 203 276 153 264 249 168 217 204 215 146 177 181 217 227 260 284 155 229 191 265
 17.8   50   73  97  86  56  93 122 109 109 157 138 198 223 149 191 235 195 189 144 242 141 257 204 278 257 385 462 206 261 227 245
 17.2   52   71  94  76  86  64 124  75 103 142 145 204 215 153 195 231 195 167 153 240 120 234 247 208 221 341 551 167 206 204 225
 16.9   49   71 106  81  71  87 116 126 115 153 133 217 207 161 183 227 175 202 117 219 152 262 221 255 252 358 455 202 258 235 240
 13.5   20   48  71  51  69  41 114  88  47  44  73 141 114 108 100 133  88 116 129 147 147 104 140 144 135 198 202  93 155  74 141
 13.3   37   48  70  72  83  61  72  67  67 152 102 172 189 110 179 206 153 139 113 200  98 188 212 369 377 301 287 159 248 181 190
 13.3   31   48  23  87  22  60  92  80  44  83 101 129 124 109 128 165 129 108  85 138  83 123 168 215 188 180 262 130 219  88 114
 12.8   39   46  75  92  41  92  96  83  84 152 138 165 196 106 167 199 153 139 138 164  97 202 169 298 302 302 270 123 277 158 194
 12.5   32   45  25 106  32  83  79  80  69 106 103 142 117  97 130 167 126 124  75 110 111 152 153 214 195 198 275 102 225  81 121
 11.7   53   57  51  78  25  55  88  60 112 141 104 160 185 116 169 196 163 200  84 174 126 224 171 233 240 301 413 127 206 162 199
 11.7   27   14  92  69  48  33  73  46 111 146 103 159 189 124 163 148 115 176 110 136 173 115 147 169 158 204 217 106 198 121 181
 11.5    1   13   1  28 -48  -5 -15 -10   4  50  36  38  76  55  46  56  46  40  -2  60  49  13  31  29  32  37  60 -23  53 -14  53
 10.7   26   54  77  71  48  46  80  85 113 147 118 175 189 137 177 189 131 196 131 133 130 156 160 196 179 236 239 172 189 188 149
 10.2    6   53  92 102  36  50  69 110 114 125 130 143 153  79 156 183 153 178  93 171 118 168 176 203 233 252 254 105 288 166 115
  8.0   54   46  48  60  34  63  83  60  75 116 108 164 165  93 128 183 133 160 101 152 134 202 165 236 242 285 367 173 234 226 207
  6.2    7   40  76  77  15  56  77  85  88 108 105 137 135  92 131 171 127 140 102 132 138 120 154 178 195 191 227  89 301 139  99
  5.4   47   37  59  60  -5  37  81  88  86 113  85 131 155  81 129 137 131 142  88 111 113 181  95 203 220 240 267  80 244 145 170
  4.9   42   14  67  45  11  32  45  54  28  75  54 109  98 106  85 108 115  91  89  52  84 135  84  90  94 160 131  90 104 171  53
  4.8   33   35  42  67  21  41  65  21  61  47  92  92  87  31  82 140  76  94  85 102  66  97  99 131 122 141 208 124 153  84 124
   .8   55   14  26  26  27   2  30   8  20  45  46  57  60  61  61  64  47  70  71  75  38  75  62  81  82  88 125  57  60  61  48
   .1   56    5  11  11  11  12  12  14 -31  19 -26  24  25  25 -14  27  27  29  30  32  37  39  40  41  42  44  53  13  55  14  58

            10  17   3   2  16  14  13  18  25   5  11  24  12  23   9  15  22  21  29  19  44   8  34  35  45  51  40   4  41  43
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FAC   ITEM                                           GRAPHS PHI CORRELATION MATRIX, N OF CASES = 1396  Contd
26.5   36
25.4   48  357
24.4   46  328 711
20.9   30  285 358 340
19.2   38  510 417 397 469
18.8   28  254 269 238 238 258
17.8   50  340 666 587 355 444 293
17.2   52  216 431 378 246 308 319 428
16.9   49  331 671 621 317 406 286 785 417
13.5   20  129 169 172 132 138 164 206 170 208
13.3   37  648 366 376 285 493 263 363 245 363 191
13.3   31  215 271 265 360 335 184 315 225 291 149 225
12.8   39  410 376 366 403 781 248 410 268 370 131 618 330
12.5   32  248 280 289 342 331 199 316 240 308 148 214 802 353
11.7   53  234 415 352 222 307 290 412 388 363 142 249 205 265 191
11.7   27  211 192 162 169 196 349 222 236 222 183 192 153 194 178 276
11.5    1   93 129  72  45 142  78 119  37 112  74  96 103 129  80  84  64
10.7   26  218 234 198 227 196 337 239 266 258 217 192 172 193 177 190 488  64
10.2    6  230 290 259 261 291 260 300 234 320 178 263 230 294 235 280 274 129 296
 8.0   54  273 406 369 298 327 278 424 434 418 225 305 244 314 248 517 239 134 292 337
 6.2    7  224 274 243 263 276 249 286 254 289 204 241 233 275 226 248 256  66 268 699 376
 5.4   47  209 356 338 260 279 234 387 287 375 194 252 234 307 245 347 229  33 257 527 411 573
 4.9   42  117 155 132 134 132 135 170 132 179 161 134 125 139 143 132 174  31 210 228 250 271 253
 4.8   33  146 192 184 181 145 148 184 155 193  68 148 228 124 228 199 158 -28 150 112 218 123 164  57
  .8   55   93 115 100 113 141 143 170 131 175  59 155 132 159 137 118 118  18 179 130 213  78 122  54  55
  .1   56   63  64  22  73  77  78  81  83  83  40  96  96  98 100 103 104  45 109 112 128  69  74  78  -8 210

            36  48  46  30  38  28  50  52  49  20  37  31  39  32  53  27   1  26   6  54   7  47  42  33  55  56
```

CSMS

FRACTIONS - 1st & 2nd year

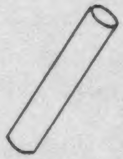
NAME .... TODAY'S DATE ....

BOY or GIRL ....   CLASS ....

SCHOOL .... DATE OF BIRTH ... .
day month year

---

1. This is a stick of rock.
It is to be shared equally
between 5 children. What
fraction of the rock does
each child get? ....



(1)

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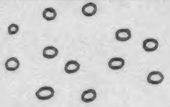
2. John wins $\frac{1}{3}$ of these marbles.
Draw a ring round his marbles.

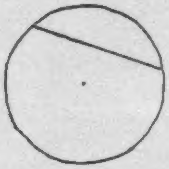


(2)

Jane wins $\frac{2}{3}$ of the marbles.
How many marbles does she win? ....

(3)

3.



Has this circle been split into 2 halves? .....(4).

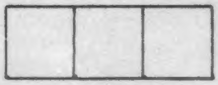
Why do you think this? ....

....

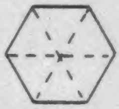
....

---

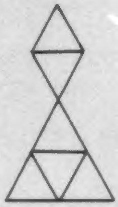
4. Shade in two-thirds of each of these shapes:

a) 

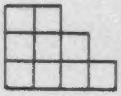
(5)

b) 

(6)

c) 

(7)

d) 

(8)

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5. A piece of ribbon 17cm long has to be cut into 4 equal pieces. Tick the answer you think is most accurate for the length of each piece.

a)

a) 4cm remainder 1 piece

b) 4cm remainder 1cm

c) $4\frac{1}{4}$ cm

d) $\frac{4}{17}$ cm

(9)

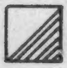
6. 5 eggs in a box of 12 are found to be cracked.

What fraction of the box of eggs is cracked? ....(10)...

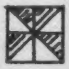
What fraction of the box of eggs is not cracked? ....(11)...

7. Put a tick under each shape in which half has been shaded. The first one is done for you:

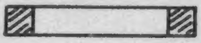
(12)



✓ ____



____



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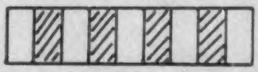
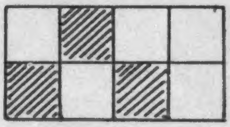
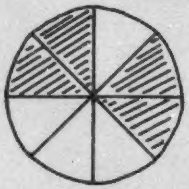


____

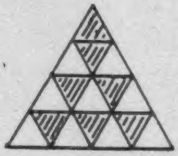
8. What fraction of each of the shapes below is shaded? Write your answer under the shape.



(13) ____



(14) ____



(15) ____

9. Two boys have the same amount of pocket money.
One decides to save $\frac{1}{4}$ of his pocket money, the other decides to save $\frac{5}{20}$ of his pocket money. Tick the answer you think correct.

a) $\frac{5}{20}$ is more than $\frac{1}{4}$

b) $\frac{1}{4}$ is more than $\frac{5}{20}$ (16)

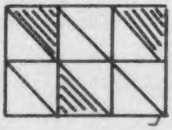
c) $\frac{5}{20}$ and $\frac{1}{4}$ are the same.

10. Put a ring round the bigger fraction in each of the pairs below. The first is done for you.

a) $\frac{1}{4}$ [✓ $\frac{3}{4}$] b) $\frac{3}{7}$ $\frac{5}{7}$ c) $\frac{3}{5}$ $\frac{3}{4}$ d) $\frac{1}{5}$ $\frac{3}{10}$ e) $\frac{89}{90}$ $\frac{90}{91}$

(17) (18) (19) (20)

11. Tick the box which has the most shaded



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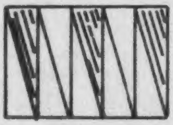
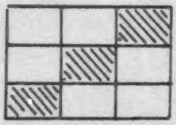


____



(21) ____

12. Put these fractions in order of size, starting with the smallest:

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{100}$ $\frac{1}{3}$ ....(22)..........

---

13. Peter and Abdul each have a bar of chocolate of the same size. Peter breaks his into 8 equal pieces and eats 4 of them. Abdul breaks his into 4 equal pieces and eats 2 of them.

Who eats the most chocolate? .... (23)

Why do you think this? ....

....

---

14. Fill in the missing numbers:

a) $\frac{1}{3} = \frac{2}{\ }$ (24) b) $\frac{6}{8} = \frac{3}{\ }$ (25) c) $\frac{5}{10} = \frac{\ }{30}$ (26) d) $\frac{2}{3} = \frac{\ }{15}$ (27) e) $\frac{4}{12} = \frac{1}{\ }$ (28)

---

15. Three bars of chocolate are to be shared equally between five children.

How much should each child get? ....(29).................

---

16. $\frac{2}{7} = \frac{\square}{14} = \frac{10}{\triangle}$

What number goes in $\square$? ...... (30)

What number goes in $\triangle$? ...... (31)

---

17. Mary and John both have pocket money. Mary spends $\frac{1}{4}$ of hers, and John spends $\frac{1}{2}$ of his.

Is it possible for Mary to have spent more than John? .... (32)

Why do you think this? ....

....

---

18. How many bicycle spokes 10½cm long can be cut from a piece of wire 40cm long? ....(33)...........

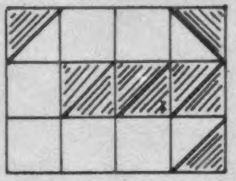
What length of wire is left over? ....(34)...................

---

19. In a baker's shop $\frac{3}{8}$ of the flour is used for bread and $\frac{2}{8}$ of the flour is used for cakes. What fraction of the flour has been used?

.....(35).........

20. I am putting tiles on the floor. They are shown shaded. What fraction of the floor has been tiled? _ _ _ _ _ _ _



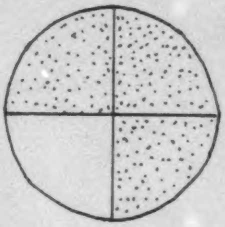
(36)

---

21. John pays $\frac{3}{5}$ of his wages in tax. What fraction of his wages does he have left? ....(37).......

He also pays $\frac{1}{10}$ of his wages on rent. What fraction of his wages does he have left after tax and rent have been paid? ....(38).....

22.



Shade in $\frac{1}{6}$ of the dotted section of the disc. What fraction of the whole disc have you shaded? ....(39).........

---

23. How many pieces of wood $1\frac{1}{4}$cm long can we get from a piece $8\frac{3}{4}$cm long? .... (40)

---

24. A relay race is run in stages of $\frac{1}{8}$ Km each. Each runner runs one stage. How many runners would be required to run a total distance of $\frac{3}{4}$ Km?

(41) ....

---

25. John saves $\frac{1}{4}$ of his own pay. Paul earns £6 and saves $\frac{1}{3}$ of it. How much must John earn to save as much as Paul? ....(42).........

CSMS

FRACTIONS - 1st & 2nd year

NAME .... TODAY'S DATE ....

BOY OR GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ... day ..... month .... year

---

Do the following questions.

1. $3 \div 5$ (43)

2. $\frac{2}{3}$ of 12 (44)

3. $15 \div 4$ (45)

4. $1 - \frac{5}{12}$ (46)

5. $\frac{1}{3}$ of $6 = \frac{1}{4}$ of $\square$. What should $\square$ be? (47)

6. $3 \times 10\frac{1}{2}$ (48)

7. $40 \div 10\frac{1}{2}$ (49)

8. $\frac{3}{8} + \frac{2}{8}$ (50)

9. $\frac{1}{10} + \frac{3}{5}$ (51)

---

10. $\frac{1}{6}$ of $\frac{3}{4}$ (52)

---

11. $1\frac{1}{4} \times 7$ (53)

---

12. $\frac{3}{4} \div \frac{1}{8}$ (54)

---

13. $2 \times \frac{1}{8}$ (55)

---

14. $\frac{1}{3} + \frac{1}{4}$ (56)

---

15. $1\frac{1}{5} - \frac{3}{5}$ (57)

---

16. $2\frac{1}{2} - \frac{3}{4}$ (58)

---

17. $32\frac{2}{3} + 5\frac{1}{4}$ (59)

---

18. $\frac{2}{7} + \frac{3}{4}$ (60)

MARKING SCHEME FRACTIONS 1 & 2

Question	CODES 1	2	3	5	6	7	8
1 (1)	1/5						
2a (2)	4				3		
2b (3)	8				6		
3 (4)	not equal						Yes 2 pieces
4 (5)	a,b	a only	b only		$\frac{1}{2}$ each	$\frac{1}{3}$ each	
(6)	c,d	c only	d only		$\frac{1}{2}$ each	$\frac{1}{3}$ each	
5 (7)	c	b				a	d
6a (8)	5/12						12/5
6b (9)	7/12						12/7
7 (10)	a	a and b,c					b or c
8a (11)	$\frac{3}{8}$				1/5	3/5	
8b (12)	4/8 or $\frac{1}{2}$					4/4	
8c (13)	7/16				7/9	9/16	1/7
9 (14)	c					a	b
10 b,c (15)	5/7,3/4	5/7 only	3/4 only				
10 d,e (16)	3/10,90/91	3/10 only	90/91 only		3/10, same	d wrong, same	same, same
11 (17)	c	c +1 other	c +2 other			b	a
12 (18)	1/100,$\frac{1}{4}$ $\frac{1}{3}$,$\frac{1}{2}$				$\frac{1}{4}$,$\frac{1}{2}$,1/100 $\frac{1}{3}$	1/100,$\frac{1}{3}$ $\frac{1}{4}$,$\frac{1}{2}$	$\frac{1}{2}$,$\frac{1}{3}$,$\frac{1}{4}$ 1/100
13 (19)	same + 4/8 =2/4					Abdul + 2/4 4/8	Peter + 4/8 2/4
14a (20)	2/6				2/4	2/3	
14b (21)	$\frac{3}{4}$				3/5	3/8	3/16
14c (22)	15/30				25/30	5/30	3/30
14d (23)	10/15			6/15	14/15	2/15	5/15
14e (24)	$\frac{1}{3}$				1/9	[illegible]/12	1/4
15 (25)	3/5			mentions squares	$\frac{1}{3}$	$\frac{2}{3}$	5/3
16a (26)	4				9	1	6
16b (27)	35				15	28	21
17 (28)	Yes +reason Mary has twice	Yes +correct numerical example			Yes +Mary has more	Yes,no reason	No + $\frac{1}{2}$ $\frac{1}{4}$
18a (29)	3				80/21		4
18b (30)	$8\frac{1}{2}$				0		$9\frac{1}{2}$
19 (31)	5/8				6/64	5/16	6/8
20 (32)	9/24 or $\frac{3}{8}$	$4\frac{1}{2}$/12				$4\frac{1}{2}$	4/12
21a (33)	2/5						5/2
21b (34)	3/10			0	11/15	7/10	1/5
22 (35)	$\frac{1}{8}$ orequivalent						1/12
23 (36)	7				$7\frac{1}{2}$ or $7\frac{3}{4}$	8	6
24 (37)	6			6/8	$\frac{5}{8}$ or $\frac{7}{8}$	32/3	1/6
25 (38)	8					6	24

COMPUTATION PAPER FRACTIONS 1&2

	CODES						
Question	1	2	3	5	6	7	8
1 (39)	3/5or .6	6		wont go	1 2/5	1 or 2	5/3 or $1\frac{2}{3}$
2 (40)	8				18	24 /36	4
3 (41)	$3\frac{3}{4}$	3 remainder 3			$12\frac{3}{4}$	$3\frac{1}{3}$ or 3.3	4/15
4 (42)	7/12				4/12	4/11	
5 (43)	8					24	6
6 (44)	$31\frac{1}{2}$				$13\frac{1}{2}$	$30\frac{1}{2}$	
7 (45)	3 17/21	80/21	3 remainder $8\frac{1}{2}$	400+	3	4	21/80
8 (46)	5/8				6/64	5/16	6/8
9 (47)	7/10	$3\frac{1}{2}$/5			3/50	4/15	4/10
10 (48)	$\frac{1}{8}$				11/12	9/12	8
11 (49)	$8\frac{3}{4}$			7	$8\frac{1}{4}$	$6\frac{3}{4}$	$7\frac{1}{4}$
12 (50)	6			6/8	$\frac{5}{8}$ or $\frac{7}{8}$	32/3	1/6
13 (51)	$\frac{1}{4}$ or 2/8				$2\frac{1}{8}$	2/16	$2\frac{1}{4}$
14 (52)	7/12			2/12	1/12	2/7	1
15 (53)	3/5				1 4/5	1 2/5	1/5
16 (54)	$1\frac{3}{4}$			$1\frac{1}{4}$	$3\frac{1}{4}$	$2\frac{1}{4}$	$\frac{3}{4}$
17 (55)	37 11/12			7 11/12	27 5/12	37 3/7	11/12
18 (56)	29/28 or 1 1/28				6/28	5/11	27/28

Code 4 was assigned when the question was simply rewritten.

FRACTIONS 1 & 2 N=246 Year 1

Column Number	1	2	3	4	5	6	7	8	9	0
1	87.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.8	1.2
2	76.8	0.0	0.0	0.0	0.0	17.9	0.0	0.0	3.7	1.6
3	70.7	0.0	0.0	0.0	0.0	15.9	0.0	0.0	8.5	4.9
4	91.1	0.0	0.0	0.0	0.0	0.0	0.0	4.5	2.0	2.4
5	61.4	31.3	0.0	0.0	0.0	.4	.4	0.0	5.7	.8
6	56.5	1.6	6.1	0.0	0.0	.4	6.5	0.0	28.0	.8
7	43.1	37.4	0.0	0.0	0.0	0.0	8.5	9.3	.4	1.2
8	69.5	0.0	0.0	0.0	0.0	0.0	0.0	.4	25.2	4.9
9	66.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	27.2	6.5
10	74.8	14.2	0.0	0.0	0.0	0.0	0,0	2.4	5.2	3.3
11	79.7	0.0	0.0	0.0	0.0	0.0	1.2	0.0	13.8	5.3
12	87.4	0.0	0.0	0.0	0.0	0.0	1.2	0.0	7.7	3.7
13	72.4	0.0	0.0	0.0	0.0	4.5	1.6	2.0	13.8	5.7
14	76.0	0.0	.4	0.0	0.0	0.0	10.6	10.2	0.0	2.8
15	65.9	24.4	6.9	0.0	0.0	0.0	0.0	0.0	.8	2.0
16	32.9	27.2	10.2	0.0	0.0	4.9	2.4	0.0	19.9	2.4
17	63.4	1.6	2.4	0.0	0.0	0.0	15.0	10.2	1.6	5.7
18	66.7	0.0	0.0	0.0	0.0	0.0	13.0	5.7	13.4	1.2
19	60.2	0.0	0.0	0.0	0.0	0.0	5.3	22.8	6.5	5.3
20	71.5	0.0	0.0	0.0	0.0	5.3	7.3	0.0	7.7	8.1
21	69.9	0.0	0.0	0.0	0.0	3.7	3.7	7.3	8.5	6.9
22	65.9	0.0	0.0	0.0	0.0	0.0	4.9	2.0	19.9	7.3
23	59.3	0.0	0.0	0.0	4.5	1.2	4.5	6.9	15.9	7.7
24	56.1	0.0	0.0	0.0	0.0	2.4	2.4	11.4	19.9	7.7
25	35.8	0.0	0.0	0.0	4.1	3.7	3.3	.4	43.1	9.8
26	57.3	0.0	0.0	0.0	0.0	1.2	13.4	2.4	17.5	8.1
27	24.0	0.0	0.0	0.0	0.0	.8	6.5	13.8	36.6	18.3
28	1.6	3.7	0.0	0.0	0.0	32.1	8.1	41.5	9.3	3.7
29	69.5	0.0	.4	0.0	0.0	.4	0.0	6.9	15.0	7.7
30	52.8	0.0	0.0	0.0	0.0	.8	0.0	4.5	31.3	10.6
31	77.2	0.0	0.0	0.0	0.0	0.0	4.9	1.2	10.6	6.1
32	30.5	18.7	0.0	0.0	0.0	0.0	6.9	.8	37.8	5.3
33	76.8	0.0	0.0	0.0	0.0	0.0	0.0	.4	11.8	11.0
34	36.2	0.0	0.0	0.0	6.5	.8	1.6	11.0	26.8	17.1
35	54.1	0.0	0.0	0.0	0.0	0.0	0.0	.8	32.1	13.0
36	41.5	0.0	0.0	0.0	0.0	2.8	6.5	8.9	22.4	17.9
37	47.6	0.0	0.0	0.0	.4	.8	0.0	0.0	33.3	17.9
38	38.6	0.0	0.0	0.0	0.0	0.0	4.9	.4	32.5	23.6
39	35.0	0.0	0.0	0.0	1.2	5.3	18.3	3.3	29.7	7.3
40	60.6	0.0	0.0	0.0	0.0	0.0	0.0	2.4	32.5	4.5
41	36.2	24.4	0.0	0.0	0.0	.4	4.5	2.0	24.4	8.1
42	68.7	0.0	0.0	0.0	0.0	2.8	0.0	0.0	17.1	11.4
43	61.4	0.0	0.0	0.0	0.0	0.0	0.0	1.2	26.0	11.4
44	76.0	0.0	0.0	.4	0.0	.8	4.1	0.0	13.4	5.3
45	7.7	.4	17.1	0.0	6.5	4.9	1.2	.4	50.8	11.0
46	77.6	0.0	0.0	0.0	0.0	0.0	8.5	.4	10.2	3.3
47	54.5	0.0	0.0	0.0	0.0	1.2	15.9	1.2	19.9	7.3
48	22.8	0.0	0.0	0.0	0.0	2.8	0.0	0.0	50.0	24.4
49	53.3	0.0	0.0	0.0	.4	4.1	.4	4.1	24.8	13.0
50	39.8	0.0	0.0	0.0	6.5	.4	0.0	1.6	30.1	21.5
51	52.0	0.0	0.0	.4	0.0	2.8	17.1	0.0	16.7	11.0
52	54.1	0.0	0.0	0.0	1.2	1.2	18.3	1.6	13.0	10.6

Column Number	1	2	3	4	5	6	7	8	9	0
53	58.1	0.0	0.0	.4	0.0	2.4	3.3	.4	22.4	13.0
54	46.7	0.0	0.0	0.0	1.6	3.7	3.3	7.7	20.3	16.7
55	42.7	0.0	0.0	0.0	0.0	0.0	11.0	.4	31.3	14.6
56	43.1	0.0	0.0	.4	0.0	.4	18.7	1.6	21.5	14.2
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

FRACTIONS 1 & 2 Year 2 N=309 227

Column Number	1	2	3	4	5	6	7	8	9	0
1	90.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.7	1.3
2	77.7	0.0	0.0	0.0	0.0	17.5	0.0	0.0	4.2	.6
3	73.1	0.0	0.0	0.0	0.0	14.6	0.0	0.0	9.1	3.2
4	87.7	0.0	0.0	0.0	0.0	0.0	0.0	7.1	3.2	1.9
5	66.0	26.5	.3	0.0	0.0	1.0	2.3	0.0	3.9	0.0
6	58.6	5.2	5.5	0.0	0.0	1.0	8.7	0.0	20.7	.3
7	54.4	29.8	0.0	0.0	0.0	0.0	6.8	6.5	1.0	1.6
8	72.5	0.0	0.0	0.0	0.0	0.0	0.0	.6	22.0	4.9
9	68.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.6	5.8
10	76.1	15.2	0.0	0.0	0.0	0.0	0.0	2.6	4.9	1.3
11	79.3	0.0	0.0	0.0	0.0	0.0	2.6	0.0	15.9	2.3
12	88.[illegible]	0.0	0.0	0.0	0.0	0.0	1.9	0.0	7.8	2.3
13	72.2	0.0	0.0	0.0	0.0	6.8	.3	3.2	13.3	4.2
14	77.0	0.0	0.0	0.0	0.0	0.0	9.1	12.0	.3	1.6
15	63.4	29.4	5.2	0.0	0.0	0.0	0.0	0.0	1.3	.6
16	42.7	20.7	12.0	0.0	0.0	5.2	2.3	0.0	16.5	.6
17	67.3	0.0	2.9	0.0	0.0	0.0	17.2	9.1	.3	3.2
18	67.6	0.0	0.0	0.0	0.0	0.0	13.3	8.4	10.4	.3
19	64.4	0.0	0.0	0.0	0.0	0.0	9.7	23.0	1.3	1.6
20	77.3	0.0	0.0	0.0	0.0	6.5	4.2	0.0	6.8	5.2
21	68.6	0.0	0.0	0.0	0.0	4.2	2.9	8.7	9.1	6.5
22	68.3	0.0	0.0	0.0	0.0	.6	1.6	2.3	17.5	9.7
23	58.3	0.0	0.0	0.0	5.2	1.9	2.6	6.8	15.5	9.7
24	52.1	0.0	0.0	0.0	0.0	4.2	5.2	9.7	20.7	8.1
25	30.1	0.0	0.0	0.0	3.2	4.9	4.5	1.6	40.1	15.5
26	55.0	0.0	0.0	0.0	0.0	.6	8.7	1.3	25.9	8.4
27	23.9	0.0	0.0	0.0	0.0	1.3	7.8	14.2	38.5	14.2
28	2.3	3.6	0.0	0.0	0.0	38.8	11.7	34.3	6.1	3.2
29	74.1	0.0	0.0	0.0	0.0	0.0	0.0	6.5	12.0	7.4
30	56.3	0.0	0.0	0.0	0.0	1.0	0.0	3.6	29.4	9.7
31	72.8	0.0	0.0	0.0	0.0	1.0	7.4	0.0	11.7	7.1
32	28.8	22.0	0.0	0.0	0.0	0.0	2.6	2.6	35.3	8.7
33	72.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.0	15.9
34	26.2	0.0	0.0	0.0	9.1	1.0	1.3	10.7	30.1	21.7
35	51.8	0.0	0.0	0.0	0.0	0.0	0.0	1.3	30.4	16.5
36	36.2	0.0	0.0	0.0	0.0	2.6	6.5	5.5	32.7	16.5
37	44.3	0.0	0.0	0.0	.6	0.0	0.0	0.0	35.0	20.1
38	46.0	0.0	0.0	0.0	0.0	0.0	3.9	.6	25.9	23.6
39	31.1	0.0	0.0	.3	.3	9.4	17.5	8.7	27.2	5.5
40	58.9	0.0	0.0	.3	0.0	1.3	0.0	3.9	29.1	6.5
41	37.2	17.2	0.0	0.0	0.0	1.3	8.1	.6	26.5	9.1
42	65.0	0.0	0.0	0.0	0.0	1.9	2.9	0.0	15.2	14.9
43	61.8	0.0	0.0	0.0	0.0	0.0	0.0	1.9	17.8	18.4
44	81.6	0.0	.3	0.0	0.0	.3	4.9	0.0	10.0	2.9
45	8.1	1.3	10.7	.3	7.1	3.9	1.0	.3	53.4	13.9
46	66.3	0.0	0.0	0.0	0.0	.6	19.7	.3	10.0	2.9
47	38.2	.6	0.0	0.0	0.0	1.0	27.2	3.6	22.0	7.4
48	23.0	0.0	0.0	0.0	0.0	2.3	0.0	0.0	44.3	30.4
49	44.0	0.0	0.0	0.0	0.0	5.8	0.0	6.5	25.6	18.1
50	24.9	0.0	0.0	0.0	6.1	2.6	0.0	3.6	35.0	27.8
51	48.2	0.0	0.0	0.0	0.0	1.9	16.5	0.0	15.9	17.5
52	37.9	0.0	0.0	0.0	2.9	.6	29.1	.3	16.8	12.3
53	51.5	0.0	0.0	0.0	0.0	1.0	3.9	1.0	22.0	20.7
54	42.1	0.0	0.0	0.0	2.3	1.3	3.6	8.7	21.7	20.4
55	25.2	0.0	0.0	0.0	.3	.3	17.5	.3	35.0	21.4
56	28.5	0.0	0.0	0.0	0.0	.3	30.1	1.0	23.0	17.2
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

PHI CORRELATION MATRIX FRACTIONS 1&2 N=555

```
92.6   5
91.7  17  89
89.2   4 101 126
88.6   1 441 119 131
87.7  14 356  86 135 455
79.5  13 367 122 138 478 612
79.1  48 227 118 134 304 334 265
77.3   2 323 195 171 361 374 373 282
76.6  16 234 126 136 338 273 319 270 390
75.5  12 271 117  84 337 298 373 253 321 268
74.8  24 264  66 185 419 390 474 262 467 305 411
74.8  35 343 156 171 446 415 402 303 378 364 315 379
74.2  37 337  61 179 529 408 465 294 437 364 334 464 587
72.3  15 301  76 121 387 553 710 196 307 246 292 381 362 398
72.1   3 346 177 171 372 367 369 312 841 385 326 461 378 459 304
72.1  33 254 118 171 384 281 260 302 391 319 242 322 442 395 224 382
71.4  50 308 113 177 363 310 319 301 351 355 361 366 494 419 292 369 307
71.2  10 322 140 149 461 380 582 250 405 343 340 482 427 470 458 401 330 318
70.5  18 254  -8 105 328 312 306 269 412 369 255 387 351 458 260 389 318 314 372
69.2  25 274 181 119 314 310 356 204 374 322 309 466 358 383 283 358 297 284 350 311
67.6  11 319 126 143 480 410 562 287 442 352 330 492 430 489 404 460 366 344 910 403 371
67.2  22 381  68 164 403 371 366 263 464 311 351 451 407 527 304 489 317 372 402 422 348 442
67.2  26 272 151 139 367 324 414 263 464 420 360 522 424 474 355 480 335 355 470 380 548 491 460
66.7  46 340 134 196 409 388 416 322 437 376 299 442 451 544 338 428 343 473 444 379 389 465 401 426
65.6  21 186  98  77 338 272 260 271 376 306 266 277 312 336 177 385 360 304 275 278 247 316 326 301 340
64.1   6 363 156 138 383 362 391 271 518 349 421 474 344 435 291 522 346 315 419 372 404 429 470 502 459 352
63.4   8 314 151 121 412 366 382 262 517 366 341 420 394 467 323 494 402 387 449 385 408 456 457 473 478 386 719
62.5  23 293  91 138 403 312 426 251 486 406 372 415 433 463 318 513 414 390 460 347 442 473 434 513 455 371 437 455
62.8  19 -22 262 157 -11   1  33  35   9  22 -14  83  32 -20  45  50  58  20  25-133 128  36  -1  53   5 -20  33  21  83
61.6  47 272 152 130 371 292 433 286 350 377 273 437 428 433 297 384 309 360 438 349 444 426 403 506 432 317 422 454 445  38
61.3   7 326 149 151 391 357 383 246 540 372 381 440 381 461 308 527 378 355 424 384 406 436 413 523 473 373 824 801 446  32 452
59.6  44 287 125 103 342 297 381 281 500 420 376 418 376 397 294 511 281 429 408 400 413 410 411 497 470 378 502 496 463  21 498
58.7  27 253 146 156 346 311 362 298 463 426 347 473 423 443 289 506 375 432 452 403 463 443 412 693 416 348 472 457 477  74 490
56.0  30 207 194 170 289 266 340 276 438 350 314 471 338 432 286 460 354 354 389 373 478 409 401 564 374 298 458 450 409 128 420
54.8  34 255 107 184 302 289 254 334 345 283 273 388 380 408 213 378 620 313 332 331 334 352 390 414 348 332 392 399 418 123 354
54.4  57 225 157 124 390 308 358 312 419 339 294 459 393 486 297 446 349 484 423 382 376 440 400 462 557 311 401 424 449  20 430
53.9  28 249 115 166 329 337 379 235 387 375 287 452 369 405 306 406 334 365 432 351 478 439 446 485 419 280 385 430 485 131 488
52.8  39 257 121 170 344 263 394 268 409 337 317 406 381 482 292 408 368 358 354 376 353 362 346 400 441 370 429 413 483  47 396
49.9  55 144 117 150 289 241 284 202 308 237 258 330 364 349 272 364 275 377 340 283 322 375 367 382 354 275 355 354 400 106 343
49.5  20-147 167  20-202-190-209-137-176-166-155-154-229-248-198-186-114-224-164-369-111-214-259-213-277-230-123-190-185 315-173
49.4   9  85  35  76 182 138 207 153 259 188 260 200 183 210 169 269 293 171 262 228 253 283 305 259 224 222 257 233 273  97 253
48.1  53 189  67 137 299 238 275 308 366 294 280 335 335 369 216 390 326 338 318 299 314 335 273 357 359 249 283 311 365  26 329
45.8  41 231 144 168 283 232 323 249 368 371 296 375 367 441 278 410 338 374 385 365 338 420 372 433 373 330 377 382 457 153 405
45.4  51 232 156 165 280 252 311 183 356 342 300 396 346 380 209 374 277 505 340 305 365 384 390 444 391 287 342 384 414 140 422
45.0  56 250 153 163 289 249 316 180 360 363 271 359 342 342 221 377 281 477 336 292 361 395 354 447 378 320 329 379 424 157 431
44.1  58 181 109 122 283 232 227 269 299 277 152 282 282 332 178 326 310 314 325 289 271 344 288 358 397 285 263 290 350  60 276
42.7  42 174  61 136 251 211 258 237 302 253 229 325 283 333 144 326 334 208 308 239 260 333 253 323 347 280 311 353 359  83 419
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36.8  45 172 107 108 262 193 248 180 279 271 260 296 253 269 180 291 199 317 295 288 290 328 253 349 340 269 312 315 335 112 401
35.0  60 149 110 133 250 204 241 153 261 289 215 303 286 328 209 287 254 422 283 267 317 322 318 359 342 220 256 305 364 130 376
33.0  59 154  71 120 238 215 271 172 270 288 230 319 283 299 229 299 248 368 294 269 326 322 326 359 317 257 284 325 376 170 411
32.8  43 109 112 107 201 190 260 123 286 286 237 290 308 288 192 323 297 306 334 309 316 360 259 373 347 312 330 331 374 128 417
32.6  29 181 111 155 248 189 268 168 312 257 235 315 271 304 190 313 270 270 349 307 322 358 322 354 337 220 335 352 348 156 327
31.5  54 132 119 123 230 170 229 167 238 274 188 322 251 266 169 240 232 335 260 261 310 263 275 341 265 254 248 297 341  92 384
30.6  38 157 128 118 237 236 299 197 304 284 269 358 304 364 202 317 300 343 327 284 324 351 355 380 366 259 325 334 377 222 395
29.5  36 152  94 149 219 205 319 216 303 264 267 303 267 336 269 332 297 261 316 306 312 355 275 368 315 269 319 319 370 108 373
24.0  31 142 122 127 200 184 243 164 284 270 241 316 228 301 197 321 245 290 292 261 328 307 320 374 289 255 331 338 364 204 321
22.9  52  55  85 106 154 164 213 111 223 250 210 257 227 222 193 243 243 231 201 258 261 276 216 298 248 204 237 235 324 161 315
 8.8  49  87  70 108 111 116 158  66 153 142 118 180 151 154 121 179 151 112 127 118 166 147 130 203 152 158 166 170 201  99 193
 2.0  32  40  42  49 -30 -25   8 -22  46 -43  20  52  -6  24   1  30  30  90  33   7  38  43 -10  44  45  -5  -1  27  56  58   5
FACULTY VAR.
           5  17   4   1  14  13  48   2  16  12  24  35  37  15   3  33  50  10  18  25  11  22  26  46  21   6   8  23  19  47
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PHI CORRELATION MATRIX FRACTIONS 1 & 2 contd

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 61.3   7
 59.6  44 521
 58.7  27 595 511
 56.9  30 465 433 562
 54.8  34 436 337 422 376
 54.4  57 480 419 460 442 353
 53.9  28 406 491 487 491 379 394
 52.8  39 463 421 431 391 395 402 428
 49.9  55 335 387 368 361 284 479 330 366
 49.5  20-173-257-201-138-127-214-167-203-138
 49.4   9 237 254 300 264 209 209 270 247 246 -84
 48.1  53 277 402 404 402 302 461 377 339 423-160 174
 45.8  41 396 460 483 478 384 397 421 455 385-179 264 324
 45.4  51 376 477 485 472 334 478 459 362 414-179 177 353 477
 45.0  56 355 449 427 429 298 494 510 348 414-194 243 345 418 709
 44.1  58 304 339 361 275 341 514 327 302 360-155 203 378 304 362 354
 42.7  42 357 390 368 339 308 322 374 342 311 -97 225 284 435 346 323 259
 38.6  40 348 380 392 388 385 375 324 363 297 -89 291 289 446 311 294 317 348
 36.8  45 387 421 388 321 182 330 383 339 375-194 263 365 342 430 481 210 286 263
 35.0  60 272 410 384 382 294 428 443 307 371-182 183 345 365 636 726 307 314 343 389
 33.8  59 298 366 408 389 275 403 402 256 365-127 303 321 363 592 666 341 316 326 379 643
 32.8  43 326 418 445 371 295 385 415 376 423-177 292 364 398 442 486 283 374 314 542 413 416
 32.6  29 379 360 380 360 261 328 343 349 351 -74 273 276 371 361 351 271 261 278 338 328 313 406
 31.5  54 245 376 379 319 242 364 379 315 385-183 252 316 372 448 500 294 252 322 407 527 472 484 338
 30.6  38 351 347 429 438 368 411 403 377 352-158 266 283 456 453 443 314 311 381 320 373 415 368 321 323
 29.5  36 344 363 390 399 334 402 393 438 317 -96 300 332 435 369 381 314 287 403 317 386 352 414 374 325 443
 24.0  31 325 358 384 480 289 361 358 370 351 -32 222 354 390 429 407 299 343 353 316 394 378 417 356 300 405 413
 22.9  52 230 343 334 326 253 326 288 300 374 -76 234 325 403 338 334 275 301 291 340 356 329 450 279 368 345 324 397
  8.8  49 195 217 209 160 167 195 211 230 260 -54 124 183 198 200 241 119 244 131 342 238 227 364 257 308 137 202 271 314
  2.0  32  33  64  66  -4  25  52   1   4 -12  14 -11  18  25   0  27  29  60  99 106   4  37  93  38  42  73 134  41  45  46
FACILITY VAR.
              7  44  27  30  34  57  28  39  55  20   9  53  41  51  56  58  42  40  45  60  59  43  29  54  38  36  31  52  49  32
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229

CSMS

FRACTIONS - 3rd & 4th year

NAME .... TODAY'S DATE ....

BOY or GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ... .
day month year

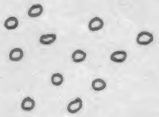
---

1. John wins $\frac{1}{3}$ of these marbles.

Draw a ring round his marbles.

Jane wins $\frac{2}{3}$ of the marbles.

How many marbles does she win? ....

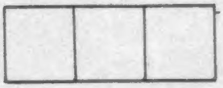


(1)

---

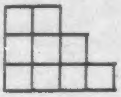
2. Shade in two-thirds of each of these shapes

A



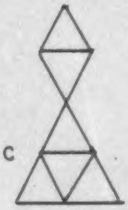
(2)

B



(3)

C



(4)

---

3. A piece of ribbon 17cm long has to be cut into 4 equal pieces.
Tick the answer you think is most accurate for the length of each piece.

a) 4cm, remainder 1 piece

b) 4cm, remainder 1 cm

c) $4\frac{1}{4}$ cm

d) $\frac{4}{17}$ cm

(5)

---

4. 5 eggs in a box of 12 are found to be cracked.

What fraction of the box of eggs is cracked? ....(6).......

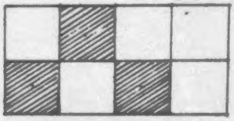
What fraction of the box of eggs is not cracked? ....(7).......

---

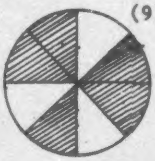
5. What fraction of each of the shapes below is shaded.

Write your answer under the shape.

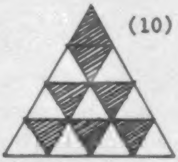
(8)



(9)



(10)



6. Two boys have equal amounts of pocket money. One decides to save $\frac{1}{4}$ of his pocket money, the other decides to save $\frac{5}{20}$ of his pocket money. Tick the answer you think correct.

a) $\frac{5}{20}$ is more than $\frac{1}{4}$
b) $\frac{1}{4}$ is more than $\frac{5}{20}$
c) $\frac{5}{20}$ and $\frac{1}{4}$ are equal. (11)

---

7. Put a ring round the bigger fraction in each of the pairs below. The first has been done for you.

a) $\frac{1}{4}$ [✓ $\frac{3}{4}$] b) $\frac{3}{5}$ $\frac{3}{4}$ (12) c) $\frac{3}{7}$ $\frac{5}{7}$ (13) d) $\frac{1}{5}$ (14) $\frac{3}{10}$ e) $\frac{89}{90}$ (15) $\frac{90}{91}$

---

8. Put these fractions in order of size, starting with the smallest:

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{100}$ $\frac{1}{3}$

............(16)..........

---

9. Fill in the missing number in each case:

a) $\frac{1}{3} = \frac{2}{\quad}$ (17) b) $\frac{6}{8} = \frac{3}{\quad}$ (18) c) $\frac{5}{10} = \frac{\quad}{30}$ (19) d) $\frac{2}{3} = \frac{\quad}{15}$ (20) e) $\frac{4}{12} = \frac{1}{\quad}$ (21)

---

10. $\frac{2}{7} = \frac{\square}{14} = \frac{10}{\triangle}$

What number goes in $\square$? ...(22)...

What number goes in $\triangle$? ...(23)...

---

11. Write down a fraction that comes between $\frac{1}{2}$ and $\frac{2}{3}$:

..(24)...........

---

12. Mary and John both have pocket money. Mary spends $\frac{1}{4}$ of hers, and John spends $\frac{1}{2}$ of his. Is it possible for Mary to have spent more than John? ....

Why do you think this? ......(25)..

....

---

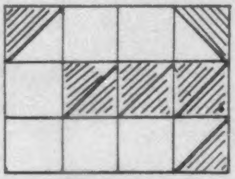
13. How many bicycle spokes 10½cm long can be cut from a piece of wire 40cm long? ....(26).......................

What length of wire is left over? ....(27).......

14. In a baker's shop $\frac{3}{8}$ of the flour is used for bread and $\frac{2}{8}$ of the flour is used for cakes. What fraction of the flour has been used? .... (28)

---

15. I am putting tiles on the floor; they are shown shaded. What fraction of the floor has been tiled?



(29)

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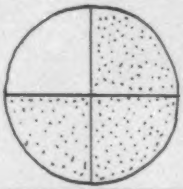
16. John pays $\frac{3}{5}$ of his wages in tax. What fraction of his wages is left? .(30)......

He also pays $\frac{1}{10}$ of his wages on rent.

What fraction of his total wages does he have left after tax and rent have been paid? .... (31)

17.



Shade in $\frac{1}{6}$ of the dotted section of the disc.

What fraction of the whole disc have you shaded?

.....(32)

---

18. How many fractions lie between $\frac{1}{4}$ and $\frac{1}{2}$?

............(33)....................

---

19. A relay race is run in stages of $\frac{1}{8}$ Km each. Each runner runs one stage.

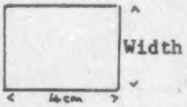
How many runners would be required to run a total distance of $\frac{3}{4}$ Km? ....(34)......

---

20. John saves $\frac{1}{4}$ of his own pay. Paul earns £6 and saves $\frac{1}{3}$ of it.

How much must John earn to save as much as Paul? ......(35)........

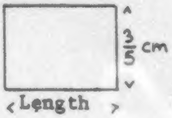
21.



Area = 10 square centimetres

Width = ...(36)..........

22.



Area = $\frac{1}{3}$ square centimetre

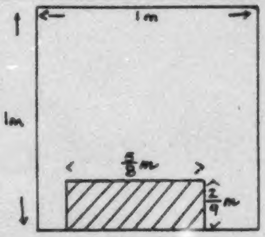
Length = ...(37)..........

23. A man is driving in France. He knows that 1 Km is the same length as $\frac{5}{8}$ mile. His hotel is $\frac{2}{3}$ Km from the petrol station. What is this distance in miles? .... (38)

---

24. What fraction of the square metre is shaded?

(39)



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25. $\frac{1}{17} \div \frac{2}{5} = \frac{5}{34}$ Then $\frac{1}{17} \div \frac{4}{5}$ is:

a) Twice $\frac{5}{34}$ b) $\frac{4}{5}$ of $\frac{5}{34}$ c) $\frac{5}{4}$ of $\frac{5}{34}$ d) Half of $\frac{5}{34}$

Tick which one you think is correct. (40)

---

26. An Australian in London converts pounds to Australian dollars when he wishes to buy something. £1 is equivalent to $1\frac{3}{5}$ Australian dollars. If he pays £45.50 for a coat, how much is this in Australian dollars? ......(41)..................

Sometimes he wants to convert the currency the other way. He knows a suit in Australia costs 65 dollars. To change this into pounds which sum would he do? Tick the one you think is correct.

a) $65 \times 1\frac{3}{5}$ b) $65 \div 1\frac{3}{5}$ c) $1\frac{3}{5} \div 65$ (42)

---

27. a,b,d, are positive whole numbers. $\frac{a}{b}$ is less than $\frac{a}{d}$ when................

(43)

....

---

28. a, b, c are positive whole numbers. $\frac{a}{b} \times \frac{c}{b} = \frac{a \times c}{b}$ when...............

(44)

....

---

29. $3\frac{1}{3} \div 2\frac{1}{5}$ is equal to: a) $(3 \div 2) + (\frac{1}{3} \div \frac{1}{5})$ b) $(3\frac{1}{3} \div 2) + (3\frac{1}{3} \div \frac{1}{5})$

c) $(3\frac{1}{3} \div 2) + \frac{1}{5}$ d) $(3 \div 2\frac{1}{5}) + (\frac{1}{3} \div 2\frac{1}{5})$

Tick which one you think is correct. (45)

CSMS

FRACTIONS - 3rd & 4th year

NAME .... TODAY'S DATE ....

BOY or GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ... .
day month year

Do the following questions.

1. $\frac{2}{3}$ of 12 (46)

2. $15 \div 4$ (47)

3. $1 - \frac{5}{12}$ (48)

4. $\frac{1}{3}$ of 6 = $\frac{1}{4}$ of ☐ . What should ☐ be ? (49)

5. $3 \times 10\frac{1}{2}$ (50)

6. $40 \div 10\frac{1}{2}$ (51)

7. $\frac{3}{8} + \frac{2}{8}$ (52)

8. $\frac{1}{10} + \frac{3}{5}$ (53)

9. $\frac{1}{6}$ of $\frac{3}{4}$ (54)

10. $\frac{3}{4} \div \frac{1}{8}$ (55)

11. $\frac{1}{3} \div \frac{3}{5}$ (56)

12. $\frac{2}{3} \times \frac{5}{8}$ (57)

13. $45\frac{1}{2} \times 1\frac{3}{5}$ (58)

14. $3\frac{1}{3} \div 2\frac{1}{5}$ (59)

15. $\frac{1}{3} + \frac{1}{4}$ (60)

16. $1\frac{1}{5} - \frac{3}{5}$ (61)

17. $2\frac{1}{2} - \frac{3}{4}$ (62)

18. $3\frac{3}{4} \times 2\frac{1}{2}$ (63)

19. $32\frac{2}{3} + 5\frac{1}{4}$ (64)

20. $4\frac{1}{2} \div 12$ (65)

MARKING SCHEME FRACTIONS 3 & 4

QUESTION	1	2	3	4	5	6	7	8
1a,b(1)	4,8					3,6	3,8	4,6
2(2)	A,B,C	A,B	A only	B only	C only	½ for ea	1/3 for ea	A,C
3(3)	C	B					A	D
4(a) (b)(4)	(a) (b) 5/12 7/12	(a) 5/12 only	(b) 7/12 only			$\frac{12}{5}$, $\frac{12}{7}$	$\frac{12}{5}$, $\frac{7}{12}$	$\frac{5}{12}$, $\frac{12}{7}$
5(a) (b)(5)	⅜, ⅝	A only	B only			3/5, 5/3	3/5, 5/8	3/5, 5/3
5c (6)	7/16					7/9	9/16	1/7
6(7)	C						A	B
7(b) (c)(8)	¾, 5/7.	¾ only	5/7 only					
7(d) (e)(9)	3/10 90/91	3/10 only	90/91 only			Same(e) 3/10 corr.	Same(e) 3/10 incorr	Same both
8(10)	1/100, ¼, 1/3 ½					¼, ½, 1/100 1/3	1/100 1/3 ¼, ½	½, 1/3, ¼ 1/100
9a(11)	2/6					2/4	2/3	
9b(12)	¾					3/5	3/8	3/16
9c (13)	15/30					25/30	5/30	3/30
9d(14)	10/15				6/15	14/15	2/15	5/15
9e(15)	1/3					1/9	1/12	¼
10 □ (16)	4					9	1	6
10 △ (17)	35					15	28	21
11(18)	7/12	Any other correct				3½/6	1½/2½	
12(19)	Yes + Reason when Mary has twice more	Yes + Correct EG				Yes + reason Mary has more	Yes only	No + reason ½ > ¼
13a,b(20)	3, 8½	3 only	8½ only			80/21 0 or equiv.	4, 0	3, 9½
14(21)	⅜					6/64	5/16	6/8
15(22)	9/24 or ⅜	4½/12					4½	4/12
16(23)	2/5 3/10	2/5 only right	3/10 only right				2/5 7/10	2/5 1/5
17(24)	1/8 or early							1/12
18(25)	Infinity many lots					1	$1 < x < 20$	$20 < x \le 100$
19(26)	6				6/8	5/8 or 7/8	32/3 or 10 2/3	1/6
20(27)	8						6	24
21(28)	$\frac{10}{4}$ or equiv.						40	
22(29)	5/9				2/3	1/5	5	9/5
23(30)	5/12 or equiv.				7/11	1/24 or 31/24	16/15 or 15/16	12/5 or 2 2/5
24(31)	5/36 or equiv.			⅝ x 2/9		29/72 or 61/72		
25(32)	D				5/68	A	B	C
26a(33)	£72.80	£72 4/5		1 3/5 x 45½		47 1/10 or 43 9/10	45 3/10	

QUESTION	1	2	3	4	5	6	7	8
26b(34)	B					A	C	
27(35)	B > D					Reason involving A	D > B	Correct Numerical Example Given
28(36)	B = 1					When B's same	D	Numerical Example Given
29(37)	D					A	B	C

↑
Column
number

COMPUTATION FRACTIONS 3 & 4

QUESTION	1	2	3	4	5	6	7	8
1(38)	8			Rewrite question		18	$\frac{24}{36}$	4
2(39)	$3\frac{3}{4}$	3 rem 3		Rewrite question		$12\frac{3}{4}$	3.3 or $3\frac{1}{3}$	$\frac{4}{15}$
3(40)	$\frac{7}{12}$			Rewrite question		$\frac{4}{12}$	$\frac{4}{11}$	
4(41)	8			Rewrite question			24	6
5(42)	$31\frac{1}{2}$			Rewrite question		$13\frac{1}{2}$	$30\frac{1}{2}$	
6(43)	$3\frac{17}{21}$	$\frac{80}{21}$	3 rem 3	Rewrite question	400+	3	4	$\frac{21}{80}$
7(44)	$\frac{5}{8}$			Rewrite question		$\frac{6}{64}$	$\frac{5}{16}$	$\frac{6}{8}$
8(45)	$\frac{7}{10}$			Rewrite question		$\frac{3}{50}$	$\frac{4}{15}$	$\frac{4}{10}$
9(46)	$\frac{1}{8}$			Rewrite question		$\frac{11}{12}$	$\frac{9}{12}$	8
10(47)	6			Rewrite question	$\frac{6}{8}$	$\frac{5}{8}$ or $\frac{7}{8}$	$\frac{32}{3}$ or $10\frac{2}{3}$	$\frac{1}{5}$
11(48)	$\frac{5}{9}$			Rewrite question	$\frac{4}{8}$	$\frac{14}{15}$ or $\frac{4}{15}$	5	$\frac{9}{5}$
12(49)	$\frac{5}{12}$ or equiv			Rewrite question	$\frac{7}{11}$	$\frac{1}{24}$ or $\frac{31}{24}$ or equiv.		$\frac{12}{5}$ or $2\frac{2}{5}$
13(50)	$72\frac{4}{5}$			Rewrite question		$47\frac{1}{10}$ or $43\frac{9}{10}$	$45\frac{3}{10}$	
14(51)	$1\frac{17}{33}$			Rewrite question	$3\frac{1}{16}$	$1\frac{2}{15}$ or $5\frac{8}{15}$	$\frac{3}{22}$	$\frac{33}{50}$
15(52)	$\frac{7}{12}$			Rewrite question	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{2}{7}$	1
16(53)	$\frac{3}{5}$			Rewrite question		$1\frac{4}{5}$	$1\frac{2}{5}$	$\frac{1}{5}$
17(54)	$1\frac{3}{4}$			Rewrite question	$1\frac{1}{4}$	$3\frac{1}{4}$	$2\frac{1}{4}$	$\frac{3}{4}$
18(55)	$9\frac{3}{8}$			Rewrite question		$1\frac{2}{8}$ or $6\frac{2}{8}$	$6\frac{3}{8}$	
19(56)	$37\frac{11}{12}$			Rewrite question	$7\frac{11}{12}$	$27\frac{5}{12}$	$37\frac{3}{7}$	$\frac{11}{12}$
20(57)	$\frac{3}{8}$ or equiv	$\frac{4\frac{1}{2}}{12}$		Rewrite question	Wont go	$16\frac{1}{2}$ or $7\frac{1}{2}$		$\frac{8}{3}$ or $2\frac{2}{3}$

Column number

FREQUENCY OF CODES FRACTIONS 3 & 4 YEAR 3 n = 308

COL. NO.	1	2	3	4	5	6	7	8	9	0
1	79.2	1.6	.3	0.0	0.0	7.1	0.0	1.0	10.4	.3
2	59.7	17.2	15.3	1.6	0.0	0.0	0.0	1.3	4.9	0.0
3	61.4	26.6	0.0	0.0	0.0	0.0	4.2	5.8	.6	1.3
4	68.8	4.2	.6	0.0	0.0	0.0	0.0	0.0	21.8	4.5
5	80.2	.6	1.9	0.0	0.0	.3	0.0	0.0	15.6	1.3
6	77.6	.3	0.0	0.0	0.0	1.9	.3	1.0	15.6	3.2
7	81.8	0.0	0.0	0.0	0.0	0.0	5.5	11.4	.6	.6
8	68.8	5.2	21.9	0.0	0.0	0.0	.3	0.0	1.9	1.9
9	41.9	27.6	8.1	0.0	0.0	3.2	1.3	0.0	15.6	2.3
10	63.6	.3	0.0	0.0	0.0	0.0	13.0	6.6	15.3	1.0
11	78.2	0.0	0.0	0.0	0.0	4.5	7.5	0.0	4.9	4.9
12	75.0	0.0	0.0	0.0	0.0	1.9	2.9	7.8	6.6	5.5
13	70.5	0.0	0.0	0.0	0.0	1.0	2.6	2.6	16.9	6.5
14	62.7	0.0	0.0	0.0	4.2	1.6	1.3	6.8	15.6	7.8
15	61.4	0.0	0.0	0.0	0.0	1.3	3.2	11.4	16.2	6.5
16	63.3	0.0	0.0	0.0	0.0	.6	7.5	1.9	19.5	7.1
17	31.5	0.0	0.0	0.0	0.0	.3	4.9	13.0	33.1	17.2
18	10.7	15.3	0.0	0.0	0.0	.3	.3	0.0	61.7	11.7
19	1.9	4.9	0.0	0.0	0.0	36.7	3.9	27.6	20.5	4.5
20	64.0	10.4	.6	0.0	0.0	0.0	.6	3.2	14.9	6.2
21	80.8	0.0	0.0	0.0	0.0	.6	6.8	.6	7.5	3.6
22	37.0	21.4	0.0	0.0	0.0	0.0	4.2	3.6	28.6	5.2
23	34.4	29.2	.6	0.0	0.0	0.0	2.3	7.5	9.7	16.2
24	56.5	0.0	0.0	0.0	0.0	.3	0.0	.3	27.6	15.3
25	14.9	0.0	0.0	0.0	0.0	29.5	25.6	0.0	16.9	13.0
26	56.5	0.0	0.0	0.0	0.0	1.0	0.0	0.0	24.4	18.2
27	47.7	0.0	0.0	0.0	0.0	0.0	2.3	1.3	29.2	19.5
28	49.7	0.0	0.0	0.0	0.0	6.6	1.6	1.9	25.6	14.3
29	7.1	0.0	.3	0.0	2.3	3.9	.3	1.6	34.1	50.3
30	8.1	0.0	0.0	0.0	.3	.3	1.9	0.0	51.0	38.3
31	12.7	0.0	0.0	5.2	0.0	1.3	0.0	0.0	39.0	41.9
32	21.1	.3	0.0	0.0	6.5	24.4	11.4	11.4	6.2	18.8
33	9.1	1.0	0.0	.3	0.0	.3	1.3	0.0	46.4	41.6
34	37.3	0.0	0.0	0.0	0.0	22.1	15.6	1.6	1.0	22.4
35	13.6	0.0	0.0	0.0	0.0	1.3	10.4	4.2	15.6	54.9
36	3.6	0.0	0.0	0.0	0.0	4.2	0.0	.6	20.5	71.1
37	10.7	0.0	0.0	0.0	0.0	34.7	11.0	14.0	1.3	28.2
38	63.0	0.0	0.0	0.0	0.0	0.0	2.9	3.9	22.4	7.8
39	45.5	13.0	0.0	0.0	0.0	1.3	2.9	.6	28.6	8.1
40	65.6	.3	0.0	0.0	0.0	1.6	1.9	0.0	14.0	16.6
41	66.2	0.0	0.0	0.0	0.0	.3	.3	1.3	15.6	16.2
42	79.9	0.0	0.0	0.0	0.0	0.0	2.9	0.0	9.7	7.5
43	8.8	0.0	12.1	.3	7.5	3.6	.6	.3	50.6	16.2
44	71.8	0.0	0.0	0.0	.3	0.0	14.3	.3	8.4	4.9
45	46.7	0.0	0.0	0.0	0.0	.3	23.7	1.9	19.8	5.5
46	25.6	0.0	0.0	.3	0.0	1.3	.6	.3	43.9	27.9
47	28.2	0.0	0.0	0.0	4.5	3.2	0.0	3.2	38.0	22.7
48	21.8	0.0	.6	0.0	1.0	1.9	1.3	2.6	39.0	31.6
49	47.4	0.0	.6	0.0	1.9	2.6	0.0	.3	29.9	17.2
50	12.3	0.0	.6	1.0	0.0	.6	13.6	0.0	39.5	32.1
51	12.7	0.0	.6	.3	0.0	0.0	.3	.3	46.8	39.0
52	35.1	0.0	.6	0.0	3.2	2.9	21.8	.3	18.8	17.2
53	47.4	0.0	.6	0.0	0.0	.3	4.2	.6	20.8	26.0
54	44.5	0.0	.6	0.0	2.6	.6	3.9	3.9	20.1	23.7
55	10.4	0.0	.6	.3	0.0	1.0	17.2	.3	44.8	25.3
56	23.4	0.0	.6	.3	.3	0.0	16.9	.3	30.5	27.6
57	14.0	.3	.6	.3	.3	.6	0.0	1.0	44.5	38.3

TEST 8 YEAR 4 NUMBER OF CHILDREN = 215 FRACTIONS 3 & 4

COL. NO.	1	2	3	4	5	6	7	8	9	0
1	82.8	0.0	.5	0.0	0.0	8.4	0.0	1.9	6.0	.5
2	61.4	14.0	14.	.5	0.0	0.0	1.4	2.8	5.5	.5
3	61.4	27.4	.5	0.0	0.0	0.0	3.7	6.0	.9	0.0
4	79.5	.9	0.0	0.0	0.0	0.0	.5	0.0	15.3	3.7
5	84.2	1.9	2.8	0.0	0.0	.9	0.0	0.0	7.4	2.8
6	83.7	0.0	0.0	0.0	0.0	1.4	.5	.5	9.8	4.2
7	84.2	0.0	0.0	0.0	0.0	0.0	4.7	9.8	.5	.9
8	72.1	6.5	18.6	0.0	0.0	0.0	0.0	0.0	1.9	.9
9	39.1	21.9	12.6	0.0	0.0	5.1	2.3	.5	17.2	1.4
10	68.8	0.0	0.0	0.0	0.0	0.0	13.5	6.0	10.7	.9
11	79.1	0.0	0.0	0.0	0.0	4.2	4.7	0.0	5.1	7.0
12	71.6	0.0	0.0	0.0	0.0	3.7	1.4	10.2	6.0	7.0
13	74.9	0.0	0.0	0.0	0.0	.9	.5	1.4	12.1	10.2
14	64.2	0.0	0.0	0.0	4.7	2.3	.5	6.5	10.2	11.6
15	63.3	0.0	0.0	0.0	0.0	.9	.9	11.6	12.6	10.7
16	67.4	0.0	0.0	0.0	0.0	0.0	7.0	1.9	17.7	6.0
17	30.7	0.0	0.0	0.0	0.0	.9	6.5	9.3	33.5	19.1
18	8.4	13.0	0.0	0.0	0.0	.5	.9	0.0	65.1	12.1
19	3.7	4.7	0.0	0.0	0.0	46.0	7.0	19.1	16.7	2.8
20	63.3	12.1	1.9	0.0	0.0	0.0	0.0	3.7	14.9	4.2
21	77.2	0.0	0.0	0.0	0.0	.5	6.0	.5	11.2	4.7
22	42.3	20.5	0.0	0.0	0.0	0.0	4.2	2.8	24.2	6.0
23	42.8	23.7	0.0	0.0	0.0	0.0	1.9	7.4	14.4	9.8
24	56.7	0.0	0.0	0.0	0.0	0.0	0.0	1.4	27.4	14.4
25	15.8	0.0	0.0	0.0	0.0	30.2	22.3	.9	14.0	16.7
26	56.7	0.0	0.0	0.0	.5	0.0	0.0	0.0	21.9	20.9
27	57.7	0.0	0.0	0.0	0.0	0.0	1.4	.5	20.5	19.5
28	56.7	0.0	0.0	0.0	0.0	5.6	1.4	.5	20.0	15.3
29	5.6	0.0	0.0	0.0	1.9	3.3	0.0	1.4	30.7	57.2
30	11.2	0.0	0.0	0.0	0.0	0.0	1.4	0.0	47.0	40.5
31	19.1	0.0	0.0	4.7	0.0	1.9	0.0	0.0	31.2	43.3
32	19.5	0.0	0.0	0.0	5.6	28.8	14.0	6.5	2.8	22.8
33	9.3	2.3	0.0	2.8	0.0	0.0	.5	0.0	34.9	50.2
34	37.2	0.0	0.0	0.0	0.0	19.1	9.3	2.8	.5	30.7
35	16.3	0.0	0.0	0.0	0.0	1.9	8.8	2.8	10.2	60.0
36	7.0	0.0	0.0	0.0	0.0	5.6	.5	.5	16.3	70.2
37	5.1	0.0	0.0	0.0	.5	36.3	10.2	11.2	.5	36.3
38	66.0	0.0	0.0	.5	0.0	0.0	1.4	7.9	15.8	8.4
39	48.4	10.7	0.0	.5	0.0	.5	4.2	.9	27.4	7.4
40	66.0	0.0	.5	0.0	0.0	2.8	0.0	0.0	14.0	16.7
41	62.3	0.0	0.0	0.0	0.0	0.0	.5	0.0	14.4	22.8
42	84.2	0.0	0.0	.5	0.0	0.0	2.3	0.0	7.9	5.1
43	16.7	0.0	10.7	0.0	6.0	6.0	1.9	.5	45.6	12.6
44	67.9	0.0	0.0	0.0	0.0	0.0	16.7	.5	7.0	7.9
45	45.1	0.0	0.0	0.0	0.0	0.0	21.4	2.8	19.5	11.2
46	30.2	0.0	0.0	.5	0.0	2.3	.9	.9	34.4	30.7
47	33.5	0.0	0.0	0.0	4.2	1.4	.9	3.7	30.7	25.6
48	28.8	0.0	0.0	.9	.5	2.3	3.7	0.0	31.2	32.6
49	48.8	0.0	0.0	0.0	.5	2.3	0.0	0.0	28.4	20.0
50	12.6	0.0	0.0	.9	0.0	1.4	10.7	0.0	37.2	37.2
51	14.0	0.0	0.0	2.3	.5	1.4	.5	.5	36.7	44.2
52	44.7	0.0	0.0	0.0	1.9	0.0	14.1	0.0	14.4	20.0
53	46.0	0.0	0.0	0.0	0.0	.9	2.3	.5	14.0	36.3
54	44.7	0.0	0.0	.5	1.4	1.9	2.3	1.4	12.6	35.3
55	13.5	0.0	0.0	0.0	0.0	1.4	15.8	0.0	34.0	35.3
56	23.7	0.0	0.0	.5	0.0	0.0	12.1	.9	25.6	37.2
57	17.7	.5	0.0	.9	0.0	0.0	0.0	1.4	34.9	44.7

PHI CORRELATION MATRIX FRACTIONS 5 & 4 n = 523

92.9	2																														
92.6	13	64																													
84.1	9	472	165																												
83.0	8	479	151	875																											
82.8	11	210	235	273	278																										
81.6	50	158	84	199	179	163																									
80.7	1	289	108	324	358	328	281																								
80.1	10	474	195	793	781	280	221	326																							
79.3	28	356	241	476	472	380	258	384	479																						
78.6	17	292	184	436	420	367	222	429	465	390																					
77.4	3	404	93	316	315	190	228	361	327	334	398																				
76.1	6	300	112	578	581	326	267	270	540	411	395	308																			
75.9	12	332	110	379	411	323	194	426	357	408	457	294	418																		
74.8	26	363	130	277	299	259	258	340	261	366	351	276	303	341																	
73.6	7	342	135	571	560	347	310	311	527	444	406	320	915	443	331																
73.6	18	223	75	238	283	301	269	344	245	316	438	258	315	352	301	320															
72.3	19	328	167	409	435	373	269	421	387	380	582	297	434	410	367	423	472														
72.2	52	292	163	391	372	322	317	369	324	514	393	277	379	297	323	425	263	333													
66.8	14	6	253	19	39	89	65	37	76	102	38	-1	108	-64	102	82	55	77	140												
65.8	16	335	141	458	466	311	230	402	413	388	497	295	455	479	313	473	381	480	357	26											
65.8	48	335	155	458	477	429	345	535	428	557	507	372	474	451	387	500	391	480	516	94	490										
65.6	22	219	264	317	350	366	273	412	357	408	516	284	406	392	339	424	452	584	397	116	476	510									
64.8	27	233	147	249	263	236	240	339	254	356	337	282	328	343	733	349	276	384	281	79	346	371	340								
64.8	30	285	134	381	433	353	229	460	405	484	405	292	422	427	309	431	431	411	333	138	447	498	466	337							
64.6	49	310	160	357	377	340	331	367	372	363	432	308	363	312	315	373	301	435	356	59	410	528	429	317	376						
64.2	46	245	129	379	341	410	336	463	367	388	398	274	320	382	374	331	368	411	359	87	411	521	422	344	394	566					
63.3	20	269	149	396	404	388	284	411	395	444	550	319	419	424	324	435	453	698	422	97	445	545	539	360	444	448	458				
62.3	4	354	115	321	307	241	192	409	314	344	392	599	267	355	284	295	313	320	269	24	312	387	314	270	336	307	333	324			
62.1	21	276	127	313	328	364	169	367	312	390	476	276	348	426	326	364	453	458	335	71	417	417	410	291	390	362	396	509	304		
61.4	5	194	162	268	303	241	181	298	293	235	284	173	328	342	271	318	255	289	229	134	346	379	323	303	287	349	334	283	201	239	
56.6	32	269	131	373	393	316	272	402	404	382	389	284	395	417	290	394	394	431	331	71	465	531	441	380	461	457	489	437	330	422	343
56.6	34	254	168	430	414	305	242	412	414	439	473	302	422	399	343	447	438	414	457	185	474	522	449	340	518	376	401	477	338	406	335
52.6	36	230	115	258	293	287	192	321	313	291	437	229	311	378	303	317	308	395	293	77	372	388	411	334	382	338	378	380	297	371	253
51.8	35	241	110	293	316	320	284	400	324	349	354	211	365	351	329	386	282	377	341	155	425	449	407	299	395	455	414	440	229	391	343
50.7	15	-123	181	-187	-182	-64	-62	-124	-117	-153	-160	-75	-149	-233	-106	-182	-156	-175	-200	308	-252	-203	-162	-62	-198	-90	-137	-172	-48	-139	-20
48.0	57	160	-6	113	119	123	168	130	152	187	193	179	125	156	152	132	149	201	124	59	112	184	183	154	170	222	189	207	170	134	38
47.2	53	216	185	327	326	339	260	385	365	406	409	217	386	380	302	409	340	431	524	178	391	504	501	328	416	403	442	537	316	390	302
46.8	61	229	157	365	353	285	257	362	381	412	434	277	346	340	316	379	335	453	394	83	426	547	447	330	410	446	468	492	313	329	233
46.7	47	168	116	311	341	284	275	302	321	335	329	202	362	320	172	377	298	399	341	89	319	424	404	159	375	379	345	441	228	350	253
44.6	62	187	24	231	242	215	266	282	273	286	270	189	330	343	290	353	204	280	357	148	330	403	318	297	281	349	387	347	268	263	229
39.2	29	206	150	365	321	272	198	323	292	361	333	199	321	351	250	329	311	366	318	130	372	397	400	304	411	331	296	383	268	287	266
39.0	60	144	169	293	309	271	186	321	310	320	369	169	365	349	230	389	283	398	427	135	378	428	430	236	375	329	383	470	176	341	231
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31.2	23	185	131	269	282	273	191	287	394	292	341	205	319	301	267	327	318	416	276	265	346	354	476	288	400	376	381	443	216	389	296
30.4	55	133	55	236	243	213	163	270	256	234	274	237	243	236	211	244	282	251	285	123	266	362	310	199	312	297	362	339	256	250	174
27.5	54	153	124	232	244	224	148	225	242	240	217	189	255	257	190	262	213	219	205	117	219	327	290	221	274	241	272	300	222	225	172
24.7	56	123	16	187	211	166	156	234	218	182	233	223	237	218	174	251	231	235	198	64	263	319	289	180	254	274	315	334	179	208	162
24.1	24	138	150	195	219	209	93	218	247	243	228	143	242	254	255	235	245	298	220	225	302	312	282	274	283	295	299	308	161	301	299
23.5	64	117	85	216	227	265	158	225	242	238	223	148	268	238	156	260	199	272	312	103	257	285	274	181	229	202	291	328	142	284	180
20.5	40	29	97	90	90	93	81	92	110	153	68	114	139	130	32	142	131	144	144	54	176	106	203	56	175	137	141	199	110	200	120
15.5	65	97	28	171	179	147	121	182	200	205	210	167	202	216	175	209	172	217	209	129	219	264	258	182	260	239	275	315	169	236	133
15.3	33	117	81	176	192	179	91	167	198	203	170	115	213	189	197	218	206	203	160	193	250	216	211	212	246	236	228	257	133	287	228
15.3	39	96	9	172	164	165	132	186	198	164	182	140	188	164	136	182	145	251	126	103	205	216	267	157	212	203	239	301	176	233	162
14.7	43	93	96	180	173	146	113	203	267	171	203	133	207	221	154	224	224	245	188	184	254	231	236	147	249	273	253	260	155	190	196
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12.4	58	58	1	116	139	171	73	169	115	120	140	161	170	144	138	159	172	142	169	136	198	198	215	119	131	205	208	226	125	198	96
12.0	51	102	-42	165	167	137	114	136	184	116	135	87	179	139	147	194	168	163	112	117	180	155	222	112	161	212	239	220	130	204	160
11.7	63	130	14	157	164	134	79	162	181	141	146	153	175	135	115	176	163	171	171	135	186	174	216	105	205	231	233	214	159	136	141
10.7	41	95	47	151	156	157	109	153	172	161	165	142	194	195	158	207	151	200	185	131	223	223	215	138	216	217	232	250	179	206	173
FACILITY	VAR.																														

```
 9.4  38   88  80 139 145 146  84 140 143 131 151 157 164 165  96 177 177 169 166 120 190 190 194  99 195 182 198 217 128 237 106
 8.4  45    3  73  37  45  29  50  61  12  86  23  64  24  41  80   9 -21  64  92  86  44  73  49  93  79  65  -3  45  22  23  70
 6.5  37   42  84 114 119  99  44 209 111 115 137 105 147 130  46 140 140 163 104 156 141 124 177  80 178 162 164 184 124 189 161
 5.0  44   63  73  99 103 104  63 111  91  94 119 102 128 128 132 136  97 141  52 108 127  90 149 131 150 114 133 155 105 160  73
 2.7  25   45  53  72  75  44  17  51  82  84  86 -23  37  93  96  45  45 102  56 119 119  44  96 122 122  48  74  77   6 129 107

            2  13   9   8  11  50   1  10  28  17   3   6  12  26   7  18  19  52  14  16  48  22  27  30  49  46  20   4  21   5

FAC  ITEM                                                  PHI CORRELATION MATRIX, N OF CASES =  523      contd
56.6  32
56.6  34  462                                               FRACTIONS   3 & 4
52.6  36  412 427
51.8  35  437 491 394
50.7  15 -138-146-117-140
48.0  57  153 146 199 145 -54
47.2  53  411 488 392 452-131 156
46.8  61  429 489 415 445-169 317 516
46.7  47  362 401 297 364-104 152 420 397
44.6  62  319 389 342 363 -46 209 415 577 380
39.2  29  434 434 385 390-163 145 370 368 371 241
39.0  60  360 415 413 386-159 212 609 498 422 395 321
38.2  31  458 530 392 443 -73 157 500 404 328 323 480 403
37.3  42  196 236 154 213  33 106 173 203 174 207 174 242 198
31.2  23  414 431 374 475 -62 138 421 369 421 310 372 375 472 155
30.4  55  335 369 261 304-129 280 365 354 331 285 303 383 352 152 300
27.5  54  297 228 268 285  -8 247 240 296 281 196 259 296 237 162 287 318
24.7  56  286 313 223 285-109 373 311 396 300 299 249 397 307 191 275 586 332
24.1  24  312 376 302 328  10  67 282 268 324 277 307 218 403 166 402 230 193 217
23.5  64  276 294 264 273 -66 207 423 355 276 319 191 582 305 169 249 270 243 362 193
20.5  40  185 195 196 166  16  44 156 160 190 174 223 158 156 167 231  35 143  50 135 143
15.5  65  300 289 311 275 -74 339 346 381 320 254 251 459 315 216 305 475 268 551 228 385 162
15.3  33  243 297 222 314  -5  48 225 197 262 260 279 237 299 111 379 227 201 175 193 177 205 214
15.3  39  222 222 212 271  -5 336 225 271 230 185 257 302 277 221 241 354 308 409 182 240 153 376 203
14.7  43  298 320 318 325 -86 162 266 280 249 235 274 276 339 181 361 276 239 250 283 253 270 329 288 318
13.2  59  216 250 234 195 -56 281 276 313 258 241 149 394 251 143 237 454 215 576 137 290  96 645  85 289 252
12.4  58  119 189 195 177 -10 310 258 285 251 280 196 340 180 117 234 343 299 470 194 337  96 415 226 322 268 401
12.0  51  205 181 222 227 -34 197 238 229 266 211 172 294 156 127 296 304 126 306  93 196 118 393 218 332 244 393 323
11.7  63  173 234 237 231 -58 306 288 315 245 273 208 368 253 175 282 394 242 400 185 304  51 470 226 342 235 439 513 377
10.7  41  228 240 254 234  -4 125 254 244 271 261 279 344 325 244 341 295 284 318 253 289 268 364 265 333 344 358 319 213 278
 9.4  38  215 228 200 231   2 242 221 224 225 200 185 280 246 118 279 301 330 318 156 223 194 315 227 373 310 281 356 223 373 376
 8.4  45   43  43  94  85 -45  25  58  60  89  47 123  40  73 150 152  69  59  50  54  43 102  60  81  62 146  24  94 120 104 117
 6.5  37  168 199 219 223  27 150 232 187 204 169 201 250 239 117 274 264 254 298 250 182 154 294 254 318 371 286 300 188 435 234
 5.0  44  164 164 146 185  67 132 153 173 191 148 104 159 218 169 244 250 233 195 261 205 167 315 220 293 351 196 233 185 136 290
 2.7  25  121 121  86 112  21  30  80 105  82 137 182  85 186  92 169  19 -22  42 183 159  92  27 159 126  98 -29  45 -24  87 134

           32  34  36  35  15  57  53  61  47  62  29  60  31  42  23  55  54  56  24  64  40  65  33  39  43  59  58  51  63  41

FAC  ITEM                                                  PHI CORRELATION MATRIX, N OF CASES =  523   contd
 9.4  38                                                   FRACTIONS   3 & 4
 8.4  45   91
 6.5  37  394 115
 5.0  44  318  89 225
 2.7  25  109  35  52  71

           38  45  37  44
```

CSMS

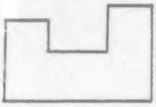
MEASUREMENT

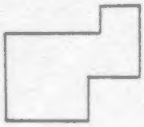
NAME .... TODAY'S DATE ....

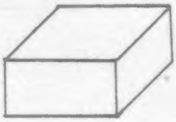
BOY or GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ... .
day month year

INTRODUCTION:


A


B


C

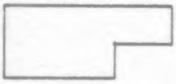
1. Trace your pencil all round the edge of Figure A.
We call the distance round the edge its PERIMETER.

2. Shade in the whole of Figure B.
We call the flat space which it contains its AREA.

3. Figure C shows a box.
We call the amount of room inside the box its VOLUME.

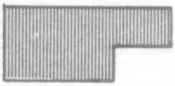
a) To find the PERIMETER of figure D
you add the lengths of its six sides.

D



b) We have shown the AREA of Figure D
by shading.

D



c) What do we mean by the VOLUME of a sugar cube?
Tick (✓) the correct answer:

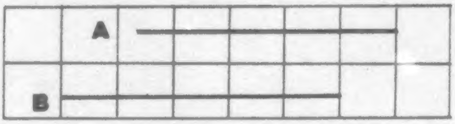
1. The shape of the cube ....
2. How big the outside faces are ....
3. The amount of sugar in the cube ....
4. How far it is all round the cube ....

Part 3 is the correct answer.

1. The lines A, B, C, D, E, F are the dark lines on the squared paper below.

For each pair of lines, tick (✓) the answer you think is true.

a)



Var.1

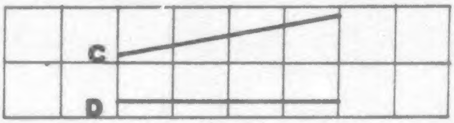
(i) Line A is longer .....

(ii) Line B is longer .....

(iii) A and B are the same length .....

(iv) You cannot tell .....

b)



(i) Line C is longer .....

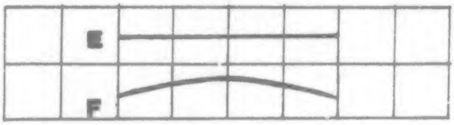
(ii) Line D is longer .....

Var.2

(iii) C and D are the same length .....

(iv) You cannot tell .....

c)



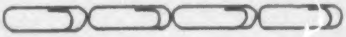
(i) Line E is longer .....

(ii) Line F is longer .....

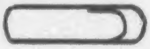
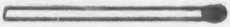
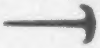
Var.3

(iii) E and F are the same length .....

(iv) You cannot tell .....

2. I can measure how long an umbrella is by putting paperclips in a row next to the umbrella like this:  and then counting them when the line of clips is as long as the umbrella.

I measure the same umbrella 3 times, using

1. paperclips: 
2. matches: 
3. tacks: 

The answers are: 38 15 26

Which number goes with which object?

a) Number of paperclips: Var.4

b) Number of matches: Var.5

c) Number of tacks: Var.6

3. John measures how long paths A and B are, using a walking stick. Then he measures how long paths C and D are, using a metal rod.

The answers are:

Path A:	13 walking sticks
Path B:	14½ walking sticks
Path C:	15 rods
Path D:	12½ rods

Draw a ring round the answer you think is true in each question:

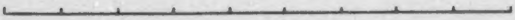
a) Path B is longer than Path A: True / False / Cannot Tell Var.7

b) Path C is longer than Path B: True / False / Cannot Tell Var.8

c) Path D is longer than Path C: True / False / Cannot Tell Var.9

4. The marks on the line show centimetres.

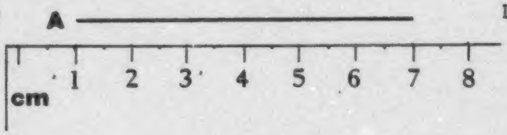
How long is the line?



Length of line: Var.10

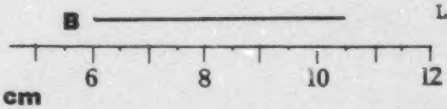
5. How long is each line in centimetres (cm):

a)



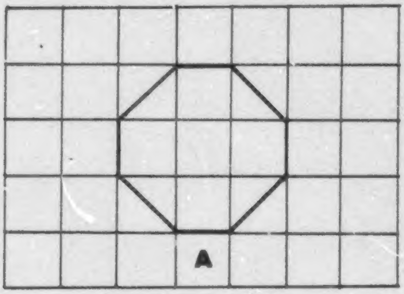
Length of line A: Var.11

b)



Length of line B: Var.12

6. The 8-sided figure A is drawn below on centimetre square paper.

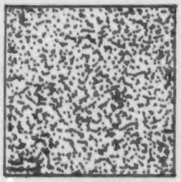


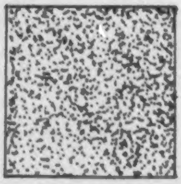
Draw a ring round the correct answer: Var.13

The distance all round the edge of A is:

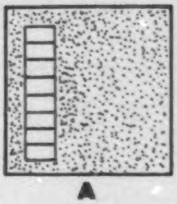
8cm more than 8cm less than 8cm you cannot tell

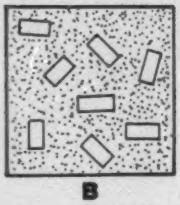
7. This picture shows two tin squares which are the same size:





A machine makes 8 equal holes in each tin square:





Tick (✓) the answer you think is true.

1. Sheet A now has more tin .....
2. Sheet B now has more tin .....
3. A and B now have the same amount of tin Var.14
4. You cannot tell if one now has more tin or not .....

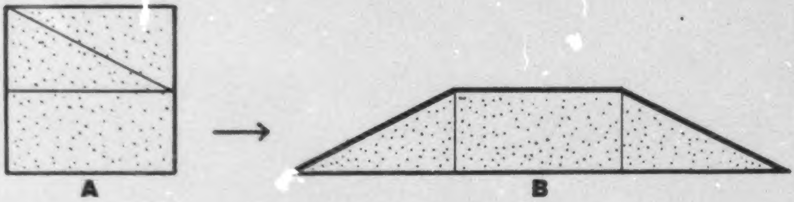
Give a reason for your answer: ....

....

....

....

8. I cut a square A into 3 pieces and arrange the pieces without overlapping to make a new shape B like this:

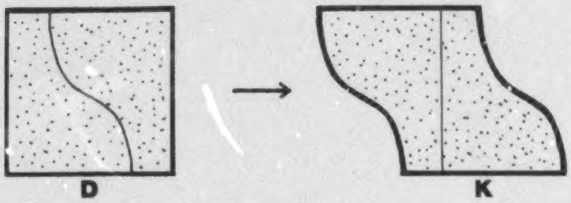


Tick (✓) the answer you think is true in each question.

a) 1. A has the bigger AREA .....
   2. B has the bigger AREA Var.15
   3. A and B have equal AREA .....
   4. You cannot tell if one AREA is bigger or not .....

b) 1. A has the bigger PERIMETER .....
   2. B has the bigger PERIMETER Var.16
   3. A and B have equal PERIMETER .....
   4. You cannot tell if one PERIMETER is bigger or not .....

9. I cut a square D and arrange the pieces to make a new shape K like this:



Tick (✓) the answer you think is true in eqch question.

a) 1. D has the bigger AREA .....

2. K has the bigger AREA .....

3. D and K have equal AREA Var.17

4. You cannot tell if one AREA is bigger or not .....

Give a reason for your answer: ....

....

....

b) 1. D has the bigger PERIMETER .....

2. K has the bigger PERIMETER .....

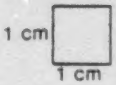
3. D and K have equal PERIMETER .....

4. You cannot tell if one PERIMETER is bigger or not Var.18

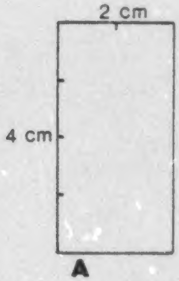
Give a reason for your answer: ....

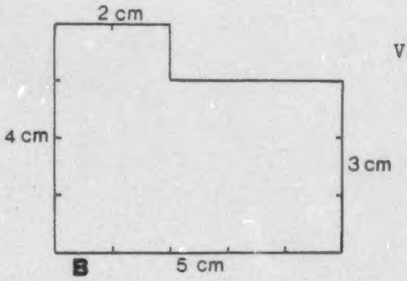
....

....

10. a) How many tiles like this→  would be needed to cover each shape A and B?

Write your answer inside each shape.

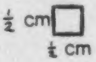




2 cm
4 cm
3 cm
B
5 cm

Var.19

Var.20

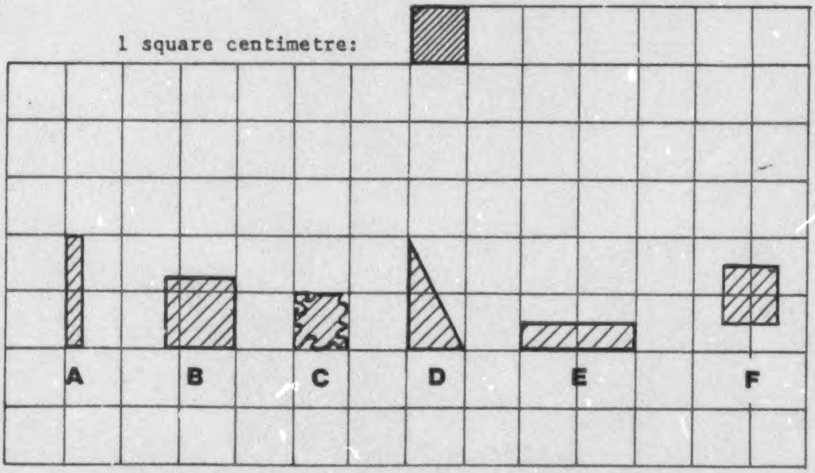
b) If I used smaller tiles like this→  how many would be needed to cover each shape?

Shape A: Var.21

Shape B: Var.22

11. The shaded area measures 1 square centimetre.

Draw a ring round each shape which has area equal to 1 square centimetre.



1 square centimetre:
A
B
C
D
E
F

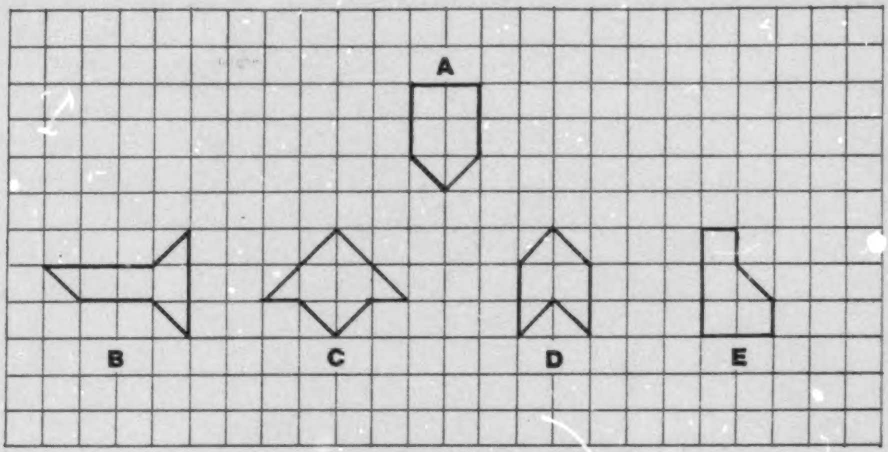
Var.23

Var.24

Var.25

12. Look at all the shapes A, B, C, D, E.

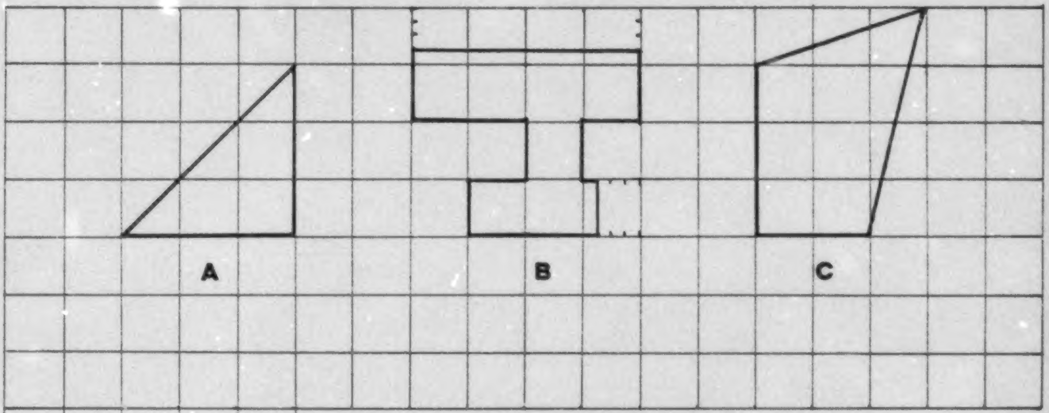
Draw a ring round the shape which has the same area as shape A. Var. 26

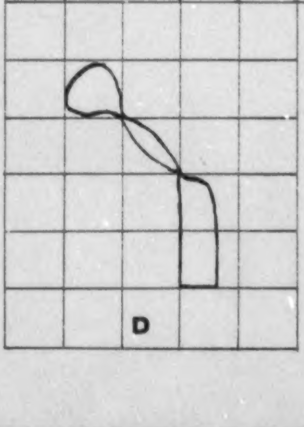


13. The area of the shaded figure measures 1 square centimetre. ▨

Find the area in square centimetres of each shape A, B, C.

Write down your best guess for the area of D.





Area of A = ... Var. 27

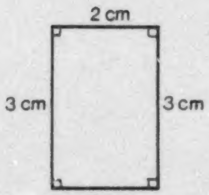
Area of B = ... Var. 28

Area of C = ... Var. 29

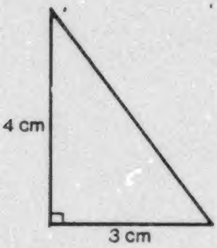
(My best guess) Area of D = ... Var. 30

14. The area of the shaded figure measures 1 square centimetre. ▨

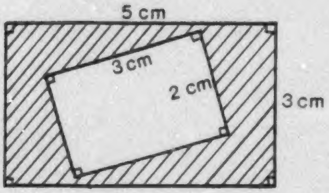
Find the area in square centimetres of each shape.


2 cm
3 cm
3 cm
2 cm

A



B


5 cm
3 cm
2 cm
3 cm

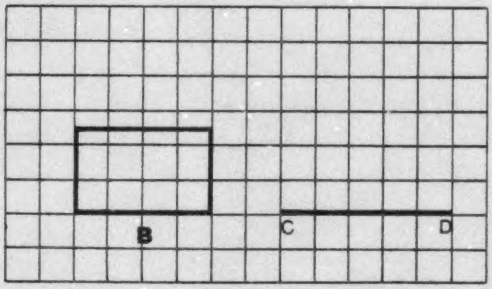
Area of A = ... Var.31

Area of B = ... Var.32

Shaded Area = ... Var.33

15.

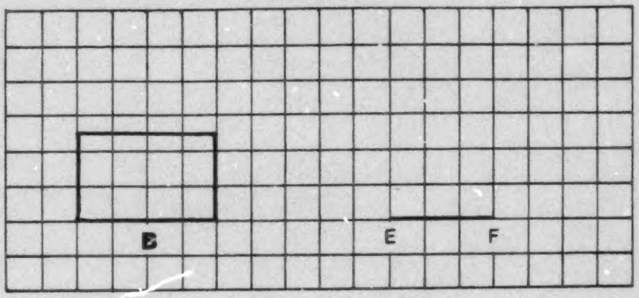
a). Using the line CD as base, draw a RECTANGLE which has the same area as shape B. Put a large cross if you think it is impossible.



Var.34

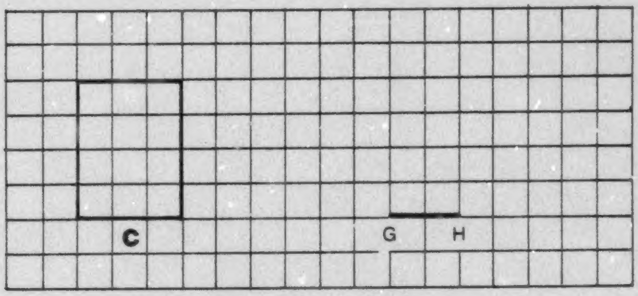
15.

b) Using the line EF as base, draw a RECTANGLE which has the same area as shape B. Put a large cross if you think it is impossible.



Var.35

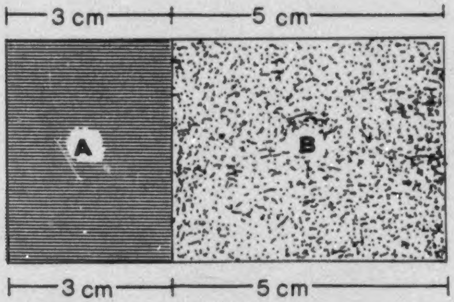
c) Using the line GH as base, draw a RECTANGLE which has the same perimeter as shape C. Put a large cross if you think it is impossible.



Var.36

16. The area of the striped rectangle A is 12 square centimetres.

What is the area of the dotted rectangle B?



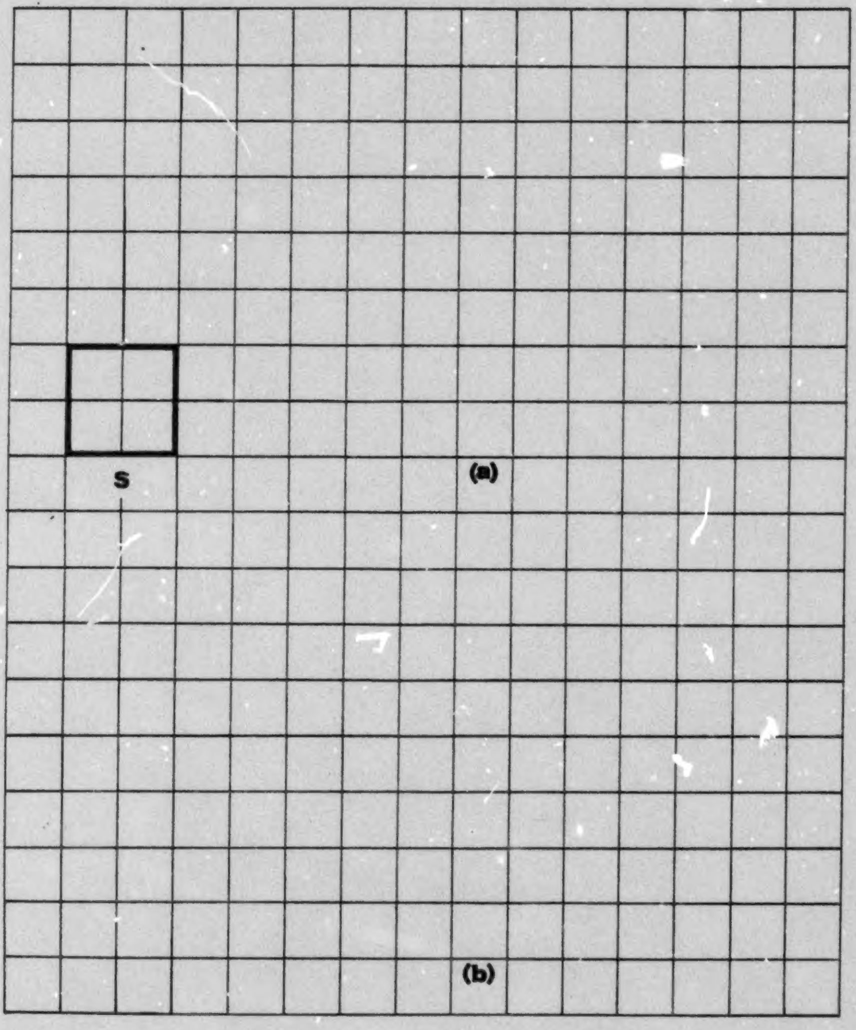
Var.37

Area of dotted rectangle B: ....

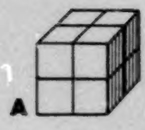
17. On the square paper below, draw:

a) a square whose area is twice (double) the area of square S. Put a cross on (a) if you think it is impossible. Var. 38

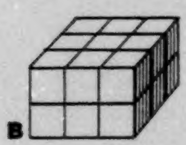
b) a square whose perimeter is twice (double) the perimeter of square S. Put a cross on (b) if you think it is impossible. Var. 39



18. A Block 'A' is made by putting 8 small cubes like this together:

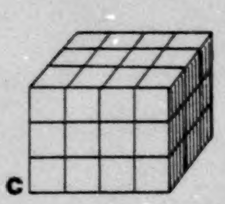


a) How many cubes make this block 'B' (there are no gaps inside)? ....



Var.40

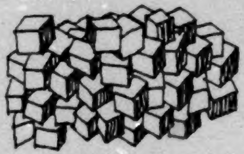
b) Block 'C' is made by putting some small cubes together:



Var.41

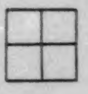
How many cubes make this block 'C' if there are no gaps inside? ....

c) All the cubes from block 'C' are put in a pile:



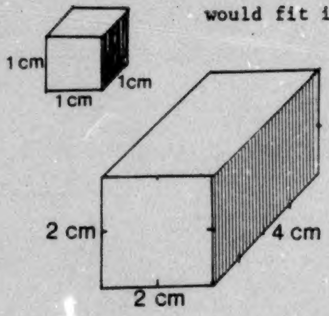
Var.42

I am now going to use all these cubes from block 'C' to build a "sky-scraper" so that the bottom floor is 4 cubes



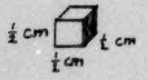
How many cubes high would this "sky-scraper" be from the ground? ....

19. How many 1 centimetre cubes would fit inside this box?


1 cm
1cm
1 cm
2 cm
4 cm
2 cm

Number of 1 centimetre cubes: Var.43

How many small cubes like this would fit into the same box?

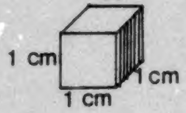


Number of small cubes: Var.44

20. The amount of room inside a block is called its VOLUME.

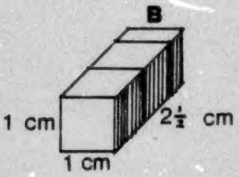
The VOLUME of block A measures 1 cubic centimetre:

A



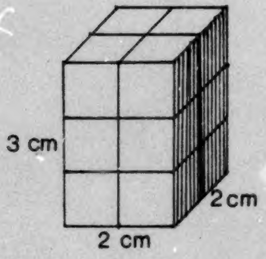
The VOLUME of block B measures 2½ cubic centimetres:

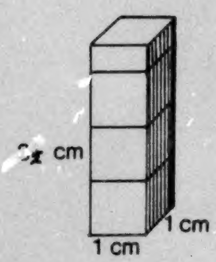
B


1 cm
2½ cm
1 cm

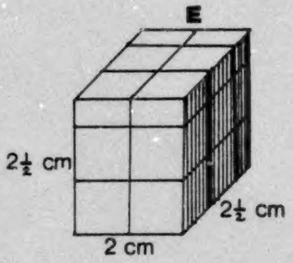
Find the VOLUME in cubic centimetres of each block C, D, E.

C



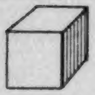

2½ cm
1 cm
1 cm

E

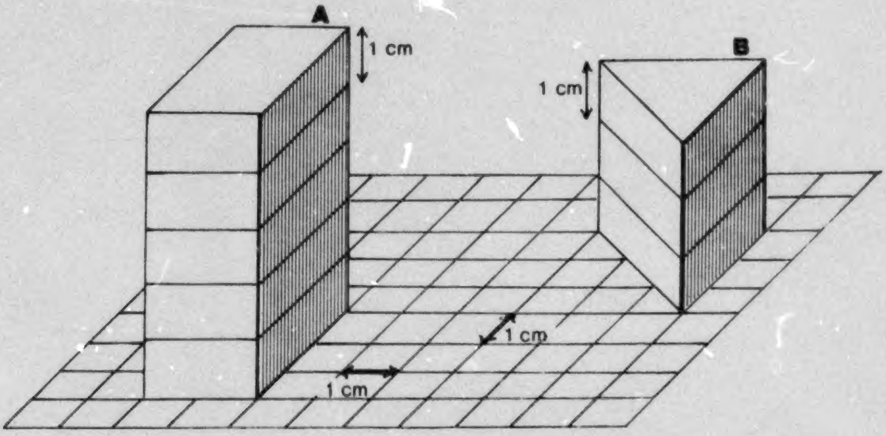

2½ cm
2½ cm
2 cm

Volume of C .... Volume of D .... Volume of E ....

Var.45 Var.46 Var.47

21. The VOLUME of this block  measures 1 cubic centimetre.

Find the VOLUME in cubic centimetres of each block A and B.

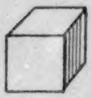

A
1 cm
B
1 cm
1 cm
1 cm

Volume of A = ...

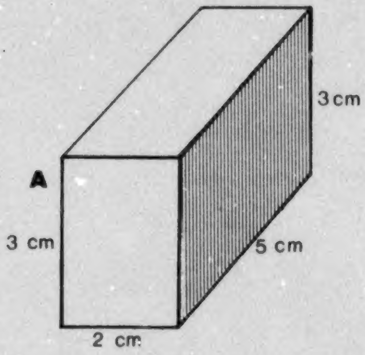
Var.48

Volume of B = ...

Var.49

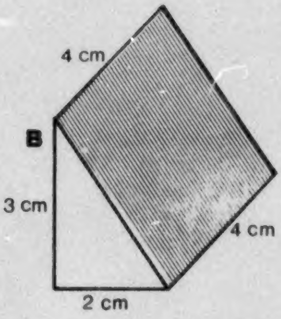
22. The VOLUME of ice in this ice-cube  measures 1 cubic centimetre.

Find the volume in cubic centimetres of each block A and B.


3cm
A
3 cm
5 cm
2 cm

Volume of A = ...

Var.50

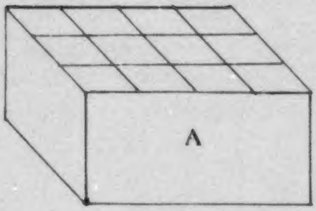

4 cm
B
3 cm
4 cm
2 cm

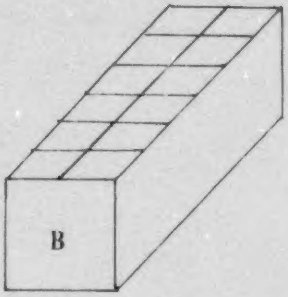
Volume of B = ...

Var.51

23.

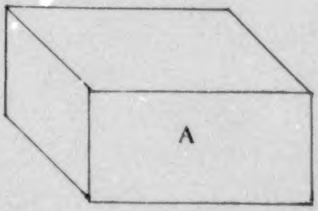
I have 24 cubes which will just fit into a box A, leaving no spaces.

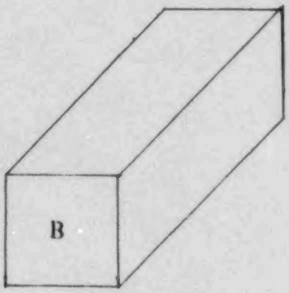




The same 24 cubes will also just fit into box B, leaving no spaces.

Tick (✓) the statement which is true about the volume of air space in the two boxes when the cubes are taken out.





Var.52

1. Box A has more air space ......
2. Box B has more air space ....
3. Box A and B have the same amount of air space ......
4. You cannot tell if one has more air space or not ......

Explain your answer: ....

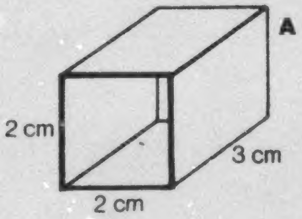
....

....

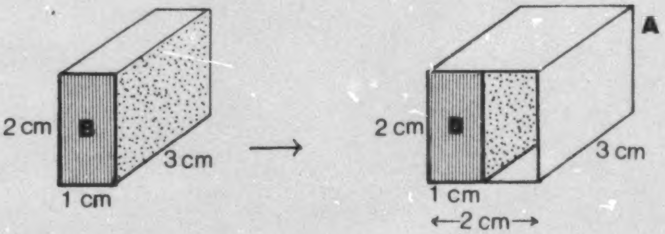
....

24. This question is about the volume of air-space left in a box when I put something in the box.

When box A is empty, the volume of air-space in box A measures 12 cubic centimetres.



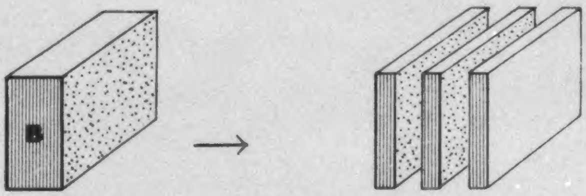
a) I put a block of plasticine B into the box:



Var.53

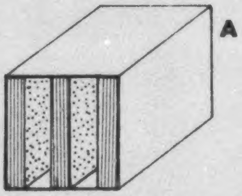
What do you think is the volume of air-space left in box A now that B is in place: ....

b) I take block B out of the box and cut it into 3 'slices':



Var.54

Then I put the 3 'slices' into box A:



What do you think is the volume of air-space left in box A: ....

MARKING SCHEME CODES **MEASUREMENT**

	1	2	3	5	6	7	8
1a	(ii)B				(iv)	(iii) same	(i)
1b	(i)C				(iv)	(iii) same	(ii)
1c	(ii)F				(iv)	(iii) same	(i)
2a	26				15	38	
2b	15				26	38	
2c	38				15	26	
3a	T				CT		F
3b	Cannot tell					F	T
3c	F				CT		T
4	9					10	
5a	6	12				7	13
5b	4.5 or 4½	4.1 or 4.3 4.2		$2\frac{1}{4}$	4	5½	10
6	more				CT	8cm	less
7	3) same + answer	3) same no reason		equal wrong reason	CT	A(i)	B(2)
8a	3) equal				CT	B(2)	A(i)
8b	2)B				CT	equal (3)	A(1)
9a	3) equal + reason	3) equal no reason		equal wrong reason	CT	K(ii)	D(i)
9b	2)K + reason	2)K no reason		2)K wrong reason	CT	equal(3)	D(i)
10a[A]	8					12	6
[B]	17			23	14	18	20
10b[A]	32	4 x 10a[A]				2x10a[A] or 16	same as 10a[A] or 8
10b[B]	68	4x10a[B]				2x10a[B] or 34	same as 10a[B] or 17
11	D E only F	D + E incorr. F ones			A B C		
12	C only				B	D	E
13A	4.5 or 4½					4	5
B	$8\frac{1}{4}$ or 8.25			9½	$8\frac{1}{3}$, 8½ $8\frac{3}{4}$, $8\frac{2}{3}$	7	8

Measurement	1	2	3	5	6	7	8
13C	8½			9 > x > 8	9½	8	9
D	2 < x < 3				2	3	4
14A	6					10	12
B	6				7	12	4½
C	9			10	3or 6	15	16
15a	height 2			impossible	area correct shape non-rect.	changes base	2½
b	3 < x < 4			impossible	area correct shape non-rect.	changes base	2½
c	height 5			impossible	area correct shape non-rect.	changes base	6
16	20			24	15	25	16
17a	square side x 2 < x < 3	rect sides 2 < x < 3		impossible	square side 3	square side 4	rect. sides 2 x 4
b	square side 4			impossible			rect. 2 x 4
18a	18				42	15	21
b	36				66	24	33
c	9	$\frac{1}{4}$x18b if integer			6	12	
19a	16			12	40	24	20
b	128			4x19a or 64	80	2x19a or 32	16
20C	12		6 , 4		32	10	16
D	3½			4½			7 or 6½
E	12½		5	13	8 or 10	15	16½
21A	30			5		20	331
21B	13½			3	9	27	18
22A	30			13	62		31
B	12			13		24	

Measurement

	1	2	3	5	6	7	8
23	3) same + reason	3) same no reason	Same wrong reason		CT	A(i)	B(ii)
24a	6 or half original			1	4		3
b	6 or half original	same answer as a)		9	4		3

TEST 9 YEAR 1 NUMBER OF CHILDREN = 169 MEASUREMENT

VAR	1	2	3	4	5	6	7	8	9	0
1	86.4	0.0	0.0	0.0	0.0	.6	9.5	1.8	1.8	0.0
2	42.0	0.0	0.0	0.0	0.0	3.6	51.5	0.0	3.0	0.0
3	71.6	0.0	0.0	0.0	0.0	5.9	19.5	1.2	1.8	0.0
4	83.4	0.0	0.0	0.0	0.0	4.7	3.6	0.0	2.4	5.9
5	81.7	0.0	0.0	0.0	0.0	3.0	6.5	0.0	2.4	6.5
6	79.3	0.0	0.0	0.0	0.0	5.3	5.9	0.0	3.6	5.9
7	91.1	0.0	0.0	0.0	0.0	2.4	0.0	3.6	0.0	3.0
8	37.9	0.0	0.0	0.0	0.0	0.0	7.7	50.9	.6	3.0
9	95.9	0.0	0.0	0.0	0.0	1.8	0.0	0.0	0.0	2.4
10	79.3	0.0	0.0	0.0	0.0	0.0	18.3	0.0	1.8	.6
11	49.1	0.0	0.0	0.0	0.0	0.0	46.2	0.0	4.1	.6
12	66.3	0.0	0.0	0.0	1.2	1.8	13.0	0.0	17.2	.6
13	38.5	0.0	0.0	0.0	0.0	3.6	43.2	13.6	0.0	1.2
14	79.9	5.9	0.0	4.7	1.8	1.8	3.0	0.0	1.8	1.2
15	80.5	0.0	0.0	0.0	0.0	3.6	10.7	1.8	2.4	1.2
16	59.0	0.0	0.0	0.0	0.0	3.0	36.1	2.4	1.8	1.8
17	60.4	5.3	0.0	4.1	5.3	1.8	14.2	5.3	1.2	2.4
18	26.6	3.6	0.0	8.9	2.4	4.1	43.2	4.1	2.4	5.3
19	85.2	0.0	0.0	0.0	0.0	0.0	2.4	1.2	3.6	7.7
20	59.8	0.0	0.0	0.0	7.7	5.3	.6	1.2	16.0	9.5
21	12.4	0.0	0.0	0.0	0.0	0.0	66.3	0.0	10.7	10.7
22	9.5	.6	0.0	0.0	0.0	0.0	62.7	0.0	14.8	12.4
23	68.0	1.2	0.0	0.0	0.0	.6	0.0	0.0	0.0	30.2
24	74.6	1.2	0.0	0.0	0.0	4.1	0.0	0.0	0.0	20.1
25	77.5	11.2	0.0	0.0	0.0	2.4	0.0	0.0	0.0	8.9
26	74.6	0.0	0.0	0.0	0.0	.6	8.9	3.6	9.5	3.0
27	77.5	0.0	0.0	0.0	0.0	0.0	2.4	.6	13.0	6.5
28	45.0	0.0	0.0	0.0	0.0	13.0	3.6	7.1	23.1	8.3
29	24.3	0.0	0.0	0.0	10.7	1.8	21.3	11.2	21.9	8.9
30	29.6	0.0	0.0	0.0	0.0	10.7	17.8	8.9	26.6	6.5
31	67.5	0.0	0.0	0.0	0.0	0.0	10.1	5.3	12.4	4.7
32	31.4	0.0	0.0	0.0	0.0	9.5	29.0	.6	22.5	7.1
33	44.4	0.0	0.0	0.0	4.7	8.3	10.1	3.6	21.3	7.7
34	43.8	0.0	0.0	0.0	29.0	.6	1.8	0.0	18.3	6.5
35	13.0	0.0	0.0	0.0	52.1	2.4	4.7	0.0	17.8	10.1
36	17.2	0.0	0.0	0.0	22.5	.6	4.7	37.9	7.7	9.5
37	62.1	0.0	0.0	0.0	1.8	1.2	7.7	5.3	13.6	8.3
38	1.8	0.0	0.0	0.0	46.2	1.8	17.8	16.6	8.3	7.7
39	35.5	0.0	0.0	0.0	32.5	0.0	0.0	8.9	9.5	13.6
40	55.6	0.0	0.0	0.0	0.0	1.2	5.9	3.0	29.6	4.7
41	52.1	0.0	0.0	0.0	0.0	0.0	10.7	3.0	29.6	4.7
42	36.7	11.2	0.0	0.0	0.0	0.0	1.8	0.0	40.2	10.1
43	46.2	0.0	0.0	0.0	8.3	1.2	4.1	2.4	30.2	7.7
44	.6	.6	0.0	0.0	4.7	0.0	71.6	0.0	12.4	10.1
45	56.2	0.0	5.9	0.0	0.0	0.0	4.7	1.2	20.7	11.2
46	65.7	0.0	0.0	0.0	2.4	0.0	0.0	3.0	16.6	12.4
47	14.2	0.0	3.6	3.0	5.9	8.3	8.9	.6	41.4	14.2
48	32.0	0.0	0.0	0.0	2.4	0.0	3.6	.6	46.7	14.6
49	6.5	0.0	0.0	0.0	3.0	5.9	9.5	8.9	46.2	20.1
50	35.5	0.0	0.0	0.0	3.6	0.0	0.0	1.2	44.4	15.4
51	15.4	0.0	0.0	0.0	7.1	0.0	16.6	0.0	37.9	23.1
52	50.9	4.7	0.0	10.1	11.2	2.4	6.5	1.8	1.8	10.7
53	55.0	0.0	0.0	0.0	1.2	1.8	0.0	6.5	18.3	17.2
54	50.3	4.1	0.0	0.0	.6	3.6	0.0	9.5	13.0	18.9
55	1.8	13.6	6.5	49.7	0.0	0.0	0.0	8.9	19.5	0.0
56	6.5	4.1	30.1	13.0	0.0	2.4	4.7	1.2	31.4	0.0
57	.6	.6	4.7	47.3	3.6	0.0	0.0	0.0	43.2	0.0

TEST 9 YEAR 2 NUMBER OF CHILDREN = 444 MEASUREMENT

VAR	1	2	3	4	5	6	7	8	9	0
1	89.9	0.0	0.0	0.0	0.0	.7	6.5	.7	1.8	.5
2	45.0	0.0	0.0	0.0	0.0	2.3	50.0	.7	1.4	.7
3	77.7	0.0	0.0	0.0	0.0	4.5	15.5	.2	1.6	.5
4	86.0	0.0	0.0	0.0	0.0	2.9	4.3	0.0	3.4	3.4
5	84.0	0.0	0.0	0.0	0.0	4.1	5.6	0.0	2.9	3.4
6	84.2	0.0	0.0	0.0	0.0	6.5	2.9	0.0	2.7	3.6
7	93.7	0.0	0.0	0.0	0.0	3.2	.2	1.8	0.0	1.1
8	56.8	0.0	0.0	0.0	0.0	.2	7.7	33.8	0.0	1.6
9	92.8	0.0	0.0	0.0	0.0	2.7	0.0	2.9	0.0	1.6
10	85.1	0.0	0.0	0.0	0.0	0.0	12.6	0.0	1.4	.9
11	64.9	.2	0.0	0.0	0.0	0.0	30.6	0.0	3.4	.9
12	66.2	0.0	0.0	0.0	2.0	.7	9.7	.5	20.0	.9
13	36.7	0.0	0.0	0.0	0.0	3.6	43.2	14.9	0.0	1.6
14	80.4	2.3	0.0	2.7	.7	1.4	9.2	2.0	1.1	.2
15	84.9	0.0	0.0	0.0	0.0	2.7	9.0	2.5	.9	0.0
16	66.2	0.0	0.0	0.0	0.0	1.8	28.6	2.0	.7	.7
17	66.0	4.1	0.0	6.3	4.1	2.9	10.6	3.8	.9	1.4
18	36.0	6.1	0.0	6.1	7.7	3.4	33.8	2.9	1.6	2.5
19	83.6	0.0	0.0	0.0	0.0	0.0	2.0	1.4	3.2	9.9
20	69.1	0.0	0.0	0.0	6.8	2.0	2.0	1.4	8.1	10.6
21	20.7	.9	0.0	0.0	0.0	0.0	59.0	2.3	9.9	7.2
22	20.0	1.1	0.0	0.0	0.0	0.0	56.1	1.6	14.0	7.2
23	75.5	2.3	0.0	0.0	0.0	.2	0.0	0.0	1.6	20.5
24	82.2	.7	0.0	0.0	0.0	1.4	0.0	0.0	.5	15.3
25	84.7	9.2	0.0	0.0	0.0	1.8	0.0	0.0	.2	4.1
26	76.1	0.0	0.0	0.0	0.0	.5	8.8	.9	10.1	3.6
27	86.9	0.0	0.0	0.0	0.0	.2	1.4	.2	7.0	4.3
28	59.7	0.0	0.0	0.0	2.7	7.0	2.0	7.4	16.4	4.7
29	28.8	0.0	0.0	0.0	16.0	1.1	19.4	14.0	15.5	5.2
30	40.3	0.0	0.0	0.0	.2	15.5	15.8	5.0	16.2	7.0
31	71.6	0.0	0.0	0.0	0.0	0.0	9.0	1.4	14.2	3.8
32	38.7	0.0	0.0	0.0	.5	7.9	30.4	1.4	16.4	4.7
33	47.7	0.0	0.0	0.0	2.5	7.0	8.8	2.5	23.4	8.1
34	61.0	0.0	0.0	0.0	22.5	0.0	1.4	.2	10.6	4.3
35	20.3	0.0	0.0	0.0	53.4	4.3	2.9	0.0	12.4	6.6
36	27.0	0.0	0.0	0.0	17.3	.2	4.1	36.0	7.4	7.9
37	61.3	0.0	0.0	0.0	4.3	2.7	5.6	2.9	16.0	7.2
38	4.1	.2	0.0	0.0	40.3	2.7	22.5	15.5	6.1	8.6
39	50.2	0.0	0.0	0.0	22.1	.5	0.0	6.1	8.6	12.6
40	59.7	0.0	0.0	0.0	0.0	1.4	9.0	1.8	23.6	4.5
41	57.2	0.0	0.0	0.0	.2	1.6	13.5	1.8	20.9	4.7
42	43.5	11.5	0.0	0.0	.2	.9	2.3	0.0	30.9	10.8
43	57.4	0.0	.2	0.0	7.4	.7	1.1	3.4	22.7	7.0
44	8.3	.2	0.0	0.0	9.0	.7	61.3	.7	11.7	8.1
45	57.0	.2	6.5	0.0	0.0	.2	2.9	2.0	20.5	10.6
46	70.7	0.0	0.0	0.0	.5	0.0	0.0	1.6	15.5	11.7
47	18.7	0.0	2.7	3.4	5.9	7.4	7.9	.5	39.2	14.4
48	44.1	0.0	0.0	0.0	2.5	0.0	4.3	.9	32.9	15.3
49	16.9	0.0	0.0	.2	2.9	8.6	8.3	4.3	37.2	21.6
50	44.4	0.0	0.0	0.0	1.8	0.0	0.0	.5	37.4	16.0
51	21.2	0.0	0.0	0.0	2.0	.2	14.9	0.0	38.5	23.2
52	59.5	5.4	0.0	7.0	4.1	2.3	6.5	2.0	1.1	12.2
53	67.1	0.0	0.0	0.0	.7	.9	0.0	4.3	11.5	15.5
54	59.7	3.4	0.0	0.0	.7	1.4	0.0	5.4	13.1	16.4
55	1.8	15.1	5.0	48.4	0.0	0.0	0.0	12.2	17.6	0.0
56	5.4	9.0	39.2	9.0	0.0	3.8	1.4	5.0	26.8	.5
57	.7	0.0	1.6	57.0	2.0	1.4	0.0	.2	36.9	.2

TEST 9 YEAR 3 NUMBER OF CHILDREN = 373 MEASUREMENT

VAR	1	2	3	4	5	6	7	8	9	0
1	93.0	0.0	0.0	0.0	0.0	.3	5.4	.3	.8	.3
2	52.0	0.0	0.0	0.0	0.0	.8	46.1	.3	.8	0.0
3	82.3	0.0	0.0	0.0	0.0	3.2	13.4	.3	.8	0.0
4	94.6	0.0	0.0	0.0	0.0	2.1	1.6	0.0	.5	1.1
5	92.5	0.0	0.0	0.0	0.0	2.4	3.5	0.0	.5	1.1
6	93.6	0.0	0.0	0.0	0.0	3.8	1.1	0.0	.5	1.1
7	96.2	0.0	0.0	0.0	0.0	1.9	0.0	.8	0.0	1.1
8	65.4	0.0	0.0	0.0	0.0	0.0	6.2	27.3	0.0	1.1
9	94.1	0.0	0.0	0.0	0.0	2.4	0.0	2.4	0.0	1.1
10	89.5	0.0	0.0	0.0	0.0	0.0	9.7	0.0	.5	.3
11	76.1	0.0	0.0	0.0	0.0	0.0	22.8	0.0	.5	.5
12	78.3	0.0	0.0	0.0	4.0	.8	4.3	0.0	12.1	.5
13	46.9	0.0	0.0	0.0	0.0	1.9	41.6	9.1	.3	.3
14	84.2	2.7	0.0	2.1	2.4	.8	6.7	.3	.3	.5
15	84.5	0.0	0.0	0.0	0.0	.8	8.8	4.6	.8	.5
16	73.2	0.0	0.0	0.0	0.0	.8	20.4	4.0	1.1	.5
17	71.0	2.9	0.0	9.4	3.5	1.9	7.0	2.4	.5	1.3
18	44.5	5.1	.3	7.8	5.9	2.7	30.0	.8	.5	2.4
19	91.4	0.0	0.0	0.0	0.0	0.0	1.6	.8	1.1	5.1
20	79.9	0.0	0.0	0.0	5.6	2.4	.3	0.0	6.7	5.1
21	22.8	.5	0.0	0.0	0.0	0.0	62.7	2.4	5.1	6.4
22	22.0	.3	0.0	0.0	0.0	0.0	59.8	1.9	9.7	6.4
23	81.8	.8	0.0	0.0	0.0	.3	0.0	0.0	.8	16.4
24	88.7	.5	0.0	0.0	0.0	.8	0.0	0.0	.5	9.4
25	88.5	8.3	0.0	0.0	0.0	.8	0.0	0.0	0.0	2.4
26	83.6	0.0	0.0	0.0	0.0	1.1	4.8	.8	6.2	3.5
27	91.4	0.0	0.0	0.0	0.0	0.0	.8	.8	4.6	2.4
28	60.9	0.0	0.0	0.0	1.9	7.2	.8	10.5	15.8	2.9
29	26.8	0.0	0.0	0.0	22.0	1.1	22.0	8.8	15.0	4.3
30	44.2	0.0	0.0	0.0	0.0	12.3	19.3	3.2	16.9	4.0
31	86.9	0.0	0.0	0.0	.3	0.0	3.8	1.3	5.4	2.4
32	47.5	0.0	0.0	0.0	0.0	6.2	27.9	1.9	13.1	3.5
33	61.9	0.0	0.0	0.0	.5	9.1	7.8	1.6	14.2	4.8
34	71.6	.5	0.0	0.0	13.1	1.1	.3	0.0	9.7	3.8
35	24.9	0.0	0.0	0.0	52.8	2.9	1.3	0.0	12.9	5.1
36	34.3	0.0	0.0	0.0	15.3	.5	.3	39.4	5.9	4.3
37	76.7	0.0	0.0	0.0	1.9	2.7	6.2	1.3	8.0	3.2
38	6.2	.3	0.0	0.0	45.3	2.7	23.3	13.1	2.4	6.7
39	56.6	0.0	.3	0.0	22.8	.3	0.0	2.7	8.6	8.8
40	67.3	0.0	0.0	0.0	.3	1.3	9.1	1.6	16.9	3.5
41	64.3	0.0	0.0	0.0	0.0	.5	12.6	1.6	16.6	4.3
42	50.9	10.2	0.0	0.0	0.0	1.6	1.6	0.0	28.2	7.5
43	66.0	0.0	0.0	0.0	7.8	.5	1.9	1.9	18.0	4.0
44	6.2	0.0	.3	0.0	16.4	.5	63.3	0.0	8.3	5.1
45	68.1	0.0	8.8	0.0	0.0	.8	2.9	1.3	11.5	6.4
46	81.5	0.0	0.0	0.0	1.3	0.0	0.0	1.3	8.8	7.0
47	27.9	0.0	3.5	5.4	7.5	7.0	8.6	.3	33.2	6.7
48	49.1	0.0	.3	0.0	2.1	0.0	3.8	.5	37.5	6.7
49	18.0	0.0	0.0	0.0	2.4	8.8	13.1	7.2	35.4	15.0
50	58.4	0.0	0.0	0.0	.8	0.0	0.0	.5	31.1	9.1
51	32.2	0.0	0.0	0.0	2.1	0.0	19.3	0.0	32.2	14.2
52	62.5	5.6	0.0	12.9	5.9	1.1	3.2	1.3	.8	6.7
53	74.8	0.0	0.0	0.0	.3	.3	0.0	7.0	7.8	9.9
54	70.0	.8	0.0	0.0	1.1	2.4	0.0	5.4	9.4	11.0
55	1.6	20.9	3.2	45.8	0.0	0.0	0.0	13.1	14.7	.5
56	2.9	11.0	40.5	11.3	0.0	2.9	1.1	7.2	22.3	.8
57	0.0	.3	1.3	59.5	2.1	1.9	0.0	.8	33.5	.5

PHI CORRELATION MATRIX. MEASUREMENT N=986

94.2	7																														
93.8	9	531																													
90.5	1	156	88																												
88.8	4	243	[illegible]	181																											
87.0	27	227	138	110	236																										
86.9	6	290	150	191	636	225																									
86.8	5	160	161	198	661	179	773																								
86.8	19	198	135	138	214	179	186	211																							
85.8	10	123	40	174	252	171	212	193	150																						
84.9	25	162	79	142	273	308	256	212	254	266																					
84.0	15	164	140	150	231	226	166	172	213	170	232																				
83.4	24	216	179	151	326	362	311	308	268	247	533	317																			
81.7	14	186	63	177	174	168	143	157	118	116	174	237	240																		
78.7	26	210	123	193	303	351	246	273	251	313	389	238	399	209																	
78.4	3	155	110	182	189	215	139	108	101	146	198	99	189	77	202																
76.7	31	223	107	147	246	314	198	224	352	290	363	295	378	211	339	194															
76.6	23	181	126	105	275	334	268	237	230	227	441	287	704	184	355	175	357														
73.9	46	248	183	169	251	279	242	239	246	268	349	275	380	167	345	221	431	342													
71.6	20	162	81	132	205	191	202	186	525	156	261	160	256	139	254	133	333	262	251												
70.8	12	146	85	216	211	123	207	230	164	371	208	102	216	135	237	145	246	202	284	144											
68.3	53	183	166	169	267	239	204	233	207	249	274	239	329	193	343	209	392	328	468	222	204										
67.2	37	160	116	163	219	244	184	207	277	260	266	195	303	162	344	163	396	287	387	255	240	317									
66.9	16	93	34	146	168	151	155	146	146	239	197	-7	120	92	197	175	259	140	186	178	145	191	212								
66.9	17	149	87	138	189	183	187	204	197	195	251	345	346	369	255	102	259	318	235	198	179	251	244	69							
66.4	11	145	120	156	225	147	201	224	148	350	203	169	224	125	207	138	283	215	301	219	431	266	249	212	244						
62.1	34	200	154	166	220	251	167	164	177	179	288	199	341	193	313	169	310	337	302	253	201	264	291	237	232	267					
62.0	54	128	128	158	259	238	191	225	213	255	293	232	317	197	352	213	392	311	491	215	231	782	334	222	284	296	317				
61.9	40	172	75	136	225	237	165	193	230	278	234	186	277	153	325	247	386	305	456	223	239	342	319	264	242	273	281	344			
61.1	45	158	166	158	238	260	171	205	192	217	249	184	291	144	321	207	375	265	577	216	255	409	311	256	256	304	292	449	503		
59.1	52	165	111	137	150	169	154	169	169	187	225	227	254	189	262	125	297	192	375	107	160	379	254	165	240	182	221	389	256	313	
59.0	41	164	137	178	222	236	159	169	223	246	241	209	281	166	322	249	403	284	458	220	226	360	323	247	212	276	283	354	755	539	259
58.7	43	163	143	141	213	233	163	196	227	231	250	189	245	153	313	200	331	246	426	202	231	390	301	212	212	276	316	403	516	504	283
57.6	28	148	120	168	204	389	172	144	144	250	251	218	344	168	325	188	298	305	322	197	211	299	271	164	212	289	289	317	302	333	159
56.8	8	117	98	163	269	211	238	216	156	231	226	188	292	117	261	169	312	247	288	145	196	317	285	231	218	238	221	341	272	269	216
52.5	33	156	135	175	282	255	263	265	235	276	251	232	273	171	289	196	450	275	416	284	229	361	388	268	251	296	324	397	349	415	246
50.1	39	169	97	97	213	199	172	204	168	163	196	205	262	132	268	151	274	252	280	167	170	296	288	247	247	239	310	321	273	280	238
48.2	50	143	138	140	238	227	199	219	231	252	282	271	321	150	323	215	411	288	466	224	213	449	365	228	267	337	310	483	460	536	335
47.2	2	86	57	92	186	141	107	109	85	151	205	124	171	94	198	397	189	196	223	144	119	226	170	158	124	146	173	250	269	237	173
45.1	42	119	122	155	224	216	152	184	196	205	240	223	268	148	287	198	350	270	380	218	192	338	324	260	225	286	318	345	565	456	256
43.9	48	131	83	134	209	220	143	187	199	225	213	185	263	158	280	191	343	262	455	244	208	393	343	239	243	269	312	432	425	530	299
40.9	13	46	67	66	84	81	71	67	43	154	103	36	83	88	115	130	180	84	122	148	107	146	193	215	145	197	131	166	177	177	74
40.8	32	134	127	163	228	191	162	189	189	248	263	233	276	173	280	224	389	273	351	234	183	322	328	197	258	305	287	335	332	370	219
40.0	30	139	80	102	150	259	163	140	122	142	193	153	203	106	222	101	200	172	215	178	141	174	198	137	203	159	240	204	214	222	164
37.5	18	129	112	123	168	137	151	172	159	183	163	172	205	116	203	223	254	211	269	181	184	258	246	397	215	253	256	279	272	305	218
28.1	36	106	104	149	171	160	115	103	116	183	194	100	206	114	181	141	227	207	242	173	153	255	234	209	161	205	260	252	202	263	189
27.3	29	112	43	43	94	223	89	63	50	92	112	93	175	65	151	105	154	166	114	102	98	108	97	91	91	97	164	128	63	134	69
24.3	51	89	106	111	148	148	121	123	151	190	166	157	202	139	214	136	262	224	277	163	114	298	234	162	167	208	185	332	284	302	226
21.4	47	97	92	51	137	150	114	115	174	155	164	180	173	118	198	153	264	160	292	164	150	257	269	193	214	229	224	301	272	386	197
20.8	35	84	100	81	141	130	117	111	140	115	160	87	175	67	144	171	164	177	196	156	92	201	181	105	190	157	215	221	170	204	126
20.1	21	91	76	102	113	125	66	68	180	145	147	136	142	99	161	164	198	169	216	237	143	181	220	228	163	206	204	210	252	234	123
19.0	22	97	70	95	105	148	80	73	135	144	168	147	167	101	169	184	199	198	216	258	134	210	205	235	169	228	228	245	230	253	128
15.5	49	94	98	100	134	140	91	109	142	158	165	149	168	122	195	163	216	[illegible]	235	176	108	252	209	188	164	221	190	289	249	284	213
6.2	44	63	65	54	77	99	74	87	75	92	84	77	103	77	113	93	131	122	152	105	109	131	143	180	82	137	131	157	175	170	93
4.5	38	32	14	36	45	83	40	40	69	73	77	81	56	89	100	53	107	73	105	114	30	106	98	89	120	60	128	108	119	81	89
FACILITY	VAR.	7	9	1	4	27	6	5	19	10	25	15	24	14	26	3	31	23	46	20	12	53	37	16	17	11	34	54	40	45	52

PHI CORRELATION MATRIX. MEASUREMENT. N=986 contd.

59.0	41																								
58.7	43	516																							
57.6	28	290	302																						
56.8	8	365	270	275																					
52.5	33	335	391	327	335																				
50.1	39	282	279	252	267	302																			
48.2	50	481	519	330	349	461	341																		
47.2	2	303	272	206	184	214	162	296																	
45.1	42	722	454	274	317	360	334	455	265																
43.9	48	421	459	353	317	427	282	611	257	441															
40.9	13	201	152	132	175	199	153	218	214	211	216														
40.8	32	359	381	273	323	437	262	443	286	400	405	250													
40.0	30	229	229	289	243	203	197	237	121	242	229	168	216												
37.5	18	266	275	219	261	300	308	313	237	315	322	224	277	133											
28.1	36	217	249	221	249	282	262	323	177	249	278	164	330	213	228										
27.3	29	84	125	161	138	180	137	175	32	117	146	46	135	179	75	123									
24.3	51	299	355	285	289	330	258	422	207	330	379	201	481	178	238	271	151								
21.4	47	268	266	247	265	322	298	402	180	272	365	230	326	185	279	262	119	343							
20.8	35	182	216	171	184	266	201	266	166	223	291	173	281	158	227	296	118	210	250						
20.1	21	258	245	189	222	207	272	266	211	277	255	242	289	185	207	283	85	299	300	198					
19.0	22	266	258	210	233	221	286	279	221	283	265	260	298	207	255	290	87	316	315	236	881				
15.5	49	254	291	265	255	334	231	366	274	320	377	202	431	216	263	268	83	592	377	235	323	357			
6.2	44	179	172	169	164	210	188	207	128	223	247	146	258	185	200	204	88	285	266	210	365	348	285		
4.5	38	130	141	66	79	126	97	135	100	139	125	180	140	114	116	149	55	186	138	264	222	233	192	148	
FACILITY	VAR.	41	43	28	8	33	39	50	2	42	48	13	32	30	18	36	29	51	47	35	21	22	49	44	38

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CSMS
REFLECTION and ROTATION

Name.................................. School .... Class ....

Boy or Girl .... Today's Date .... Date of Birth ....
day month year

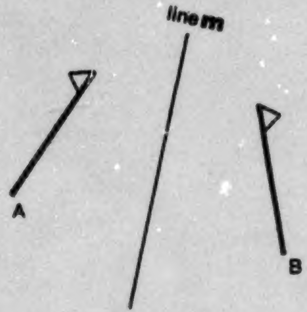
PART A
REFLECTION

Introduction

→ Imagine folding this paper along the line m:

the flag A would go exactly onto the flag B.

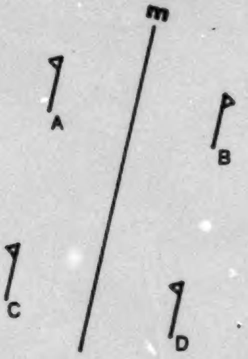
We are going to say
A is REFLECTED onto B, and
B is the REFLECTION of A,
and we are going to call
the line m the MIRROR LINE.

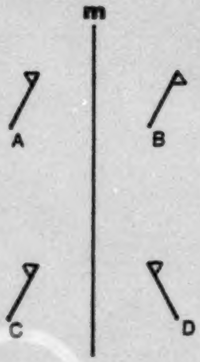


Practice Question

→ For each of these diagrams say whether B is the reflection of A and whether D is the reflection of C.

Draw a circle round the YES or the NO in each case:





B is the reflection of A. YES NO   B is the reflection of A. YES NO
D is the reflection of C. YES NO   D is the reflection of C. YES NO

→ Now check your answers by folding the paper along each mirror line,

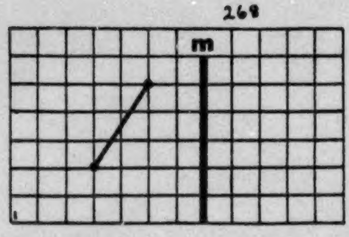
and check your answers against these:

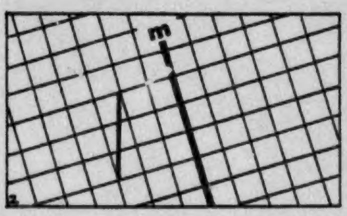
[✓ YES] NO   YES [✓ NO]
YES [✓ NO]   [✓ YES] NO

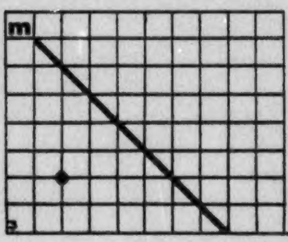
page 1

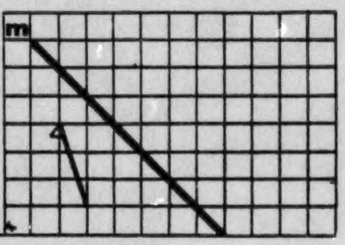
A1. Reflect in each mirror line m, and draw your answers freehand;

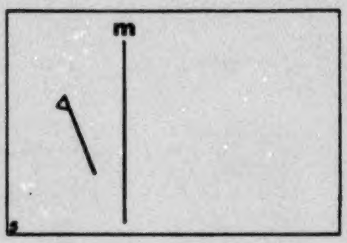
DO NOT USE A RULER.

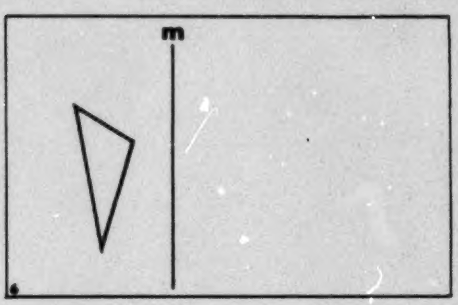


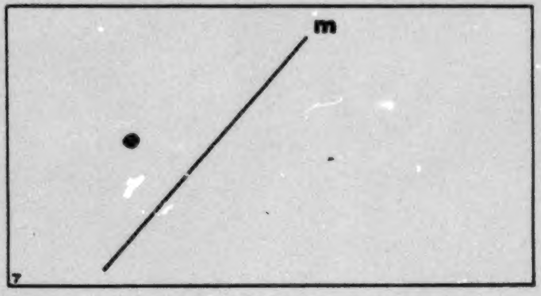


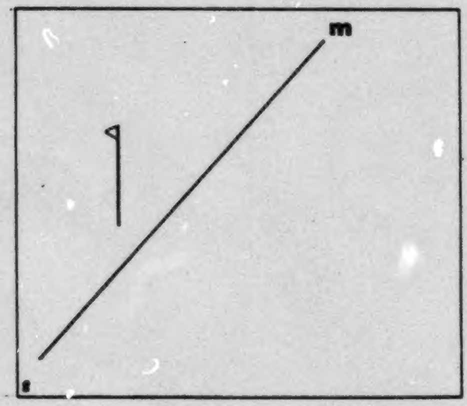


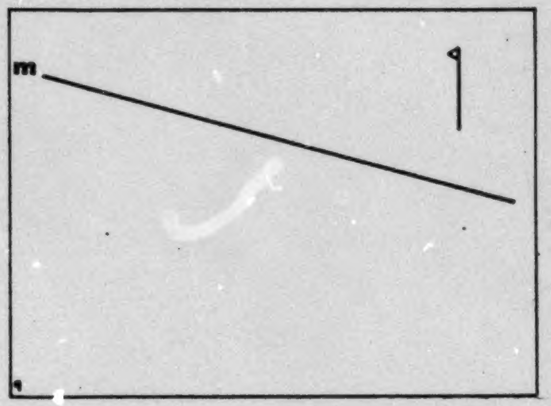




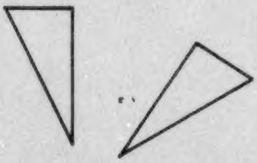




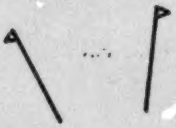




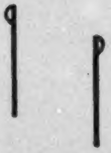
A2. Sketch the mirror lines. If you think there is no mirror line, say so.



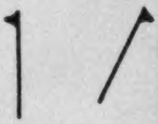
Var.10



Var.11



Var.12



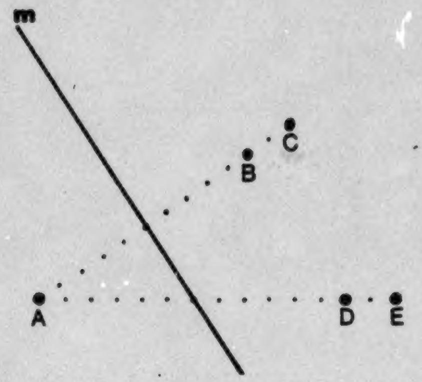
Var.13

A3. Look at the points B, C, D and E.
Which one is
the reflection
of the point A? ....Var.14.......

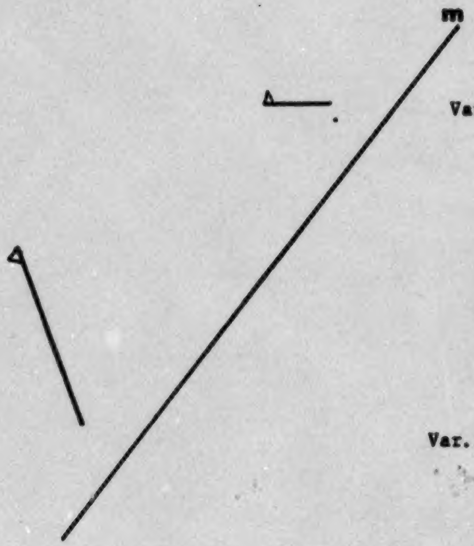
Explain why
you have chosen
this point: ....Var.15.......

....

....



A4. Now
reflect these flags in the mirror line m,
and draw your answers carefully;
use a ruler to measure with.



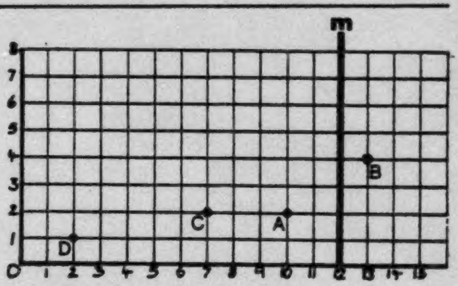
Var.16

Var.17

A5. The points in the diagram are to
be reflected in the mirror line m.

i. Draw the reflection of the point A.

ii. The point A has coordinates (10,2).
Write down the
coordinates of the
reflection of point A: .Var.18

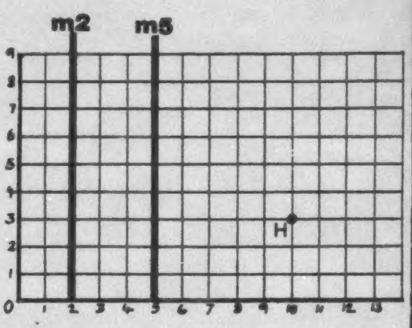


iii. Imagine where the reflections of
points B, C and D would be.
Write down the coordinates of their reflections:

B(13,4)	C(7,2)	D(2,1)
Var.19.	Var.20.	Var.21.

iv. If these points are also to be
reflected in the mirror line m,
write down the coordinates of their reflections:

E(10,3)	F(10,24)	G(20,6)
Var.22.	..Var.23	..Var.24



A6. The two mirror lines in the diagram have been called m2 and m5.

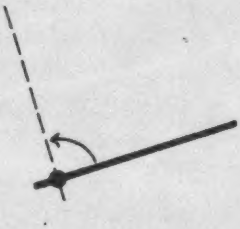
i. Draw the mirror line m7. Var.25

ii. Imagine the mirror line m40. If the points H and I are to be reflected in m40, write down the coordinates of their new positions: H(10,3) I(32,3)

Var.26 .... Var.27 ....

PART B
ROTATION

Introduction



→ In this diagram

imagine the solid line is a stick;

imagine we put a pin through the dot ●

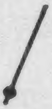
and imagine we now ROTATE the stick through a $\frac{1}{4}$ turn anticlockwise ↺ :

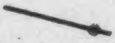
then the stick will end up on the broken line.

Practice Question

→ Rotate each of these sticks through a $\frac{1}{4}$ turn anticlockwise, using the dot as the "centre of rotation".

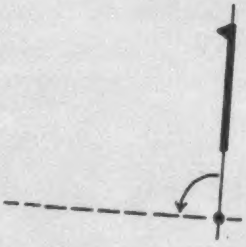
Sketch where each stick ends up.
(Do NOT use a ruler.)

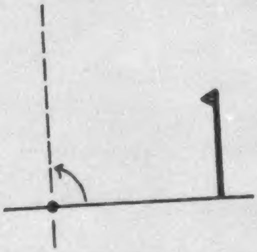




→ Rotate each of these flags through a $\frac{1}{4}$ turn anticlockwise, using the dot as the centre of rotation.

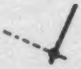
Sketch where each flag ends up.
(Do NOT use a ruler.)

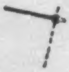




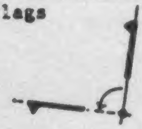
→ Now check your answers against these drawings:

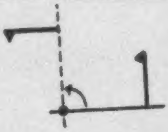
Sticks



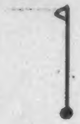


Flags

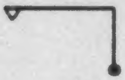




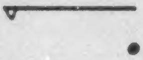
B1. Each of these shapes is to be rotated through a $\frac{1}{4}$ turn anticlockwise ↺)
using each dot as the centre of rotation.
Sketch where each shape ends up. (Do NOT use a ruler.)



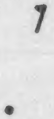
Var.28



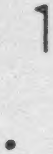
Var.29



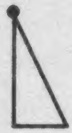
Var.30



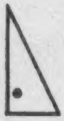
Var.31



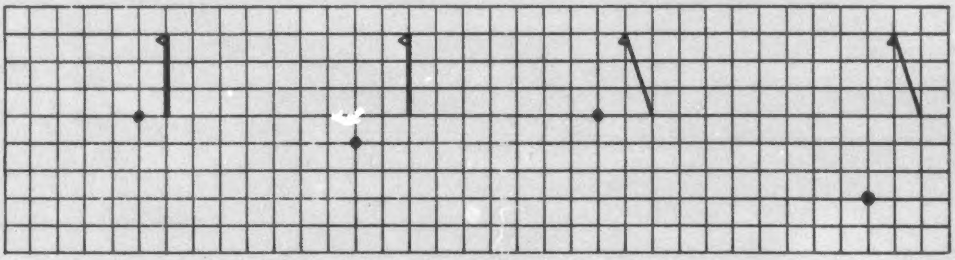
Var.32



Var.33

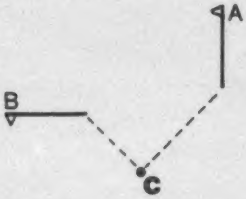


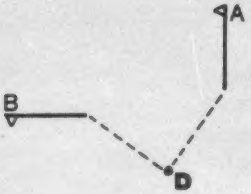
Var.34

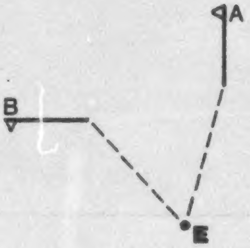


Var.35 Var.36 Var.37 Var.38

B2. Each flag has been rotated through a $\frac{1}{4}$ turn anticlockwise, so that A ends up at B.







Explain why C is NOT the centre of rotation:

....

....

....

..........Var.39......

Explain why D IS the centre of rotation:

....

....

....

..........Var.40.......

Explain why E is NOT the centre of rotation:

....

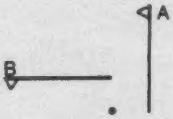
....

....

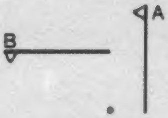
..........Var.41.......

prac 5

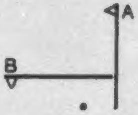
B3. Each flag has been rotated through a $\frac{1}{4}$ turn anticlockwise, so that A ends up at B. In each case state whether the dot shown IS or IS NOT the centre of rotation:



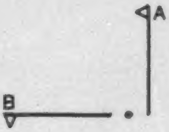
IS / IS NOT



IS / IS NOT



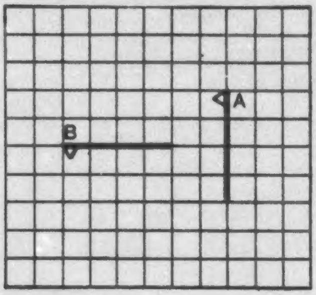
IS / IS NOT



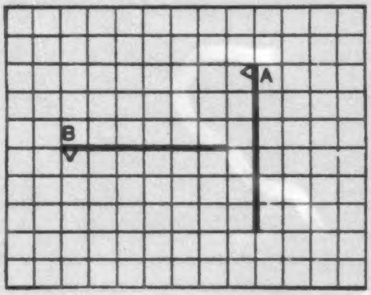
IS / IS NOT

Var.42

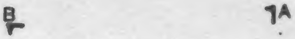
B4. Each flag has been rotated through a $\frac{1}{4}$ turn anticlockwise, so that A ends up at B. In each case draw a dot to show the centre of rotation:



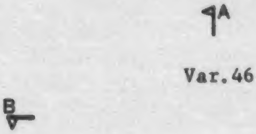
Var.43



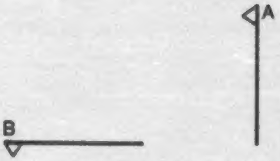
Var.44



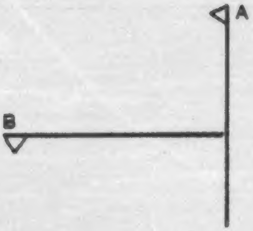
Var.45



Var.46



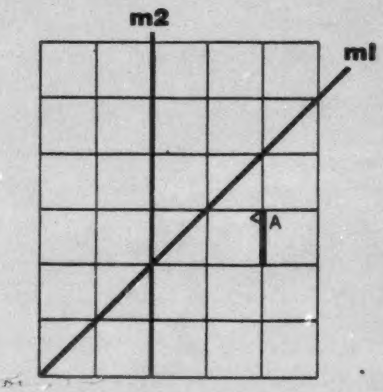
Var.47



Var.48

PART C
reflection AND rotation

C1. i. Reflect flag A in the mirror line ml, and then reflect your answer in the mirror line m2. (Draw the new positions of the flag.)

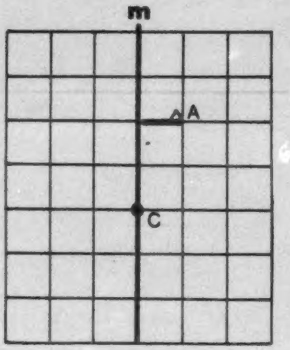


ii. Can you find a single move (a reflection or a rotation) that would take flag A to its final position? yes or no: ....

iii. If your answer was "yes", show this move on the diagram, by drawing a mirror line or a centre of rotation.

Var49

C2. i. Reflect flag A in the mirror line m, and then rotate your answer through a ¼ turn anticlockwise, using the point C as the centre of rotation. (Draw the new positions of the flag.)

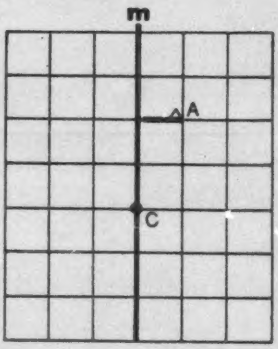


ii. Can you find a single move (a reflection or a rotation) that would take flag A to its final position? yes or no: ....

iii. If your answer was "yes", show this move on the diagram, by drawing a mirror line or a centre of rotation.

Var 50

C3. i. Rotate flag A through a ¼ turn anticlockwise, using the point C as the centre of rotation, and then reflect your answer in the mirror line m. (Draw the new positions of the flag.)



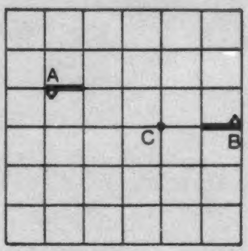
ii. Can you find a single move (a reflection or a rotation) that would take flag A to its final position? yes or no: ....

iii. If your answer was "yes", show this move on the diagram, by drawing a mirror line or a centre of rotation.

Var 51

C4. An unknown move (a reflection or a rotation) is applied to flag A, and then the answer is rotated through a $\frac{1}{4}$ turn anticlockwise, using the point C as the centre of rotation. The flag ends up at B.

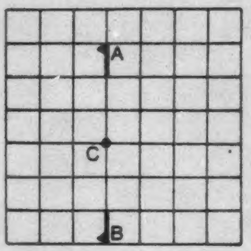
Show the unknown move on the diagram, by drawing a mirror line or a centre of rotation.



Var 52

C5. An unknown move (a reflection or a rotation) is applied to flag A, and then the answer is rotated through a $\frac{1}{4}$ turn anticlockwise, using the point C as the centre of rotation. The flag ends up at B.

Show the unknown move on the diagram, by drawing a mirror line or a centre of rotation.



Var 53

C6. Look at the 8 moves listed at the bottom of the page, and look at the diagram.

i. Write down just one of these moves which, when applied to flag A and then applied again to the answer, takes the flag back to where it started: ....

Write down ALL the single moves that would do this: .... Var. 54

ii. Write down 2 different moves which, when one is applied to flag A and the other is then applied to the answer, take the flag back to where it started: ....

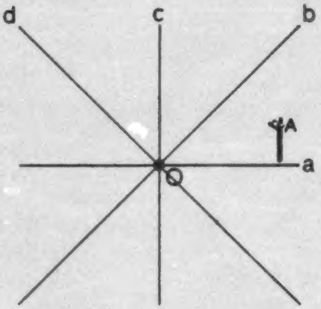
Can you find another pair of different moves that would do this? yes or no: ..... If you can, write them down: .... Var. 55

iii. Write down the single move that has the same effect as applying Move B and then applying Move E: .... Var 56

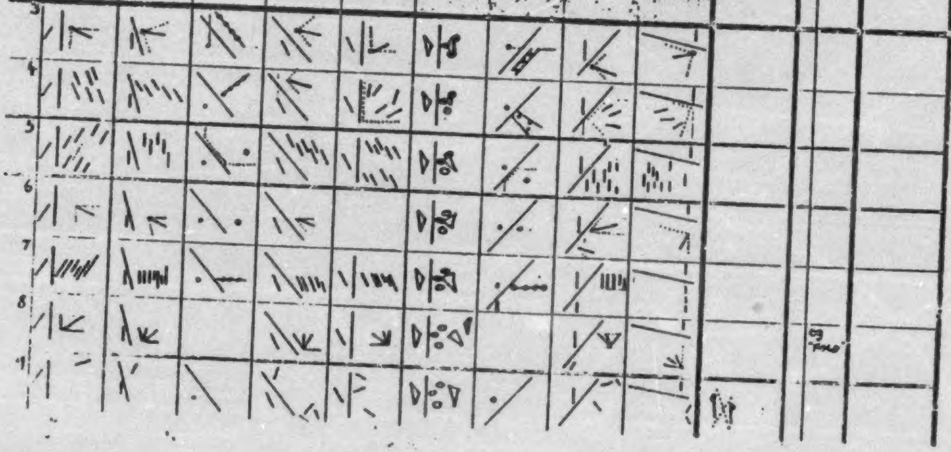
Move A is a reflection in the mirror line a.
Move B is a reflection in the mirror line b.
Move C is a reflection in the mirror line c.
Move D is a reflection in the mirror line d.

Move E is a rotation of a $\frac{1}{4}$ turn anticlockwise.
Move F is a rotation of a $\frac{1}{2}$ turn anticlockwise.
Move G is a rotation of a $\frac{3}{4}$ turn anticlockwise.
Move H is a rotation of a whole turn.

(The point O is the centre of rotation.)



	A1 1	2	3	4	5	6	7	8	9	A2	A3		A4	A 10.2	B 13,4	C 7,2	D 2,1	E 10,3	[illegible]	[illegible]
1	PRECISE	PRECISE	PRECISE	PRECISE	V GOOD	V GOOD	V GOOD	V GOOD	V GOOD	V GOOD	B	DIST + DIR	V GOOD	14,2 OR COORDS OF DRAWING	11,4	17,2	22,1	14, 3	14; 7.[illegible]	[illegible]
2	CORRECT ¼ □	CORRECT ¼ □	CORRECT ¼ □	CORRECT ¼ □	2 POINTS ✓ SLOPE ✓	3 POINTS ✓ 3 SLOPES OK	CORRECT	2 POINTS ✓ SLOPE ✓	2 POINTS ✓ SLOPE ✓	2 POINTS ✓ SLOPE ✓			2 POINTS ✓			16 18, 2	21 23, 1			3 5,
3	1 POINT ✓ SLOPE OK	1 POINT ✓ SLOPE OK	DISTANCE ✓ (REF OK)	1 POINT ✓ SLOPE OK	1+ POINTS ✓ SLOPE OK	3 POINTS ✓ SLOPE X	DISTANCE ✓ (REF OK)	1+ POINTS ✓ SLOPE OK	1+ POINTS ✓ SLOPE OK	1+ POINTS ✓ SLOPE OK		DIST	1 POINT ✓ SLOPE OK	14; X	11. X	17, X	22, X	14; X	14; X	1[illegible];
4	SLOPE ✓ include H or V	SLOPE ✓ REF OK	DISTANCE X REF ✓	SLOPE OK REF OK NOT CUT	SLOPE OK include not H or V on	2 POINTS ✓ SLOPES ✓	DISTANCE X REF ✓	SLOPE OK REF OK NOT CUT	SLOPE OK REF OK NOT CUT	SLOPE OK	C	DIR	SLOPE OK REF OK NOT CUT	X, 2	X, 4	X, 2	X, 1	X, 3	X, 2[illegible]	X, [illegible]
5	IMAGE ∥ NOT H or V	IMAGE ∥ REF OK NOT H or V	DISTANCE X REF OK	IMAGE ∥ REF OK	IMAGE ∥ NOT H or V	2 POINTS ✓ SLOPE X	DISTANCE X REF OK	IMAGE ∥ REF OK	IMAGE ∥ REF OK	SLOPE ∥			IMAGE ∥ REF OK							
6	H or V SLOPE OK	H or V SLOPE OK	H or V DISTANCE ✓	H or V SLOPE OK	⊕	1 POINT ✓ SLOPES OK	H or V DISTANCE OK	H or V SLOPE OK	H or V SLOPE OK	SLOPE VERT DOMINANT	D		H or V SLOPE OK							
7	H or V IMAGE ∥	H or V IMAGE ∥	H or V DISTANCE X	H or V IMAGE ∥	H or V IMAGE ∥	1 POINT ✓ SLOPE X	H or V DISTANCE X	H or V IMAGE ∥	H or V IMAGE ∥		E		H or V IMAGE ∥							
8	H or V NOT ∥ NOT SL OK	H or V NOT ∥ NOT SL OK		H or V NOT ∥ NOT SL OK	H or V NOT ∥ NOT SL OK	POINTS X Δ THIRD OVER		H or V NOT ∥ NOT SL OK	H or V NOT ∥ NOT SL OK	("no")		TAUT	H or V NOT ∥ NOT SL OK							
9	include NOT H or V + ref(∥ or SL ok)	include NOT H or V + not(∥ or SL)		include ROTATION 180°		IGNORE	FLAG HEAD	EXCEPT	FOR DOT POSITION	SLOPE X				THROUGHOUT,		IGNORE	ORDER	OF	CO-ORDI[illegible]	



ROTATIONS & REFLECTIONS CODING SCHEME

REFLECTIONS & ROTATIONS CONTD.

H 110,3	I 32,3	B1	Δ	L Δ	B2 C	D	E		B4	C1 C2 C3		C4	C5	C6 i	ii	iii	
70,3	48,3	V GOOD	VGD	V GOOD	L	LSA	A	4 ✓	GOOD	FLAG ✓ HEAD ✓	R	R	R	ABCD FH	E,G + "no"	C	1
69 71,3	47 49,3	1 POINT ✓ SLOPE ✓	GD	A' IN Δ L→C < [illegible] L' ABOVE M C→A' < C→B'			L ✓	✓	CORRECT	FLAG ✓ HEAD X (CORRECT END OF FLAG, WRONG SIDE)	S	S	S	5 ✓ 0X	E,G + blank blank + E,G		2
70,×	48,×	1 POINT ✓ SLOPE OK (0° < R < 180°)		L'→C < [illegible] L' "ABOVE" M C→A' < C→B' [A' IN/OUT Δ]		L		2 ✓	EQUIDIST (NOT SYM)	FLAG ✓ HEAD ✓	S			ABCD 0X	E,G + any any + E,G		3
×,3	×,3	POSIT OK (NOT SYM NOT 180) SLOPE ✓		C IN Δ ASPECT OF Δ ✓		A		✓	90° ([illegible])	FLAG ✓ HEAD X	R			FH 0X			4
		POSIT OK SLOPE OK	SOME RTTN	C IN Δ' SOME RTTN					OK	FLAG ✓ NO / TWO } HEADS	R or S			1+ REF ✓ 1+ ROT ✓ 0 ×			5
		PT THRU 180°/SYM SLOPE OK ✓	Δ TURNED OVER	Δ TURNED OVER DOMINANT					EQUIDIST BUT SYMMETRY	FLAG ✓ HEAD ✓	APPROX (with C) R (C2 C3 ONLY)		APPROX R (C5 ONLY)	1+ ✓ 0×			6
		PT THRU 180°/SYM SLOPE X	Δ TURNS 180°						SPECIFIC WRONGS	FLAG ✓ HEAD ✓	OTHERS "NO" APPROX S (STILL POINT)	APPROX S	APPROX T				7
60,3		SLOPE ∥ (NOT 180°) POINT NOT MOVED POINT AT CENTRE DOMINANT	B+C Join	Δ' AT C Δ' OUTSIDE C	TAUTOLOGY				POINT ON OBJECT	FLAG ✓ HEAD X	OTHERS "NO" APPROX R,S	T	T				8
			SHAPE LOST	SHAPE LOST	OTHERS WRONG OR UNCLEAR			4X		FLAG X				1+ X		OTHERS	9

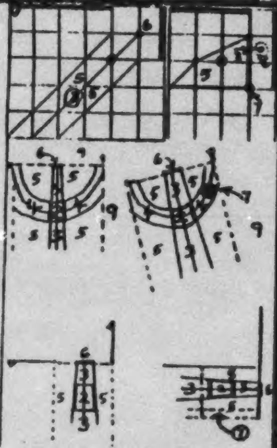
NOTES B1

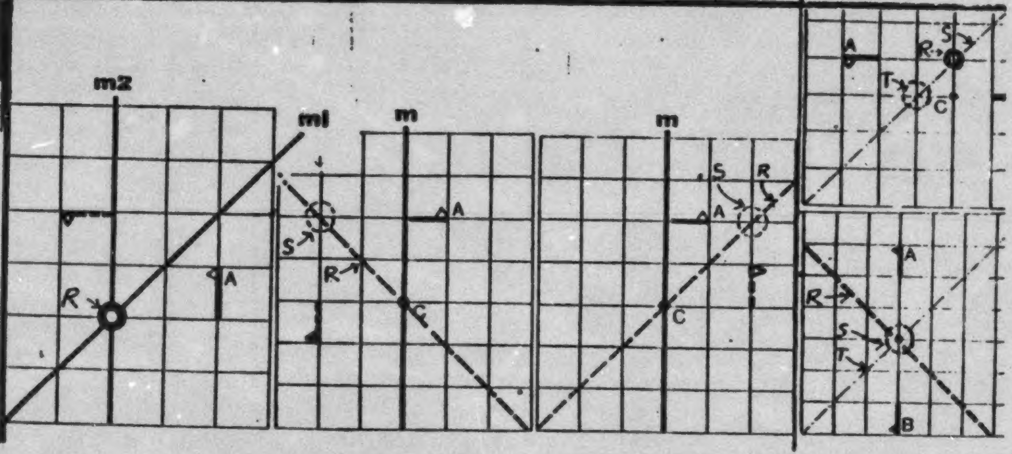
. NO CODE 3 ii

IGNORE LENGTHS & SIZE OF OBJECTS including RELATIVE LENGTH OF ll.

R CAN BE CLOCKWISE BUT SLOPE OK MUST BE MORE THAN 0° LESS THAN 180° IN ASSUMED DIRECTION.

CODES 6 + 7: SYM APPLIES ONLY TO GRID
180 APPLIES THROUGHOUT





321

276

TEST 12 YEAR 2 NUMBER OF CHILDREN = 293 REFLECTIONS & ROTATIONS

VAR	1	2	3	4	5	6	7	8	9	0
1	71.0	11.3	7.5	3.8	0.0	.3	2.4	0.0	.3	3.4
2	57.3	19.8	5.5	1.4	6.1	0.0	3.1	.3	2.4	4.1
3	77.8	1.0	1.7	.7	8.5	2.0	3.4	0.0	.7	4.1
4	32.4	15.4	5.8	4.8	4.1	1.7	3.4	13.0	11.6	7.8
5	8.2	37.2	32.8	9.2	4.1	0.0	3.1	.3	1.0	4.1
6	2.7	13.7	2.7	19.8	9.9	11.9	11.6	18.8	6.1	2.7
7	5.8	49.1	25.6	4.4	4.8	4.4	2.4	0.0	1.4	2.0
8	1.0	20.5	29.4	13.3	10.6	1.4	13.3	1.4	6.1	3.1
9	.7	6.6	23.2	10.9	18.4	3.4	26.6	0.0	5.8	4.1
10	7.8	38.2	28.7	0.0	3.8	2.7	0.0	5.8	3.1	9.9
11	9.9	48.5	14.7	0.0	4.8	5.5	0.0	5.1	2.4	9.2
12	61.4	0.0	0.0	0.0	0.0	13.3	0.0	0.0	14.7	10.6
13	71.3	5.1	3.4	0.0	2.7	3.1	0.0	0.0	4.4	9.9
14	78.8	0.0	0.0	2.7	0.0	6.8	.7	0.0	5.1	5.8
15	20.8	0.0	30.0	21.2	0.0	0.0	0.0	11.3	8.5	8.2
16	5.5	19.1	13.0	21.8	4.8	2.4	23.5	0.0	1.0	8.9
17	4.8	20.8	14.7	28.3	.3	8.2	.7	2.0	3.4	16.7
18	88.1	0.0	.7	1.7	0.0	0.0	0.0	0.0	4.4	5.1
19	88.1	0.0	.3	1.7	0.0	0.0	0.0	0.0	4.1	5.8
20	47.8	29.4	.7	9.6	0.0	0.0	0.0	0.0	5.5	7.2
21	46.1	18.1	1.0	18.1	0.0	0.0	0.0	0.0	7.8	8.9
22	75.8	0.0	.7	6.1	0.0	0.0	0.0	0.0	4.4	13.0
23	65.9	0.0	.3	7.8	0.0	0.0	0.0	0.0	7.2	18.8
24	37.2	11.9	0.0	22.2	0.0	0.0	0.0	0.0	7.5	21.2
25	82.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.9	5.5
26	31.1	0.0	0.0	28.3	0.0	0.0	0.0	5.5	3.1	32.1
27	37.2	.7	0.0	22.2	0.0	0.0	0.0	.3	3.8	35.8
28	66.9	16.0	0.0	0.0	7.2	0.0	.3	3.4	1.0	5.1
29	30.7	36.2	0.0	.7	2.7	0.0	2.4	6.8	13.0	7.5
30	20.8	16.0	1.0	6.5	5.1	14.7	1.0	22.2	2.4	10.2
31	9.6	25.9	4.4	19.8	19.8	0.0	.3	9.2	2.4	8.5
32	2.4	11.3	12.3	25.3	22.5	.3	.3	11.9	4.1	9.6
33	10.2	46.1	0.0	0.0	9.6	14.0	1.0	4.8	5.1	9.2
34	4.1	11.6	1.7	18.4	1.7	17.1	0.0	19.5	5.5	20.5
35	22.2	.7	.3	8.2	.7	30.0	3.1	13.0	6.5	15.4
36	14.3	0.0	1.0	18.1	6.1	19.1	3.1	12.6	8.2	17.4
37	13.3	1.0	9.6	1.7	10.6	4.8	11.9	11.9	13.7	21.5
38	10.6	.3	5.5	8.5	24.2	3.1	5.8	10.2	5.1	26.6
39	51.9	0.0	0.0	0.0	0.0	0.0	0.0	6.1	20.8	21.2
40	5.5	0.0	39.6	1.4	0.0	0.0	0.0	5.8	18.8	29.0
41	22.2	2.0	0.0	0.0	0.0	0.0	0.0	6.1	36.2	33.4
42	11.6	40.6	31.7	8.5	0.0	0.0	0.0	0.0	.3	7.2
43	56.7	0.0	1.0	0.0	8.9	15.0	0.0	1.7	6.5	10.2
44	25.9	0.0	7.2	0.0	20.5	2.7	19.8	3.4	5.8	14.7
45	.7	13.0	14.3	3.1	7.2	24.6	0.0	1.0	16.7	19.5
46	.7	7.8	14.7	9.6	11.6	9.2	18.4	0.0	10.9	17.1
47	3.4	9.6	9.9	0.0	5.8	40.3	0.0	3.4	11.3	16.4
48	3.8	10.6	4.8	0.0	16.4	1.7	19.8	8.9	8.5	25.6
49	15.4	0.0	0.0	0.0	0.0	0.0	16.4	1.7	51.2	15.4
50	8.5	1.4	.3	.7	0.0	.3	8.9	4.8	57.0	18.1
51	7.5	.7	.3	0.0	0.0	1.0	6.1	.3	59.4	24.6
52	4.8	7.2	0.0	0.0	0.0	0.0	.7	6.8	41.6	38.9
53	11.6	3.1	0.0	0.0	0.0	.7	.3	4.8	42.0	37.5
54	7.5	1.7	3.4	5.1	4.4	18.4	0.0	0.0	10.2	49.1
55	7.5	.7	6.5	0.0	0.0	0.0	0.0	0.0	20.8	64.5
56	9.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.5	69.6
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

TEST 12 YEAR 3 NUMBER OF CHILDREN = 449 REFLECTIONS & ROTATIONS

VAR	1	2	3	4	5	6	7	8	9	0
1	75.1	8.5	7.6	2.7	0.0	0.0	3.1	.2	1.3	1.
2	66.8	12.5	4.9	.4	6.2	0.0	2.9	.7	2.2	3.
3	84.4	1.3	1.1	1.1	5.6	.7	2.4	0.0	1.3	2.
4	36.7	12.9	9.4	2.4	4.9	.7	4.7	6.7	14.9	6.7
5	8.0	40.8	32.7	7.6	3.8	0.0	2.7	.7	.9	2.
6	3.6	20.9	3.3	20.0	7.3	11.6	9.1	17.1	4.0	2.
7	12.7	48.6	23.8	4.5	3.3	2.0	1.6	0.0	2.0	1.6
8	2.4	20.5	26.3	15.8	9.6	1.8	14.0	1.1	5.1	3.
9	2.2	9.1	16.7	13.6	20.3	3.6	27.2	.2	4.5	2.7
10	7.1	40.1	29.0	0.0	5.1	2.2	0.0	6.5	2.4	7.6
11	11.6	46.8	18.3	0.0	5.6	4.7	0.0	2.7	2.2	8.
12	62.8	0.0	0.0	0.0	0.0	11.1	0.0	0.0	15.4	10.7
13	65.9	4.5	5.6	0.0	4.9	1.8	0.0	0.0	4.9	12.
14	83.5	0.0	0.0	3.6	0.0	6.7	.4	0.0	4.7	1.1
15	20.9	0.0	33.4	19.8	0.0	0.0	0.0	12.9	6.0	6.
16	8.0	22.0	12.7	20.3	4.0	1.1	21.6	.4	4.2	5.
17	6.2	23.8	14.7	28.7	.7	7.8	.9	2.9	3.8	10.
18	87.8	0.0	.4	2.2	0.0	0.0	0.0	0.0	4.9	4.
19	80.8	0.0	1.6	6.2	0.0	0.0	0.0	0.0	5.6	5.
20	53.9	24.7	.4	7.3	0.0	0.0	0.0	0.0	7.3	6.2
21	50.6	14.7	1.6	17.6	0.0	0.0	0.0	0.0	7.8	7.
22	75.7	0.0	2.0	5.8	0.0	0.0	0.0	0.0	4.5	12.
23	67.5	0.0	.7	7.3	0.0	0.0	0.0	0.0	7.6	16.
24	41.9	9.1	0.0	23.4	0.0	0.0	0.0	0.0	6.0	19.
25	89.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.7	3.
26	35.6	.9	0.0	24.1	0.0	0.0	0.0	7.3	7.3	24.7
27	44.8	0.0	0.0	19.8	0.0	0.0	0.0	0.0	5.3	30.1
28	67.9	17.4	0.0	.2	8.7	0.0	.2	1.8	.7	3.
29	37.9	35.1	0.0	.4	3.1	0.0	3.3	5.6	8.5	5.1
30	23.2	20.5	.4	9.6	4.5	13.6	2.4	17.6	1.6	6.7
31	7.1	34.1	4.5	15.8	21.8	0.0	1.1	7.8	.7	7.1
32	2.0	16.3	12.5	24.9	22.0	.2	.7	10.0	3.3	8.
33	9.8	56.8	0.0	0.0	4.0	10.5	0.0	4.2	5.6	9.1
34	2.9	14.0	1.3	25.2	1.3	14.3	0.0	18.9	4.5	17.
35	26.1	1.1	.7	7.1	1.1	28.1	3.8	14.5	7.8	9.
36	19.4	.7	.9	17.4	3.8	22.3	2.9	14.3	6.9	11.
37	15.4	2.4	10.7	1.6	11.6	6.7	14.7	13.1	9.1	14.7
38	15.6	1.6	5.3	4.2	24.1	2.4	7.6	14.7	4.0	20.
39	49.4	0.0	0.0	0.0	0.0	0.0	0.0	7.8	19.8	22.
40	5.3	0.0	38.5	2.2	0.0	0.0	0.0	7.3	18.0	28.
41	22.0	.9	0.0	.4	0.0	0.0	0.0	6.9	35.6	34.1
42	16.7	44.5	25.2	8.9	.2	0.0	0.0	0.0	.7	3.
43	61.7	.2	.4	0.0	6.2	14.9	0.0	1.3	6.0	9.
44	30.7	0.0	8.7	0.0	20.5	1.6	13.6	2.7	8.2	14.
45	1.6	15.1	12.9	4.9	7.1	29.8	0.0	.2	16.3	12.
46	1.6	12.0	14.9	10.9	8.9	9.8	20.5	0.0	8.0	13.
47	3.8	12.9	9.4	0.0	6.9	41.9	0.0	3.1	8.7	13.
48	4.0	15.8	4.9	0.0	14.3	.4	19.2	11.8	8.9	20.
49	17.4	0.0	0.0	.4	0.0	0.0	14.5	1.6	55.2	10.
50	10.9	.9	.7	2.0	0.0	.2	9.1	7.1	57.2	11.
51	10.9	.4	.2	.7	0.0	.2	6.5	1.1	63.7	16.
52	8.5	14.3	0.0	0.0	0.0	0.0	2.0	5.8	41.9	29.
53	12.4	5.3	0.0	0.0	0.0	1.8	.9	5.8	41.0	31.
54	5.3	2.2	4.9	3.6	3.9	14.3	0.0	0.0	11.4	[illegible]
55	[illegible]	1.5	0.2	0.0	0.0	0.0	0.0	0.0	20.0	61.
56	4.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.2	[illegible]
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0	0.0	0.0	0.0	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0	0.0	0.0	0.0	[illegible]	[illegible]

TEST 12 YEAR 4 NUMBER OF CHILDREN = 284 REFLECTIONS & ROTATIONS

VAR	1	2	3	4	5	6	7	8	9	0
1	72.5	10.9	8.5	2.5	0.0	.4	3.2	.7	.4	1.1
2	66.2	16.2	1.9	.7	4.2	.4	3.9	0.0	3.2	3.5
3	83.1	3.5	1.4	.4	7.4	0.0	2.1	0.0	.4	1.[illegible]
4	44.9	17.3	4.9	4.5	2.5	0.0	2.1	3.5	11.3	4.9
5	13.9	39.4	32.4	8.1	2.5	0.0	1.4	.7	1.4	3.2
6	2.8	25.7	2.5	25.0	6.0	10.9	7.0	14.6	2.9	2.5
7	16.9	54.2	13.7	2.8	7.4	0.0	2.8	0.0	.7	2.5
8	2.8	30.6	25.7	14.8	9.9	1.8	10.2	.4	2.1	1.8
9	3.5	13.4	24.3	10.9	13.4	1.4	20.4	0.0	4.6	2.1
10	9.9	43.7	31.3	0.0	2.8	2.1	0.0	4.2	1.8	4.2
11	13.7	46.1	23.6	0.0	2.8	4.6	0.0	3.2	1.4	4.6
12	76.1	0.0	0.0	0.0	0.0	12.3	0.0	0.0	12.0	5.6
13	75.7	7.0	6.0	0.0	1.8	1.1	0.0	0.0	2.8	5.6
14	86.6	0.0	0.0	4.6	0.0	1.8	0.0	0.0	3.9	3.2
15	31.3	0.0	26.4	11.3	0.0	0.0	0.0	17.6	5.6	7.7
16	9.5	30.6	12.3	15.8	3.5	1.8	19.0	0.0	.7	6.7
17	5.6	33.8	14.9	23.2	.4	6.0	.4	1.6	4.2	9.9
18	90.5	0.0	1.4	.7	0.0	0.0	0.0	0.0	2.1	5.3
19	88.4	0.0	.7	3.2	0.0	0.0	0.0	0.0	2.8	4.9
20	65.5	18.3	0.0	6.3	0.0	0.0	0.0	0.0	5.3	4.6
21	59.5	12.3	1.1	14.8	0.0	0.0	0.0	0.0	5.6	6.7
22	80.6	0.0	.4	4.9	0.0	0.0	0.0	0.0	3.9	10.2
23	75.7	0.0	.4	4.6	0.0	0.0	0.0	0.0	5.3	14.1
24	49.6	6.3	0.0	22.5	0.0	0.0	0.0	0.0	5.6	15.[illegible]
25	91.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.2	4.[illegible]
26	44.0	0.0	.4	20.4	0.0	0.0	0.0	6.7	5.3	23.2
27	56.0	.7	0.0	18.7	0.0	0.0	0.0	0.0	4.6	26.[illegible]
28	73.2	17.6	0.0	0.0	2.1	0.0	0.0	1.4	1.1	4.[illegible]
29	37.0	39.6	.4	.4	.7	0.0	4.6	3.2	7.7	6.3
30	27.1	18.7	.4	8.1	3.5	17.6	1.1	12.3	2.1	9.[illegible]
31	7.7	37.7	4.2	17.3	15.5	.4	.7	7.0	1.8	7.7
32	1.8	15.1	11.6	33.1	15.8	0.0	.7	10.6	1.4	9.9
33	12.0	59.2	0.0	0.0	3.9	7.0	0.0	2.1	5.6	10.2
34	4.9	19.7	1.4	20.4	2.5	12.0	0.0	18.0	3.2	16.[illegible]
35	32.4	0.0	1.1	5.3	.7	31.3	2.5	7.0	6.7	13.0
36	23.2	.7	0.0	22.9	1.8	16.7	3.5	6.3	8.5	14.4
37	23.2	1.4	11.3	2.5	8.1	5.3	14.8	4.9	12.7	15.[illegible]
38	19.7	1.4	7.4	4.6	21.1	2.8	8.1	8.5	2.1	24.3
39	53.5	0.0	0.0	0.0	0.0	0.0	0.0	5.6	19.7	21.1
40	8.8	0.0	45.4	1.4	0.0	0.0	0.0	6.3	11.6	26.4
41	27.5	1.4	0.0	.4	0.0	0.0	0.0	4.9	35.2	30.6
42	20.9	42.3	22.2	5.3	0.0	0.0	0.0	0.0	.4	9.2
43	61.3	0.0	.4	0.0	5.3	13.0	0.0	2.8	3.9	13.[illegible]
44	34.5	.4	7.0	0.0	17.3	2.5	11.6	3.2	6.0	17.6
45	2.5	18.3	12.7	4.6	5.6	22.9	0.0	.7	12.0	20.[illegible]
46	1.4	14.8	13.4	8.1	7.0	9.2	19.4	0.0	7.0	19.7
47	3.2	16.5	8.1	0.0	7.0	33.1	0.0	1.8	10.2	20.1
48	4.6	10.9	2.5	0.0	16.5	.4	13.0	8.8	10.9	32.[illegible]
49	21.5	0.0	0.0	0.0	.4	0.0	18.7	1.1	44.0	14.[illegible]
50	16.5	1.1	0.0	1.4	.4	.4	10.9	6.0	46.5	16.[illegible]
51	14.4	.7	.7	0.0	0.0	0.0	6.7	1.1	53.5	22.[illegible]
52	6.3	13.0	0.0	0.0	0.0	0.0	1.4	6.7	33.1	39.[illegible]
53	19.7	2.8	0.0	0.0	0.0	1.4	0.0	5.6	29.6	40.[illegible]
54	7.0	3.2	3.9	3.5	9.2	16.2	0.0	0.0	9.9	47.[illegible]
55	8.5	1.4	4.9	0.0	0.0	0.0	0.0	0.0	27.5	57.[illegible]
56	10.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.5	[illegible]
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.[illegible]
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10[illegible]

PHI CORRELATION MATRIX. REFLECTIONS & ROTATIONS N=1026

88.6	18																														
87.3	25	272																													
86.2	26	[illegible]	[illegible]																												
85.0	19	673	[illegible]	301																											
84.6	3	337	[illegible]	241	[illegible]																										
83.1	1	[illegible]	[illegible]	334	311	[illegible]																									
83.0	14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]																								
82.9	5	[illegible]	113	228	295	316	218	230																							
79.6	20	579	[illegible]	435	317	343	[illegible]	[illegible]	215																						
79.5	2	296	157	244	265	352	467	195	275	331																					
78.2	13	248	166	273	233	247	234	295	241	324	223																				
77.5	11	276	137	244	256	274	257	253	295	335	310	585																			
77.1	22	570	159	465	276	343	[illegible]	291	169	655	236	239	278																		
72.7	29	324	244	514	295	335	265	301	314	412	339	343	319	353																	
70.2	13	255	116	[illegible]	221	135	[illegible]	136	214	255	144	289	347	273	241																
69.3	23	479	248	332	347	292	242	234	263	502	248	272	283	733	393	263															
64.9	33	295	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	273	[illegible]	[illegible]	339	298	327	318	544	248	356														
64.4	12	232	84	258	217	[illegible]	252	195	218	291	248	360	389	234	280	543	246	264													
60.5	7	275	124	[illegible]	[illegible]	[illegible]	[illegible]	236	297	275	311	293	266	224	324	201	231	283	257												
64.2	43	241	181	227	193	216	142	184	207	226	234	211	219	253	275	188	244	274	182	227											
59.2	42	232	220	245	244	241	245	221	206	273	261	213	235	283	394	212	311	357	248	237	341										
59.1	6	216	129	217	213	253	207	204	341	263	285	274	282	211	295	193	206	297	259	349	174	227									
53.7	4	244	139	267	244	303	317	282	314	320	401	318	351	284	365	296	331	369	343	421	288	357	383								
51.8	21	315	143	337	376	270	220	254	160	441	240	273	250	392	393	201	414	336	239	217	251	336	247	331							
51.3	39	297	220	204	207	234	232	242	185	296	273	235	254	294	370	152	306	349	228	233	307	395	227	346	323						
47.1	40	243	226	242	254	240	206	212	165	273	231	229	269	304	358	175	297	333	219	202	271	382	217	303	328	790					
44.1	27	243	314	[illegible]	202	265	237	274	231	351	274	245	266	390	407	243	445	372	282	250	271	373	287	410	416	389	362				
42.7	24	276	132	369	302	206	215	253	268	356	262	236	267	428	347	256	484	345	266	241	270	364	241	394	431	321	334	508			
42.3	30	220	183	297	216	263	216	261	265	271	287	304	277	250	502	221	270	439	294	316	308	398	339	450	337	388	362	404	353		
40.7	31	243	184	274	192	231	150	237	251	246	238	260	290	248	356	228	255	356	226	288	227	329	278	391	288	300	294	331	319	382	
36.6	26	272	277	203	245	259	113	257	194	317	245	235	255	342	352	195	400	338	214	225	233	335	238	365	386	373	380	722	504	360	303
32.5	9	189	151	175	165	205	209	157	257	257	212	295	293	214	219	251	213	252	302	274	120	224	327	393	236	251	259	261	306	257	289
31.4	17	163	175	191	196	191	197	232	243	227	220	265	263	223	277	238	240	290	287	374	223	309	332	463	312	340	334	377	354	398	341
31.3	16	182	168	191	195	242	197	215	263	247	232	275	278	237	285	223	253	328	277	360	239	299	339	466	256	338	315	353	339	383	356
30.5	44	191	176	196	177	219	[illegible]	214	165	208	189	232	225	230	306	214	225	340	218	257	335	365	237	339	305	383	363	345	305	418	299
27.4	35	179	168	226	166	208	177	201	181	216	214	218	219	235	351	166	257	350	218	236	298	385	213	346	273	336	322	370	353	407	335
25.4	8	[illegible]	[illegible]	175	164	193	113	160	221	206	196	254	250	195	237	185	204	260	261	242	172	225	313	331	219	197	211	279	246	279	253
25.0	41	143	145	225	166	177	163	188	154	213	208	184	192	207	277	149	227	272	169	184	247	300	168	282	259	381	435	345	312	331	264
23.8	15	142	116	144	151	161	134	215	207	160	141	217	186	179	198	148	178	199	180	212	149	235	199	270	200	260	239	265	221	281	245
19.5	36	145	157	182	151	187	129	156	157	193	176	194	206	209	262	164	204	284	201	211	264	353	219	308	287	327	334	321	336	415	323
18.7	34	156	130	177	152	161	130	159	169	205	169	217	198	208	282	143	216	310	184	223	246	261	221	315	278	282	273	309	293	363	273
18.7	37	149	152	185	145	188	122	176	175	180	169	187	196	196	271	187	227	289	205	234	277	352	226	355	303	307	308	354	358	418	293
17.9	49	159	162	172	175	176	[illegible]	184	162	204	136	204	215	218	257	177	228	263	220	242	265	305	229	352	268	343	342	357	377	350	243
17.1	48	113	132	151	153	148	126	128	147	145	140	170	176	142	184	154	183	262	185	148	161	239	150	239	220	265	252	291	263	251	256
17.0	45	129	117	135	[illegible]	140	127	128	143	153	152	175	168	159	236	158	171	212	189	232	234	264	164	278	197	232	255	190	229	296	227
16.6	32	126	106	148	99	165	95	152	169	153	141	191	171	180	249	181	165	261	221	182	222	274	221	298	188	229	230	232	240	308	340
16.5	38	126	161	176	157	165	129	165	155	191	153	196	201	198	260	168	215	298	236	197	274	326	220	327	302	291	281	341	339	380	278
16.5	47	117	145	139	127	143	129	144	135	178	101	190	170	154	201	168	198	243	214	229	215	294	188	254	197	259	276	256	254	295	225
14.6	53	165	[illegible]	133	[illegible]	135	[illegible]	179	123	140	162	198	190	133	222	155	167	246	169	204	217	265	198	284	239	298	295	282	267	276	229
12.9	46	92	120	145	[illegible]	127	103	127	157	143	129	153	158	112	196	155	142	215	182	173	222	242	177	240	219	217	238	221	268	254	250
12.9	50	124	120	128	153	127	97	134	142	143	129	174	179	167	209	142	192	233	188	185	246	271	183	304	225	304	302	321	303	313	261
11.5	51	129	137	144	103	143	47	146	159	159	129	146	165	174	200	148	193	220	184	197	211	268	175	273	274	302	302	326	269	309	254
9.4	56	[illegible]	92	[illegible]	97	[illegible]	[illegible]	107	113	112	113	121	141	119	166	128	163	159	161	95	131	164	110	204	203	212	233	233	209	172	108
9.1	55	142	69	97	113	120	69	77	92	117	126	117	137	139	162	116	165	203	156	143	131	213	138	204	168	212	239	232	214	217	159
6.6	54	95	111	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	106	102	115	126	136	79	143	138	107	94	136	189	70	200	163	204	211	252	245	208	121
5.9	52	90	33	130	62	93	[illegible]	113	101	106	36	126	96	127	144	100	158	141	117	110	128	158	75	183	193	162	192	225	232	226	152
Facility	Var.	18	35	28	19	3	1	14	5	20	2	13	11	22	29	13	23	33	12	7	43	42	6	4	21	39	40	27	24	30	31

PHI CORRELATION MATRIX REFLECTIONS & ROTATIONS CONTD

36.6	[illegible]																										
32.5	9	274																									
31.4	17	[illegible]	374																								
31.3	16	322	430	357																							
30.5	44	357	241	311	[illegible]																						
27.4	35	362	241	314	334	[illegible]																					
25.4	[illegible]	217	438	351	511	[illegible]	[illegible]																				
25.0	41	314	141	289	276	332	337	185																			
23.8	15	244	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	270																		
19.5	36	335	247	319	336	460	[illegible]	144	347	233																	
18.7	34	293	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	318	201	451																
18.7	37	276	218	359	364	419	594	259	347	283	669	384															
17.9	49	334	427	422	340	414	[illegible]	258	362	234	424	401	446														
17.1	48	310	178	268	269	448	261	217	289	233	207	200	300	341													
17.0	45	174	174	237	312	332	318	153	274	198	289	222	315	276	306												
16.6	32	243	236	288	264	256	261	245	221	255	363	310	391	228	243	196											
16.5	38	343	274	339	344	395	[illegible]	307	351	256	577	447	669	429	280	275	388										
16.5	47	256	236	265	261	356	275	229	278	177	325	238	312	367	315	366	212	341									
14.6	53	315	221	326	309	266	277	227	328	241	346	353	346	424	311	180	245	344	277								
12.9	46	257	265	317	388	371	292	169	289	222	384	315	367	374	354	431	275	386	370	318							
12.9	50	348	[illegible]	336	343	407	425	272	336	270	369	412	315	526	297	221	212	402	331	425	321						
11.5	51	340	272	342	303	418	326	245	363	257	362	367	367	468	356	252	233	399	292	378	345	518					
9.4	56	221	199	222	194	215	185	168	278	197	272	257	266	303	192	104	135	263	191	321	216	336	335				
9.1	55	246	179	225	211	233	232	135	257	134	324	248	266	321	217	182	206	308	189	359	304	365	333	399			
6.6	54	244	133	216	251	223	240	150	172	182	244	243	243	304	181	130	134	251	145	333	178	272	210	358	393		
5.9	52	236	160	220	185	243	224	184	226	130	240	228	270	269	258	193	131	299	255	385	260	334	361	301	250	297	
Facility	Var.	26	9	17	16	44	35	8	41	15	36	34	37	49	48	45	32	38	47	53	46	50	51	56	55	54	52

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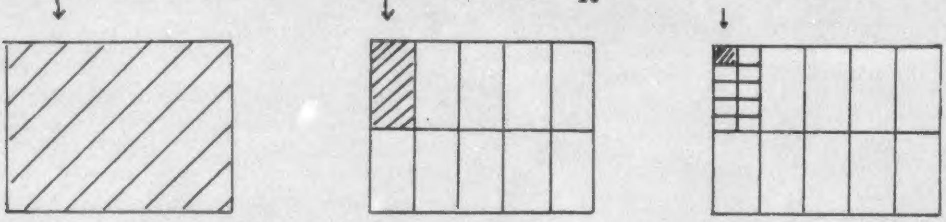
CSMS

NUMBERS 2 : PLACE-VALUE & DECIMALS

NAME .. . .
TODAY'S DATE ....
BOY OR GIRL ....
CLASS ....
SCHOOL ....
DATE OF BIRTH ... day ..... month .... year

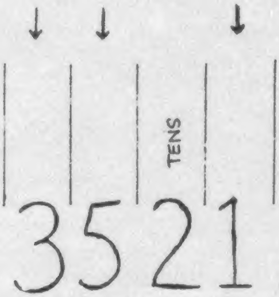
IF THIS IS ONE UNIT (1), then THIS IS ONE TENTH ($\frac{1}{10}$), and THIS IS ONE HUNDREDTH ($\frac{1}{100}$)



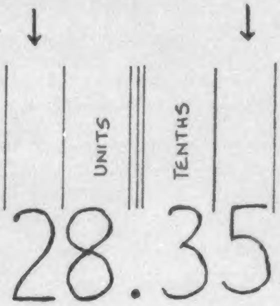
Trial Items

Fill in the empty columns:

(1)



(2)



Notes

These mean the same:

(A) 30 000 ↔ 30,000

(B) 4 . 21 ↔ 4·21

NOW TURN OVER

CHECK YOUR ANSWERS TO THE TRIAL ITEMS

THOUSANDS	HUNDREDS	TENS	UNITS (ONES)
3	5	2	1

TENS	UNITS	TENTHS	HUNDREDTHS
2	8.	3	5

NOW DO THESE:

Working

1) 5214 The 2 stands for 2 HUNDREDS

521 The 2 stands for 2 ....Var.1..........................

521 400 The 2 stands for 2 ....Var.2..........................

2) 0.126 The 2 stands for 2 HUNDREDTHS

0.2 The 2 stands for 2 ....Var.3..........................

0.260 The 2 stands for 2 ....Var.4..........................

0.412 The 2 stands for 2 ....Var.5..........................

3) Write in figures:

Two hundred and fifty ....Var.6....

Four hundred thousand and seventy three ....Var.7.....

4) Write in words, as you would say it:

8030 ....Var.8...

140 000 ....Var.9...

0.29 ....Var.10..

5) Add ten

3597 → .Var.11

0.15 → .Var.12

Add one hundred

21 534 → Var.13

19 930 → Var.14

Add one tenth

4.254 → .Var.15

2.9 → .Var.16

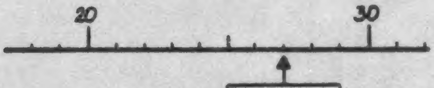
Take away one hundred

583 → ..Var.17

30 000 → ..Var.18

Working

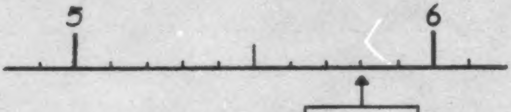
6)



This number is

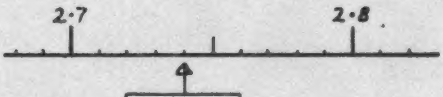
Var.19

GIVE THE REST OF YOUR ANSWERS AS DECIMALS



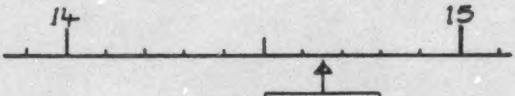
This number is

Var.20



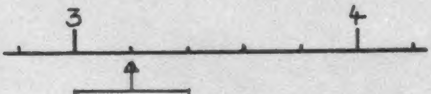
This number is

Var.21



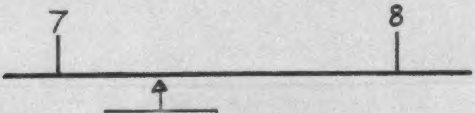
This number is about

Var.22



This number is

Var.23

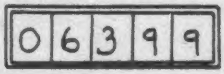


This number is about

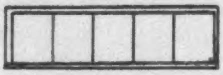
Var.24

Working

7) This meter counts the people going into a football stand.



After one more person has gone in, it will read:



Var. 25

---

8) The number that is 2 less than 17 000 is: .... Var. 26

---

9) For the answer to a maths question Dave got 4.9 and Lynn got 4.90

Is there any difference between them? .... Var. 27

Why? ....

....

....

---

10) Ring the BIGGER of the two numbers: 0.75 or 0.8

Why is it bigger? .... Var. 28

....

....

For each pair, ring the BIGGER number:

20 100 or 20 095 Var. 29

7.55 or 7.5 Var. 30

4.06 or 4.5 Var. 31

---

11) Six tenths as a decimal is 0.6

How would you write as decimals: three hundredths .... Var. 32

eleven thousandths .... Var. 33

eleven tenths .... Var. 34

Four tenths is the same as .... hundredths Var. 35

Working

12) Write down any number between:

4000 and 5000 ....Var.36...

4100 and 4200 ....Var.37....

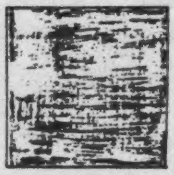
0.4 and 0.5 ....Var.38....

0.41 and 0.42 ....Var.39....

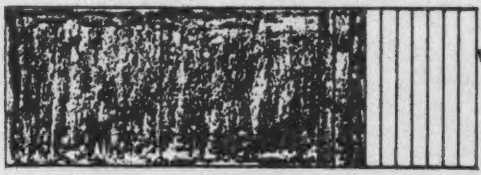
How many different numbers could you write down which lie between 0.41 and 0.42? ....Var.40........

---

13)



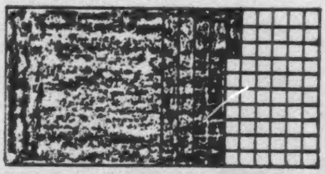
This is 1 square unit



Var.41

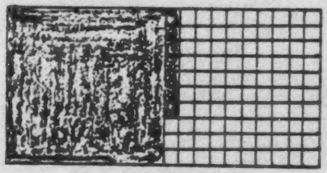
The area shaded is . square units

(Give your answers as DECIMALS)



Var.42

The area shaded is . square units



Var.43

The area shaded is . square units

Working

14) Multiply by ten

4 → ..Var.44

5.13 → ..Var.45

Multiply by one hundred

317 → ..Var.46

2.3 → ..Var.47

Write 'NO' if you think there is no answer

Divide by one hundred

1600 → ..Var.48

3.7 → ..Var.49

Divide by twenty

24 → ..Var.50

16 → ..Var.51

---

15) Ring the one in each pair which gives the BIGGER answer:

(a) 8 x 4 or 8 ÷ 4

(b) 8 x 0.4 or 8 ÷ 0.4

(c) 0.8 x 0.4 or 0.8 ÷ 0.4

Var 52

---

16) Add

$$\begin{array}{r} 263 \\ +\ 978 \\ \hline \end{array}$$

Var.53

13.4 + 2.7 = ...

Var.54

Subtract

$$\begin{array}{r} 2312 \\ -\ 547 \\ \hline \end{array}$$

Var.55

8.44 − 6.37 = ...

Var.56

---

17) 20 x 500 = ...Var.57...

60 ÷ 3 = .Var.60.

5 x 0.2 = ...Var.58...

60 ÷ 0.3 = Var.61

0.2 x 0.4 = ...Var.59...

18) Ring the number NEAREST IN SIZE to:

$\boxed{182}$ → 100 / 82 / 180 / 150 / 200 / 190 Var.62

$\boxed{2.9}$ → 3 / 30 / 2 / 20 / 0 / 1 Var.63

$\boxed{0.18}$ → 0.1 / 10 / 0.2 / 20 / 0 / 1 / 2 Var.64

Ring the number you think is NEAREST IN SIZE to the answer (do not work out the sum):

$\boxed{2.9 \times 7}$ → .002 / .02 / .2 / 2 / 20 / 200 / 2000 Var.65

$\boxed{0.29 \times 7.1}$ → .002 / .02 / .2 / 2 / 20 / 200 / 2000 Var.66

$\boxed{59 \div 190}$ → .003 / .03 / .3 / 3 / 30 / 300 / 3000 Var.67

---

19) Ring the CALCULATION you would need to do to find the answer:

A.

A table is 92.3 centimetres long.
About how many inches is this?
(1 inch is about 2.54 centimetres.)

2.54 + 92.3	92.3 ÷ 2.54	
2.54 ÷ 92.3	92.3 − 2.54	Var.68
2.54 − 92.3	92.3 x 2.54	

B.

My car tank was full after I put in 6.44 gallons of petrol.
The tank holds 8.37 gallons.
How much petrol was in it to start with?

8.37 + 6.44	6.44 ÷ 8.37	
8.37 ÷ 6.44	6.44 − 8.37	Var.69
8.37 − 6.44	6.44 x 8.37	

C.

The price of minced beef is shown as 88.2 pence for each kilogram.
What is the cost of a packet containing 0.58 kg of minced beef?

88.2 + 0.58	0.58 ÷ 88.2	
88.2 ÷ 0.58	0.58 − 88.2	Var.70
88.2 − 0.58	0.58 x 88.2	

D.

The cost of the 6.44 gallons of petrol was £4.86.
What would the price of one gallon be?

6.44 + 4.86	4.86 ÷ 6.44	
6.44 ÷ 4.86	4.86 − 6.44	Var.7
6.44 − 4.86	4.86 x 6.44	

E.

My car can go 41.8 miles on each gallon of petrol on a motorway.
How many miles can I expect to travel on 8.37 gallons?

41.8 + 8.37	8.37 ÷ 41.8	
41.8 ÷ 8.37	8.37 − 41.8	Var.7
41.8 − 8.37	8.37 x 41.8	

20) One **story** which matches this sum is:

$\boxed{6+2=8} \longrightarrow$ Linda had 6 records. She got another 2 for her birthday. So now she has 8.

Write a story which matches this sum : Var.73

$\boxed{6.4+2.3=8.7} \longrightarrow$ **Story:**

MARKING SCHEME: PLACE VALUE AND DECIMALS

QUESTION	1	2	3	4	5	7	8
1.1(1)	Tens					Tenths	Other 'This'
1.2(2)	Ten Thousands		Ten thousandths		Thousands	Millions	--ths
2.1(3)	Tenths		Tens		Units	Hundred(th)s	Thousand(th)s
2.2(4)	Tenths		Tens		Units	Hundred(th)s	Thousand(th)s
2.3(5)	Thousandths		Thousands		Units	Hundred(th)s	
3.1)(6) 3.2)	250 400073					Correct Wrong	Wrong Correct
4.1)(7)) 4.2)	Eight thousand thirty (A) Hundred & forty thousand					Correct)) Wrong)	Wrong)) Correct)
4.3(8)	(Nought) point two nine		Twenty nine hundredths			Nought point twenty nine	Twenty nine
5.1a(9)	3607						35107
5.1b(10)	10.15					0.25	25
5.2a)(11) 5.2b)	21634 20030					Correct Wrong	Wrong Correct
5.3a(12)	4.354				4.264	4.255	14.254
5.3b(13)	3.0				2.10	2.19	3.9
5.4a)(14) 5.4b)	483) 29900)					Correct Wrong	Wrong Correct

QUESTION	1	2	3	4	5	7	8
6.1)))(15))) 6.2)	27 correct)omitt-)ing No.)before)dec.pt)after 5.8 .8)pt.OK			Correct 13, 1.3	Wrong 13, 1.3	Correct Wrong other	Wrong Correct
6.3(16)	2.74				2.7.4	27.4	31 or 3.1 or 2.11
6.4(17)	14.65				14.6.5	$14.6\frac{1}{2}$	14.6 or 14.7
6.5(18) 6.6(19)	3.2 7.3	7.25 7.35				7.2 7.4	3.1 $7.2\frac{1}{2}$ $7.3^{1}/_{3}$
7.0) (20)) 8.0)	06400)) 16996)		Correct) 16.998) 15.000) 15)			Correct) Other)	Wrong)) Correct)
9.0(21)	Yes + reasoning involving accuracy or sig. figs.	No + reason which is not wrong, or no reason					No + wrong reason (clearly wrong)
10.1)))(22)) 10.2)	0.8 + reason which is not clearly wrong, or no reason 20 100				Omit Correct	correct Wrong	0.75; or 0.8 + clearly wrong reason Correct
10.3))(23) 10.4)	7.55)) 4.5)					Correct Wrong	Wrong)) Correct)
11.1(14) 11.2(15)	.03 .011				.11	.300 .11000	.0011
11.3)(16) 11.4)	1.1) 40)			.11) correct)	.11) Wrong)	Correct) Wrong)	Wrong) Other) Correct)

QUESTION	1	2	3	4	5	7	8
12.1) 12.2)(17)	4---) 41--)					Correct) Wrong)	Wrong) Correct)
12.3) 12.4)(18)	0.4-) 0.41-)			½ in either answer eg .41½		Correct) Wrong)	Wrong) Correct)
12.5(19)	Infinitely many, as many as you like		Hundred) Lots)	100 1000,99 etc.	10,8,9	1	None
13.1) (20) 13.2	2.3)) 1.43)			Correct $1.4^{3}/10$ or 1.4.3	Wrong(including $2.^{3}/10$) $1.4^{3}/10$ or 1.4.3	Correct Wrong	Wrong Correct
13.3(21)	1.07						1.7
14.1a) (4))(22) 14.1b)	40)) 51.3) (no working))	Correct Correct & Working on part 2 shown			Correct 5.130 50.130 50.13	Correct Wrong Other	Wrong Correct
14.2a) (317))(23) 14.2b)	31700) 230 (no)) working)	Correct Correct & working			Correct 2.300 2.003 200.300	Correct Wrong	wrong Correct
14.3a) (1600))(24) 14.3b)	16)) .037) (no working))	Correct Correct & working		Correct "No"	Wrong 'No"	Correct Wrong Other	Wrong
14.4a) (24))(25)	1.21(no working)	Correct) Work)	$^{4}/_{20}$ $^{1}/_{5}$	1	1.4	1 or 4	"No"
14.4b(26)	0.8(no working)	Correct)) Work	$^{16}/_{20}$ $^{4}/_{5}$	0	0.16	0 or 16	"No"

QUESTION	1	2	3	4	5	7	8
15a) 15b)(27) 15c)	8x 4 8 ÷ 0.4 0.8 ÷ 0.4						8 x 4 8 x 0.4 0.8 x 0.4
16.1)(28) 16.2)	1241 16.1 (no working)	Correct Correct Written under each other		Correct 15.11	Wrong 15.11	Correct Wrong Other	Wrong Correct
16.3(29)	1765						2235
16.4(30)	2.07 (no working)	Correct written under each other				2.13	2.7
17.1(31)	(20 x 500) 10000(no working)	Correct Working				1000 No working	Wrong Working
17.2(32)	(5 x .02) 1 or 1.0 (no working)	Correct Working				0.10 no working	Wrong working
17.3(33)	(0.2 x 0.4) .08(no working)	Correct Working				0.8 No working	Wrong Working
17.4))(34) 17.5)	20)) 200)			Correct)) 2)	Correct)) 20)	Correct) Wrong) Other)	Wrong Correct
18.1))(35) 18.2)	180)) 3)			Correct)) 30	Correct)) 2	Correct) Wrong) Other)	Wrong
18.3(36)	0.2					20	0.1
18.4))(37) 18.5)	20)) 2)			Correct)) .002)	Wrong)) .002)	Correct) Wrong) Other)	Wrong)) Correct
18.6(38)	.3						3
19.1(39)	92.3 ÷ 2.54			2.54 ÷ 92.3			92.3 x 2.54

QUESTION	1	2	3	4	5	7	8
19.2(40)	8.37 - 6.44			6.44 - 8.37			8.37 + 6.44
19.3(41)	0.58 x 88.2					88.2 ÷ 0.58	0.58 ÷ 88.2
19.4(42)	4.86 ÷ 6.44			6.44 ÷ 4.86			4.86 x 6.44
19.5(43)	8.37 x 41.8					41.8 ÷ 8.37	8.37 ÷ 41.8
(44)	Quantities reasonably measured in tenths (usually involves units) e.g. 6.4kg flour.		Money correct but figures altered e.g. £6.40		Money, ambiguous amounts e.g. £6.4	Objects which could be split into 10, but are not sensibly or usually split e.g. 6.4 apples, 6.4 cakes	Objects which cannot reasonably be split into 10, e.g. 6.4 books, 6.4 sweets, 6.4p

( ) column numbers

TEST 10 YEAR 1 NUMBEP CF CHILDREN = 170 DECIMALS

VAR	1	2	3	4	5	6	7	8	9	0
1	84.1	0.0	0.0	0.0	0.0	0.0	6.5	1.2	7.1	1.2
2	21.8	0.0	1.2	0.0	31.2	0.0	8.2	4.7	31.2	1.2
3	63.5	0.0	7.1	0.0	14.7	0.0	5.3	5.3	2.9	1.8
4	52.9	0.0	4.7	0.0	4.7	0.0	22.9	10.0	1.8	1.2
5	48.2	0.0	6.5	0.0	21.2	0.0	7.1	0.0	14.1	2.9
6	41.2	0.0	0.0	0.0	0.0	0.0	49.4	.6	6.5	2.9
7	49.4	0.0	0.0	0.0	0.0	0.0	21.2	9.4	18.8	2.4
8	25.9	0.0	10.0	0.0	0.0	0.0	25.3	18.8	15.9	1.2
9	55.9	0.0	0.0	0.0	0.0	0.0	0.0	1.2	40.6	4.1
10	22.9	0.0	0.0	0.0	0.0	.6	38.8	11.2	22.9	2.4
11	48.2	0.0	0.0	0.0	0.0	.6	14.1	1.2	31.2	3.5
12	42.4	0.0	0.0	0.0	16.5	.6	5.3	1.2	20.6	4.7
13	37.6	0.0	0.0	0.0	12.9	.6	10.6	9.4	14.7	13.5
14	36.5	0.0	0.0	0.0	0.0	.6	48.2	.6	9.4	14.1
15	58.2	0.0	0.0	5.9	0.0	4.1	19.4	4.1	5.3	4.7
16	31.2	0.0	0.0	0.0	4.1	1.2	6.5	27.1	20.6	2.9
17	24.1	0.0	0.0	0.0	0.0	1.2	18.2	14.1	33.5	9.4
18	22.9	0.0	0.0	0.0	0.0	1.8	0.0	47.6	22.4	8.8
19	29.4	1.2	0.0	0.0	0.0	2.9	31.8	1.2	27.1	5.3
20	44.1	0.0	6.5	0.0	0.0	.6	17.6	7.6	22.9	6.5
21	2.4	58.2	0.0	0.0	0.0	.6	0.0	1.2	31.2	.6
22	54.1	0.0	0.0	0.0	1.8	.6	2.9	31.8	6.5	6.5
23	49.4	0.0	0.0	0.0	0.0	.6	25.3	16.5	3.5	2.4
24	50.0	0.0	0.0	0.0	0.0	.6	9.4	1.8	34.1	4.7
25	30.0	0.0	0.0	0.0	14.7	.6	5.9	16.5	25.3	4.1
26	14.7	0.0	0.0	11.8	37.1	.6	13.5	1.2	17.6	7.1
27	65.3	0.0	0.0	0.0	0.0	.6	10.0	3.5	16.5	3.5
28	35.3	0.0	0.0	17.6	0.0	.6	6.5	1.2	28.8	4.1
29	7.1	0.0	4.7	.6	21.8	.6	17.1	8.8	17.1	10.0
30	57.1	0.0	0.0	0.0	1.2	.6	11.8	1.2	26.5	22.4
31	31.8	0.0	0.0	0.0	0.0	.6	0.0	38.8	25.9	1.8
32	32.9	2.4	0.0	0.0	8.8	.6	30.0	1.8	16.5	2.9
33	21.2	0.0	0.0	0.0	5.3	1.2	10.0	6.5	46.5	7.1
34	16.5	.6	0.0	22.9	21.2	1.2	12.4	3.5	12.9	9.4
35	7.1	1.8	1.8	7.1	8.2	1.8	12.4	14.7	31.8	8.8
36	4.7	2.4	.6	2.4	4.1	1.8	.6	50.6	18.8	13.5
37	12.9	0.0	0.0	0.0	0.0	1.8	0.0	48.8	28.2	14.1
38	29.4	21.2	0.0	8.8	1.8	1.8	25.3	4.7	4.1	8.2
39	61.2	0.0	0.0	0.0	0.0	1.8	0.0	3.5	26.5	2.9
40	15.3	30.6	0.0	0.0	0.0	2.4	2.4	13.5	23.5	7.1
41	35.3	13.5	0.0	0.0	0.0	2.4	8.2	10.6	21.2	12.4
42	38.2	12.9	0.0	0.0	0.0	2.4	17.1	1.8	14.7	8.8
43	10.6	4.1	0.0	0.0	0.0	2.9	43.5	9.4	11.2	12.9
44	12.9	0.0	0.0	12.9	5.9	3.5	35.9	0.0	12.9	18.2
45	46.5	0.0	0.0	13.5	11.2	3.5	15.9	3.5	3.5	15.9
46	43.5	0.0	0.0	0.0	0.0	3.5	30.0	4.1	15.9	2.4
47	12.4	0.0	0.0	1.2	3.5	4.1	28.8	2.9	38.8	2.9
48	15.3	0.0	0.0	0.0	0.0	4.1	0.0	27.1	42.4	8.2
49	26.5	0.0	0.0	17.1	0.0	5.3	0.0	16.5	25.3	11.2
50	51.8	0.0	0.0	11.2	0.0	5.3	0.0	2.4	19.4	9.4
51	17.6	0.0	0.0	.6	0.0	5.3	28.2	8.8	29.4	10.0
52	19.4	0.0	0.0	33.5	0.0	5.3	0.0	3.5	28.2	10.0
53	32.4	0.0	0.0	.6	0.0	5.3	21.2	7.1	23.5	10.0
54	32.9	0.0	1.8	0.0	1.8	5.3	10.0	14.1	20.6	10.0
55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.5
56	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

TEST 10 YEAR 2 NUMBER OF CHILDREN = 294 DECIMALS

VAR	1	2	3	4	5	6	7	8	9	0
1	85.7	0.0	0.0	0.0	0.0	0.0	6.8	.7	6.1	.7
2	32.3	0.0	1.0	0.0	24.1	0.0	5.1	5.1	27.2	5.1
3	69.7	0.0	4.9	0.0	12.9	0.0	2.0	3.7	2.7	4.1
4	65.3	0.0	3.7	0.0	2.7	0.0	14.6	6.1	3.1	4.4
5	57.8	0.0	8.5	0.0	15.3	0.0	4.1	0.0	6.2	6.1
6	50.7	0.0	0.0	0.0	0.0	0.0	45.2	.3	3.7	0.0
7	65.6	0.0	0.0	0.0	0.0	0.0	16.0	7.5	9.5	1.4
8	32.3	0.0	10.5	0.0	0.0	0.0	32.0	13.3	8.5	3.4
9	64.6	0.0	0.0	0.0	0.0	0.0	0.0	2.4	31.0	2.0
10	24.8	0.0	0.0	0.0	0.0	0.0	42.2	3.7	26.2	3.1
11	56.5	0.0	0.0	0.0	0.0	0.0	18.4	1.7	19.7	3.7
12	49.0	0.0	0.0	0.0	11.9	0.0	3.7	0.0	19.0	16.3
13	44.2	0.0	0.0	0.0	10.9	0.0	5.4	6.8	16.0	16.7
14	41.2	0.0	0.0	0.0	0.0	0.0	43.5	4.4	7.1	3.7
15	70.4	0.0	0.0	4.1	.3	1.4	12.6	4.1	6.8	.3
16	48.3	0.0	0.0	0.0	2.4	0.0	3.1	15.3	23.8	7.1
17	35.7	0.0	0.0	0.0	2.0	.7	19.0	13.9	23.8	4.8
18	36.7	0.0	0.0	0.0	0.0	.3	.3	42.2	16.0	4.4
19	35.4	7.1	0.0	0.0	0.0	0.0	32.0	1.0	20.4	4.1
20	53.7	0.0	4.8	0.0	0.0	0.0	18.4	5.1	17.0	1.0
21	.7	69.7	.3	0.0	0.0	0.0	0.0	0.0	25.5	3.7
22	62.6	0.0	0.0	0.0	1.4	0.0	2.0	26.2	5.8	2.0
23	57.1	0.0	0.0	0.0	0.0	0.0	19.0	15.3	4.4	4.1
24	60.2	0.0	0.0	0.0	0.0	0.0	6.1	2.0	26.5	5.1
25	33.3	0.0	0.0	0.0	9.5	0.0	4.1	24.1	22.4	6.5
26	21.1	0.0	0.0	8.2	39.1	0.0	13.6	1.4	11.6	5.1
27	75.9	0.0	0.0	0.0	0.0	0.0	6.8	4.1	10.2	3.1
28	49.6	0.0	0.0	15.3	0.0	0.0	5.1	.7	20.4	9.9
29	7.1	0.0	2.7	3.1	39.1	0.0	8.2	5.1	12.9	21.8
30	57.5	0.0	0.0	1.4	2.4	0.0	16.7	2.7	14.6	4.8
31	36.1	0.0	0.0	0.0	0.0	0.0	0.0	34.7	21.1	8.2
32	36.1	5.1	0.0	0.0	7.1	0.0	34.7	.7	8.8	7.5
33	25.5	2.4	0.0	0.0	5.4	0.0	11.9	3.7	41.5	9.5
34	23.8	.3	0.0	21.4	13.9	0.0	11.2	2.7	13.9	12.6
35	8.8	4.4	3.4	4.1	15.6	0.0	7.5	11.6	28.2	16.3
36	6.1	5.4	.3	2.7	5.1	.3	.3	46.6	17.7	15.3
37	7.5	0.0	0.0	0.0	0.0	0.0	.3	58.2	28.2	5.8
38	34.4	31.6	0.0	6.8	0.0	.3	16.3	3.4	4.4	2.7
39	60.9	.3	0.0	0.0	0.0	.7	0.0	5.4	25.5	7.1
40	15.6	38.1	0.0	0.0	0.0	1.0	3.4	6.1	24.8	10.9
41	35.7	23.5	0.0	0.0	0.0	1.7	7.1	4.8	20.4	6.8
42	36.4	11.9	0.0	0.0	0.0	2.0	13.6	4.4	20.4	11.2
43	11.6	7.1	0.0	0.0	0.0	2.0	41.5	9.5	14.3	13.9
44	22.1	0.0	0.0	10.2	8.2	2.0	35.0	0.0	10.5	11.9
45	66.7	0.0	0.0	9.2	5.8	2.4	8.2	2.4	2.0	3.4
46	47.6	0.0	0.0	0.0	0.0	3.1	25.2	3.1	17.3	3.7
47	16.0	0.0	0.0	3.4	5.8	3.4	24.1	3.7	35.4	8.2
48	9.9	0.0	0.0	0.0	.3	3.7	.3	25.9	49.7	10.2
49	35.4	0.0	0.0	14.6	0.0	7.8	0.0	18.4	16.7	7.1
50	52.0	0.0	0.0	15.3	0.0	8.2	0.0	5.1	12.9	6.5
51	17.3	0.0	0.0	0.0	0.0	8.5	28.6	10.5	27.6	7.5
52	23.1	0.0	0.0	38.9	0.0	9.2	0.0	2.7	19.4	6.8
53	42.5	0.0	0.0	0.0	0.0	9.2	12.6	7.1	20.7	7.8
54	39.5	0.0	2.7	0.0	2.0	9.2	11.9	15.0	10.9	8.8
55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
56	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

TEST 10 YEAR 3 NUMBER OF CHILDREN = 247 DECIMALS

VAR	1	2	3	4	5	6	7	8	9	0
1	86.6	0.0	0.0	0.0	0.0	0.0	6.1	0.0	5.7	1.6
2	31.2	0.0	1.6	0.0	26.3	0.0	6.9	4.5	26.7	2.8
3	73.3	0.0	5.3	0.0	8.5	0.0	2.4	6.9	.4	3.2
4	64.4	0.0	3.2	0.0	2.4	0.0	14.2	11.7	.8	3.2
5	53.0	0.0	9.3	0.0	17.4	0.0	6.1	0.0	10.1	4.0
6	57.1	0.0	0.0	0.0	0.0	0.0	41.7	.4	.4	.4
7	70.0	0.0	0.0	0.0	0.0	0.0	12.6	8.1	7.7	1.6
8	40.5	0.0	10.1	0.0	0.0	0.0	29.6	8.1	6.5	5.3
9	74.5	0.0	0.0	0.0	0.0	0.0	0.0	2.8	19.8	2.6
10	30.4	0.0	0.0	0.0	0.0	0.0	42.1	4.0	19.4	4.0
11	66.0	0.0	0.0	0.0	0.0	0.0	18.2	.8	11.7	3.2
12	48.2	0.0	0.0	0.0	17.4	0.0	3.6	1.2	11.3	18.2
13	51.0	0.0	0.0	0.0	6.1	0.0	6.1	6.5	11.7	18.6
14	49.8	0.0	0.0	0.0	0.0	0.0	34.8	4.0	6.9	4.5
15	74.9	0.0	0.0	2.4	.8	.4	6.5	8.5	5.7	.8
16	66.0	0.0	0.0	0.0	2.4	0.0	1.6	10.9	11.7	7.3
17	53.4	0.0	0.0	0.0	1.2	0.0	13.4	8.5	19.4	4.0
18	49.8	0.0	0.0	0.0	0.0	0.0	.4	32.8	12.6	4.5
19	42.9	12.6	0.0	0.0	0.0	.8	24.7	.4	15.0	3.6
20	67.2	0.0	4.9	0.0	0.0	0.0	14.2	5.7	7.7	.4
21	1.6	65.2	0.0	0.0	0.0	0.0	0.0	0.0	27.9	5.3
22	66.0	0.0	0.0	0.0	.8	0.0	2.8	24.7	3.6	2.0
23	62.3	0.0	0.0	0.0	0.0	0.0	10.9	20.6	2.0	4.0
24	58.7	0.0	0.0	0.0	0.0	0.0	8.5	0.0	25.1	7.7
25	32.8	0.0	0.0	0.0	8.1	0.0	4.0	27.5	17.8	9.7
26	24.7	0.0	0.0	16.2	31.6	0.0	11.3	1.2	9.7	5.3
27	83.0	0.0	0.0	0.0	0.0	0.0	5.7	2.0	6.1	3.2
28	65.6	0.0	0.0	10.1	0.0	0.0	5.7	0.0	10.1	8.5
29	16.2	0.0	4.5	2.0	35.6	0.0	8.1	4.0	11.7	17.8
30	66.8	0.0	0.0	1.2	0.0	0.0	13.0	5.3	9.7	4.0
31	47.8	0.0	0.0	.4	0.0	0.0	0.0	31.6	13.8	6.5
32	49.0	8.1	0.0	0.0	9.7	0.0	23.1	.8	2.8	6.5
33	29.6	2.8	0.0	0.0	8.5	0.0	17.4	3.6	30.8	7.3
34	30.0	.8	0.0	25.9	8.5	0.0	12.6	3.6	8.1	10.5
35	17.0	11.3	4.9	.8	13.8	0.0	8.9	10.5	18.6	14.2
36	13.4	11.3	1.6	.8	4.9	0.0	0.0	42.9	11.7	13.4
37	15.0	0.0	0.0	0.0	0.0	0.0	0.0	47.0	34.0	4.0
38	31.6	39.3	0.0	3.2	0.0	0.0	14.2	6.9	2.8	2.0
39	61.5	0.0	0.0	0.0	0.0	0.0	0.0	4.9	26.3	7.3
40	15.0	50.2	0.0	0.0	0.0	0.0	.8	5.7	19.4	8.9
41	39.7	28.7	0.0	0.0	0.0	0.0	6.1	8.9	10.5	6.1
42	46.2	18.6	0.0	0.0	0.0	0.0	9.7	4.0	11.3	10.1
43	8.9	10.9	0.0	0.0	0.0	0.0	38.9	19.8	8.9	12.6
44	27.5	0.0	0.0	6.9	6.9	0.0	38.9	.4	8.1	11.3
45	71.3	0.0	0.0	6.5	5.3	0.0	10.1	2.8	.8	3.2
46	61.1	0.0	0.0	0.0	0.0	0.0	17.8	4.5	13.0	3.6
47	22.7	0.0	0.0	3.6	6.5	.4	30.4	2.0	26.3	8.1
48	13.4	0.0	0.0	0.0	0.0	.4	0.0	36.0	39.7	10.5
49	44.1	0.0	0.0	18.6	0.0	2.0	0.0	14.6	13.4	7.3
50	67.6	0.0	0.0	6.5	0.0	2.4	0.0	2.4	11.3	7.7
51	20.6	0.0	0.0	0.0	0.0	2.4	36.8	10.9	19.4	9.7
52	29.6	0.0	0.0	42.9	0.0	2.4	0.0	2.0	13.8	9.3
53	54.3	0.0	0.0	0.0	0.0	2.8	13.8	9.3	10.5	9.3
54	40.1	0.0	4.5	0.0	4.0	2.8	12.1	13.0	12.1	11.3
55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
56	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

TEST 10 YEAR 4 NUMBER OF CHILDREN = 224 DECIMALS

VAR	1	2	3	4	5	6	7	8	9	0
1	86.6	0.0	0.0	0.0	0.0	0.0	6.3	.4	5.8	.9
2	42.9	0.0	1.8	0.0	21.0	0.0	3.1	1.8	25.0	4.5
3	79.0	0.0	3.6	0.0	7.1	0.0	1.8	2.2	.9	5.4
4	72.3	0.0	2.7	0.0	4.9	0.0	8.5	4.5	1.3	5.8
5	63.4	0.0	8.5	0.0	11.2	0.0	3.6	0.0	7.6	5.8
6	56.7	0.0	0.0	0.0	0.0	0.0	40.2	0.0	1.3	1.8
7	71.4	0.0	0.0	0.0	0.0	0.0	12.9	6.3	5.8	3.6
8	40.6	0.0	9.8	0.0	0.0	0.0	26.8	9.8	6.3	6.7
9	70.5	0.0	0.0	0.0	0.0	0.0	.4	1.8	22.3	4.9
10	33.5	0.0	0.0	0.0	0.0	0.0	39.3	.9	22.3	4.0
11	64.7	0.0	0.0	0.0	0.0	0.0	8.5	4.0	18.8	4.0
12	54.0	0.0	0.0	0.0	9.4	0.0	4.5	0.0	17.4	14.7
13	58.9	0.0	0.0	0.0	3.1	0.0	5.4	2.2	16.5	13.8
14	54.0	0.0	0.0	0.0	0.0	0.0	28.6	.9	12.5	4.0
15	79.9	0.0	0.0	1.3	0.0	0.0	8.0	5.4	4.5	.9
16	71.0	0.0	0.0	0.0	1.8	0.0	1.3	8.9	9.8	7.1
17	60.7	0.0	0.0	0.0	2.2	0.0	8.9	8.9	13.8	5.4
18	57.6	0.0	0.0	0.0	0.0	0.0	.4	28.1	9.8	4.0
19	42.4	17.4	0.0	0.0	0.0	0.0	24.1	1.3	10.3	4.5
20	67.9	0.0	3.6	0.0	0.0	0.0	16.5	.4	10.7	.9
21	2.2	67.9	0.0	0.0	0.0	0.0	0.0	1.8	23.2	4.9
22	74.1	0.0	0.0	0.0	.4	0.0	1.3	20.1	1.8	2.2
23	68.3	0.0	0.0	0.0	0.0	0.0	13.8	11.6	3.6	2.7
24	67.4	0.0	0.0	0.0	0.0	0.0	7.6	2.2	17.4	5.4
25	30.8	0.0	0.0	0.0	6.3	0.0	3.6	35.7	17.0	6.7
26	25.4	0.0	0.0	12.1	32.6	0.0	8.5	2.2	13.8	5.4
27	84.4	0.0	0.0	0.0	0.0	0.0	2.7	3.6	6.3	3.1
28	70.1	0.0	0.0	7.6	0.0	.4	4.5	1.3	7.6	8.5
29	15.6	0.0	4.0	3.1	38.4	.9	9.4	2.2	13.4	12.9
30	72.8	0.0	0.0	.9	.4	.9	10.7	1.8	7.6	4.9
31	55.4	0.0	0.0	0.0	0.0	1.3	0.0	25.0	12.1	6.3
32	58.5	6.3	0.0	0.0	4.9	1.8	20.1	0.0	4.9	3.6
33	46.9	3.1	0.0	0.0	4.5	1.8	12.5	3.6	21.4	6.3
34	32.6	2.7	0.0	11.2	8.9	1.8	13.8	5.8	10.3	12.9
35	21.9	12.5	2.2	1.3	15.2	1.8	3.1	5.8	22.8	13.4
36	23.2	12.5	.9	0.0	6.3	1.8	0.0	22.8	18.3	14.3
37	17.9	0.0	0.0	0.0	0.0	1.8	0.0	29.9	46.0	4.5
38	41.1	30.8	0.0	2.7	.4	3.1	13.4	3.6	2.2	2.7
39	65.6	0.0	0.0	0.0	0.0	3.1	0.0	4.5	18.8	8.0
40	24.6	38.4	0.0	0.0	0.0	3.1	.4	4.5	17.4	11.6
41	46.4	22.3	0.0	0.0	0.0	4.0	4.5	3.1	12.9	6.7
42	49.6	15.6	0.0	0.0	0.0	4.0	8.9	3.1	8.0	10.7
43	20.5	10.3	0.0	0.0	0.0	4.5	33.9	12.1	7.1	11.6
44	43.3	0.0	0.0	8.0	.9	4.5	20.3	.4	5.8	10.7
45	75.4	0.0	0.0	4.5	2.2	4.9	4.9	3.6	.4	4.0
46	58.9	0.0	0.0	0.0	0.0	5.4	17.0	3.1	10.3	5.4
47	29.5	0.0	0.0	5.8	5.4	5.8	26.8	1.3	17.0	8.5
48	21.9	0.0	0.0	0.0	0.0	5.8	0.0	25.4	37.1	9.8
49	44.6	0.0	0.0	12.1	0.0	6.7	0.0	20.5	7.1	8.[illegible]
50	63.4	0.0	0.0	6.3	0.0	6.7	0.0	3.1	11.2	9.4
51	28.6	0.0	0.0	0.0	0.0	7.6	33.9	7.6	12.9	9.4
52	27.7	0.0	0.0	40.2	0.0	7.6	0.0	4.5	9.4	10.7
53	52.7	0.0	0.0	0.0	0.0	8.0	11.2	6.3	12.5	9.4
54	41.1	0.0	6.7	0.0	3.1	8.5	11.2	8.0	8.0	13.4
55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
56	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

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FAC  ITEM                          DECIMALS  PHI CORRELATION MATRIX, N OF CASES =  950
96.0   6
90.2  29  221
89.6  62  106 165
87.9  53  138 170 317
86.1  19  119 164  82 140
86.0   1  134 193  60 110 118
85.1  44  215 289 253 296 173 230
84.4  17  149 141 138 116  45 111 153
84.2  36  235 313 201 193 168 249 304 172
81.5  37  234 262 209 196 160 229 347 153 677
80.8   8  214 181 149 139 206 189 366 130 280 304
80.5  25  238 249 136 127 155 222 397 191 377 367 368
78.1  41  203 219 202 185 162 277 377  88 343 337 292 311
77.6  20  212 247 180  94 353 226 326 103 369 386 251 366 301
76.7  30  103 355  89 108  52  72 139  86 110  90 111 106 130  74
76.4  60  202 225 305 280 142 211 420 157 371 360 252 346 359 301 140
76.0  31  187 362 147 146 151 213 310 112 324 296 246 352 328 312 104 245
74.9  13  241 161 136 157  90 137 234 374 249 249 224 329 216 213  89 228 221
73.4   9  205 210 153 148 136 161 315 128 274 270 372 305 279 287  74 299 302 239
72.2   3  197 206 180 151 205 305 286 115 304 309 301 359 376 348  25 308 388 167 280
70.8  54  199 217 258 266 177 154 380 126 262 278 240 245 287 338  74 385 290 200 278 331
69.8  63  204 238 300 212 259 216 386  96 419 406 332 342 344 415 104 412 349 228 314 390 395
68.0  27  136 229 120 118 187 249 269 128 272 311 205 300 302 285 177 294 348 176 214 410 269 330
67.4  11  190 148 137 113 129 152 256 227 314 303 254 394 261 250  57 269 255 576 220 259 240 299 220
67.3  28  201 313 121 119 199 216 336 108 343 371 315 348 335 313 104 283 527 202 330 433 287 415 468 251
66.8  42  175 211 206 143 182 237 356 116 357 337 310 302 605 312 114 367 331 186 273 381 291 413 331 258 366
64.9   4  176 218 168 146 202 288 347 122 305 336 298 340 368 393  75 288 341 135 241 741 324 395 437 241 423 401
64.0  26  193 276 146 145 167 190 331 191 354 381 381 412 334 306 178 306 359 279 386 376 266 433 303 347 388 380 372
62.6  55  153 192 192 253 142 190 292  88 274 303 182 252 259 226  94 360 289 196 184 263 294 292 183 251 240 269 285 277
62.6  57  175 192 242 266 211 164 329  88 280 298 270 290 327 294 110 380 279 206 302 327 318 401 277 288 286 306 312 313 329
61.8  38  193 294 171 179 222 282 350 146 479 511 360 401 406 460 166 405 476 245 359 508 363 529 445 316 527 481 516 497 292 426
61.7  14  181 199 135 165 190 218 313 270 300 304 293 405 300 271 129 286 312 643 279 313 289 320 280 596 280 291 287 360 246 272
60.6  48  187 176 204 229 168 239 417  93 306 292 324 294 287 290 112 359 298 195 284 332 355 375 232 234 301 320 333 347 308 326
60.2  32  163 282 145 179 208 328 358 101 373 381 287 349 375 331 147 293 404 204 317 556 316 415 493 269 486 405 610 425 305 305
59.2  69  158 237 270 243 223 275 354  85 303 325 275 299 352 354 115 411 320 201 318 388 366 400 320 294 323 360 381 389 336 398
58.0  56  152 200 184 226 132 185 319  81 257 263 192 265 281 263 127 321 220 123 162 271 406 322 234 221 206 288 268 281 361 339
57.6  58  161 211 230 203 153 193 339  65 294 308 253 298 318 304 157 416 350 127 306 342 424 404 351 261 354 368 373 336 296 362
57.3  39  181 295 164 175 231 270 365 156 437 469 379 386 386 438 194 367 446 237 331 504 377 520 456 301 485 471 520 486 273 379
56.7   5  125 234 168 171 208 327 295  93 326 343 287 316 339 355  87 270 364 127 286 648 323 392 434 226 445 377 743 367 300 309
55.9  21  164 263 168 144 231 252 335 121 342 367 310 366 344 442 183 335 359 234 318 452 307 445 399 263 442 405 480 420 247 295
53.5  64  100 175 255 171  95 207 200  82 221 272 210 207 281 232 135 262 276 137 216 382 214 300 346 197 329 279 389 329 204 274
53.1   7  184 261 100 103 164 155 214 107 234 300 297 309 246 268 126 217 246 195 330 301 203 350 295 245 310 300 312 397 202 232
52.8  46  183 220 188 198 175 232 420 105 302 325 300 344 336 331 168 349 333 130 289 378 303 402 319 192 356 360 388 341 298 403
51.8  65  168 206 222 210 197 187 269 102 246 293 258 270 283 319 116 337 311 185 281 346 303 452 327 276 368 372 350 369 256 334
51.4  45  134 232 226 194 199 220 389  93 259 305 280 292 381 395 137 407 400 181 333 426 381 414 402 243 425 374 450 419 371 397
49.3  15  147 204 115  94 182 239 265 173 322 361 308 314 328 287 193 259 331 229 279 507 229 377 450 290 436 372 560 379 204 269
49.1  18  157 160 114 112 156 177 240 270 245 289 327 317 249 285 151 267 285 319 324 336 189 306 289 368 321 310 345 490 257 283
48.8  16  167 251 133 136 167 254 297 164 336 352 309 374 323 343 209 324 346 268 302 469 283 431 435 352 457 379 479 425 249 323
48.7  24  113 207 132  97 209 138 243  47 277 291 217 272 307 367  88 333 276 155 263 308 259 370 267 233 294 315 319 305 217 300
46.6  72  136 187 222 126 174 133 291  52 265 271 235 262 244 330 125 320 298 199 281 325 307 376 342 250 350 326 346 344 264 290
44.9  22  130 233 169  95 197 224 295  40 333 338 310 326 319 399 146 327 359 204 324 428 314 428 388 281 422 393 433 391 243 374
43.5  43  124 246 174  97 149 237 314  95 304 330 302 324 346 339 166 327 348 208 268 392 305 419 351 279 376 423 395 383 269 313
43.1  23  144 236 171 120 244 203 269  86 300 294 288 320 321 406 161 307 324 169 302 368 277 414 309 197 411 341 380 390 240 315
39.2  73  141 169 153 165 160 106 275  41 193 216 187 220 221 229  79 262 223 150 175 204 234 269 194 162 242 244 196 246 218 276
38.8  68  129 197 179 130 126 184 206  80 208 224 212 290 254 257 142 264 270 196 235 320 245 326 292 218 307 240 336 291 245 294
38.1  47  115 222 189 178 189 210 298  62 262 273 238 292 326 364 154 349 349 123 256 399 303 402 398 203 399 359 458 362 350 373
36.1  10   86 188 127  84 168 176 198  57 205 228 215 225 244 246  87 216 273 110 224 324 197 322 304 181 300 282 336 253 186 222
35.9  35  119 209 133 116 141 156 209  73 245 260 230 223 284 260 230 265 312 138 257 327 243 348 343 169 363 312 342 332 228 296
34.1  34   67 147 136 193 134 143 195  39 214 257 175 213 219 248 191 221 284 128 212 302 217 289 260 178 364 299 342 257 197 239
33.5   2   99 196 117 126 188 215 247 101 252 257 282 281 262 285 195 236 304 173 286 355 248 345 386 217 395 305 408 369 229 271
```

PHI CORRELATION MATRIX DECIMALS contd.

32.4	33	72	182	111	112	122	195	202	49	244	260	160	192	247	232	115	209	299	125	188	349	236	308	364	184	349	315	405	275	232	228
31.8	49	1.4	186	158	177	169	197	279	62	264	267	223	261	306	301	172	293	320	159	273	357	293	305	366	181	384	331	402	370	321	354
28.6	12	1·5	153	63	42	139	154	141	124	204	212	231	246	205	206	144	181	252	237	260	335	180	244	324	311	342	258	367	329	166	200
27.7	61	90	187	164	121	139	161	219	90	229	246	241	262	287	276	223	332	270	173	266	336	267	366	333	224	346	320	365	361	239	322
25.6	71	70	128	97	54	96	76	157	52	141	124	156	172	141	193	105	166	171	49	184	196	174	191	128	104	177	202	182	188	108	173
23.4	66	1·0	123	115	98	106	93	175	79	198	199	155	189	208	225	115	230	199	95	186	248	195	309	272	177	247	246	301	289	190	246
22.2	50	96	150	132	97	156	171	224	13	224	248	221	230	240	226	210	255	276	145	247	269	204	307	301	209	329	268	286	311	271	297
21.6	59	88	112	145	116	99	86	184	13	163	197	170	206	184	220	107	243	192	149	206	216	240	278	222	152	240	190	235	249	235	304
20.9	70	65	73	124	56	49	6	150	21	88	98	79	89	78	121	69	151	95	28	93	128	102	85	70	98	111	93	101	111	71	124
20.7	51	64	159	131	102	152	153	214	40	221	223	216	218	226	256	207	247	263	139	249	282	259	308	306	217	318	288	315	324	277	325
16.2	40	60	116	112	84	135	136	160	86	182	180	163	194	184	209	181	203	207	142	181	228	181	233	234	178	252	230	263	252	174	221
14.9	67	55	98	85	65	91	109	92	57	108	116	151	153	143	119	132	170	180	78	139	200	158	185	224	146	191	188	215	197	207	189
13.2	52	31	44	71	77	57	40	84	72	57	81	78	89	78	74	52	91	94	88	107	144	105	147	113	71	112	88	135	149	126	146
		6	29	62	53	19	1	44	17	36	37	8	25	41	20	30	60	31	13	9	3	54	63	27	11	28	42	4	26	55	57

PHI CORRELATION MATRIX DECIMALS contd.

61.8	38																														
61.7	14	4·0																													
60.6	48	395	277																												
60.2	32	555	363	396																											
59.2	69	510	327	395	396																										
58.8	56	322	228	335	297	338																									
57.6	58	469	256	354	411	378	339																								
57.3	39	879	395	362	549	476	312	468																							
56.7	5	537	260	296	618	402	259	389	534																						
55.9	21	597	346	351	486	409	257	374	599	469																					
53.5	64	4·0	228	254	405	348	219	326	414	399	323																				
53.1	7	419	299	299	381	291	208	293	402	323	370	285																			
52.8	46	459	257	434	421	411	324	392	471	371	392	302	298																		
51.8	65	472	283	300	403	389	263	446	478	399	415	345	320	346																	
51.4	45	522	311	444	486	472	375	473	500	468	480	388	363	515	439																
49.3	15	514	326	324	598	343	219	360	523	541	459	383	395	331	352	419															
49.1	18	424	452	320	376	309	182	305	417	325	422	282	374	277	356	377	372														
48.8	16	543	410	356	527	413	234	442	537	479	473	413	395	357	403	470	646	410													
48.7	24	454	235	285	340	313	172	359	467	332	420	246	254	334	346	363	336	307	378												
46.6	72	413	294	278	363	411	261	383	423	373	358	338	308	333	361	466	332	319	395	283											
44.9	22	610	337	338	488	423	271	394	635	443	563	337	375	400	397	515	438	362	509	461	389										
43.5	43	516	324	302	452	374	294	366	512	410	483	366	365	394	378	441	388	384	413	338	367	479									
43.1	23	496	274	306	402	333	248	333	514	390	455	363	362	365	392	463	384	337	443	453	371	483	399								
39.2	73	298	266	253	277	324	197	265	287	221	260	220	188	317	243	288	171	230	199	210	265	267	288	221							
38.8	68	4·0	263	288	352	376	249	308	387	347	342	280	277	302	345	373	320	302	314	311	354	391	338	340	223						
38.1	47	478	243	388	469	422	325	445	489	462	439	397	364	567	423	676	414	305	464	392	417	480	426	416	267	370					
36.1	10	365	173	224	337	272	208	259	383	382	367	270	281	266	321	314	306	257	317	288	312	347	348	289	218	286	339				
35.9	35	434	251	302	397	309	241	375	451	338	390	328	299	381	361	442	408	323	440	337	356	448	348	412	186	317	425	287			
34.1	34	364	183	275	380	308	153	374	379	359	339	310	276	310	320	406	410	289	460	271	320	354	314	352	150	242	391	305	447		
33.5	2	452	269	265	449	367	237	351	473	443	441	375	403	357	349	440	496	370	467	334	347	457	448	396	212	317	444	349	413	374	
32.4	33	382	249	306	521	278	183	371	389	409	357	307	250	311	299	367	460	256	443	260	276	355	349	287	149	250	391	237	386	550	366
31.8	49	457	259	383	457	401	296	380	461	431	410	401	379	459	387	596	426	306	468	347	399	487	413	419	225	406	637	348	469	419	497
28.6	12	349	307	214	381	270	161	265	358	388	342	334	353	285	284	327	447	361	387	267	290	345	346	338	150	235	332	289	346	359	424
27.7	61	438	260	301	411	375	283	421	453	393	407	412	346	391	432	465	425	338	468	408	402	438	449	445	226	361	493	333	449	388	503
25.6	71	197	119	220	183	231	181	185	189	180	214	135	140	225	222	242	179	163	223	162	269	226	186	196	162	210	195	157	149	199	172
23.4	66	336	215	241	296	281	222	302	366	306	305	310	260	267	413	338	301	269	326	262	326	331	313	283	178	305	345	206	[illegible]	253	319
22.2	56	394	238	321	325	356	285	325	410	308	372	366	325	383	348	443	349	311	394	325	388	448	410	374	235	332	472	299	418	358	420
21.6	59	270	218	213	280	316	218	258	272	266	264	212	190	300	239	351	255	217	229	230	258	261	288	267	265	269	347	186	258	210	294
20.9	70	117	113	123	74	169	144	127	110	89	76	101	105	128	118	174	87	84	92	67	198	132	75	74	122	78	112	54	127	77	95
20.7	51	386	237	303	352	366	282	354	405	336	370	378	309	405	353	466	378	303	419	347	391	446	431	389	227	343	496	285	439	338	451
16.2	40	354	170	237	264	231	165	267	339	268	304	272	236	295	298	347	274	276	330	251	281	314	282	332	115	223	384	299	367	370	335
14.9	67	226	142	168	220	264	195	216	260	246	223	260	205	236	233	283	224	202	216	164	253	268	251	237	165	253	315	170	240	177	228
13.2	52	152	108	147	131	165	104	132	166	163	170	169	116	161	176	179	145	103	186	181	179	180	167	170	38	181	201	141	202	179	179
		38	14	48	32	69	56	58	39	5	21	64	7	46	65	45	15	18	16	24	72	22	43	23	73	68	47	10	35	34	2

300

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FAC  ITEM
32.4  33                                                DECIMALS PHI CORRELATION MATRIX, N OF CASES =  950    Contd
31.8  49   386
28.6  12   287 392
27.7  61   355 507 409
25.6  71   186 226  87 187
23.4  66   303 354 239 408 160
22.2  50   284 510 378 512 232 393
21.6  59   227 339 256 350 196 200 335
26.9  70    41 115  74 155 125 119 166 151
24.7  51   380 570 399 530 229 429 719 381 183
16.2  40   268 417 359 398 187 283 417 262 117 458
14.9  67   195 328 270 308 234 333 365 239 146 411 216
13.2  52   169 242 180 295 178 233 271 265  82 346 242 212

            33  49  12  61  71  66  50  59  70  51  40  67  52
```

CSMS

NUMBERS 1

(Final Draft)

NAME ....
TODAY'S DATE ....
BOY OR GIRL ....
CLASS ....
SCHOOL ....
DATE OF BIRTH ... .
day month year

Trial Item:

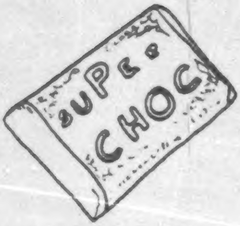
Linda has 5 LP's and Sandra has 3 more LP's than Linda.

How do you work out how many LP's Sandra has?

....

5 ÷ 3	3 + 5
8 — 5	3 × 5
5 + 3	8 — 3
5 — 3	4 + 4

303



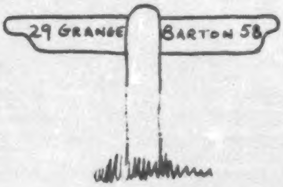
1.

A bar of chocolate can be broken into 12 squares.
There are 3 squares in a row.

How do you work out how many rows there are?

....

$12 + 3$
3×4
12×3
$3 - 12$
$6 + 6$
$12 \div 3$
$12 - 3$
$3 \div 12$



2.

The signpost shows that it is 29 miles west to Grange and 58 miles east to Barton.

How do you work out how many miles it is from Grange to Barton?

$29 + 58$
$58 \div 29$
$58 - 29$
29×2
$29 \div 58$
$58 + 29$
58×29
$87 - 29$

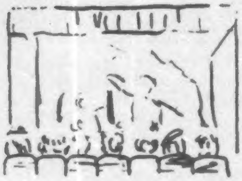
3.

John is cycling 8 miles home from school.
He stops at a sweet shop after 2 miles.

How do you work out how much further John has to go?

....

$8 + 2$
2×4
$2 \div 8$
$8 - 2$
8×2
$2 + 6$
$2 - 8$
$8 \div 2$



4.

286 people are coming to see the school play.
The chairs are arranged in rows, and there are 26 chairs in each row.

How do you work out how many rows are needed?

286×26
$26 + 26$
$26 \div 286$
$286 - 26$
$26 + 286$
26×11
$26 - 286$
$286 \div 26$

5.

A bucket holds 8 litres of water.
4 buckets of water are emptied into a bath.

How do you work out how many litres of water are in the bath.

....

8×4
$4 \div 8$
$12 - 8$
$8 \div 4$
$8 + 4$
4×8
$4 + 4$
$8 - 4$

6.

Motorway
M1

The Green family have to drive 261 miles to get from London to Leeds. After driving 87 miles they stop for lunch.

How do you work out how far they still have to drive?

87 × 3	261 + 87
87 ÷ 261	261 − 87
261 × 87	261 ÷ 87
87 − 261	87 + 174

7.

A shop makes sandwiches.
You can choose from 3 sorts of bread and 6 sorts of filling.

How do you work out how many different sandwiches you could choose?

....

3 × 6	6 − 3
6 + 3	3 − 6
18 ÷ 3	6 ÷ 3
6 × 3	3 + 3

8.

A gardener has 391 daffodils.
These are to be planted in 23 flowerbeds.
Each flowerbed is to have the same number of daffodils.

How do you work out how many daffodils will be planted in each flowerbed?

391 − 23	23 ÷ 391
23 − 391	391 × 23
391 + 23	23 + 23
23 × 17	391 ÷ 23

9.

An oven tray used for cooking little cakes will hold 56 cakes.
A baker fills 28 trays.

How do you work out how many cakes he will cook?

56 ÷ 28	84 − 28
28 ÷ 56	56 × 28
56 + 28	56 + 56
28 × 56	56 − 28

Trial Item:

$\boxed{9 + 3}$ Story:

a. $\boxed{84 - 28}$ Story:

b. $\boxed{9 \div 3}$ Story:

c. $\boxed{9 \times 3}$ Story:

d. $\boxed{84 \div 28}$ Story:

e. $\boxed{84 \times 28}$ Story:

MARKING SCHEME NUMBER OPERATIONS

QU.NO	CODE	CODE	CODE
1	12 ÷ 3	3 ÷ 12	3 x 4
2	29 + 58) 58 + 29)	-	-
3	8 - 2	2 - 8	2 + 6
4	286 ÷ 26	26 ÷ 286	26 x 11
5	8 x 4) 4 x 8)	-	-
6	261 - 87	87 - 261	87 + 174
7	3 x 6) 6 x 3)	-	-
8	391 ÷ 23	23 ÷ 391	23 x 17
9	28 x 56) 56 x 28)	-	-
10	Correct story		
11	" "		
12	" "		
13	" "		
14	" "		

FREQUENCY OF CODES NUMBER OPERATIONS

TEST 13 YEAR 9 n = 102

VAR	1	2	3	4	5	6	7	8	9	0
1	74.5	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	25.5	0.0
2	53.9	15.7	11.[illegible]	0.0	0.0	0.0	0.0	0.0	18.6	0.0
3	59.8	10.7	7.[illegible]	0.0	0.0	0.0	0.0	0.0	14.7	1.0
4	69.6	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	28.4	2.0
5	41.2	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	52.9	5.9
6	52.[illegible]	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	48.0	0.0
7	2[illegible].5	5.9	32.4	0.0	0.0	0.0	0.0	0.0	41.2	0.0
8	27.5	27.5	5.[illegible]	0.0	0.0	0.0	0.0	0.0	35.3	3.9
9	36.3	2[illegible]	1.[illegible]	0.0	0.0	0.0	0.0	0.0	40.2	2.0
10	81.4	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	16.7	1.0
11	37.3	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	53.9	8.8
12	14.7	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	68.6	16.7
13	62.7	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	33.3	3.9
14	44.1	[illegible]	[illegible]	0.0	0.0	0.0	0.0	0.0	46.1	9.8

TEST 13 YEAR [illegible] NUMBER OF CHILDREN = 497

VAR	1	2	3	4	5	6	7	8	9	0
1	74.4	0.0	.2	0.0	0.0	0.0	0.0	0.0	24.1	1.2
2	69.[illegible]	13.7	8.7	0.0	0.0	0.0	0.0	0.0	8.2	.4
3	59.6	13.9	5.2	0.0	0.0	0.0	0.0	0.0	20.5	.8
4	77.9	.4	[illegible]	0.0	0.0	0.0	0.0	0.0	20.9	.8
5	45.7	.2	[illegible]	0.0	0.0	0.0	0.0	0.0	53.3	.8
6	65.6	.6	[illegible]	0.0	0.0	0.0	0.0	0.0	32.4	1.4
7	31.6	14.1	24.[illegible]	0.0	0.0	0.0	0.0	0.0	29.2	.2
8	34.4	32.[illegible]	4.4	0.0	0.0	0.0	0.0	0.0	28.0	1.2
9	43.7	26.[illegible]	2.5	0.0	0.0	0.0	0.0	0.0	26.4	1.4
10	77.3	.6	[illegible]	0.0	0.0	0.0	0.0	0.0	20.1	2.0
11	45.3	0.0	[illegible]	0.0	0.0	0.0	0.0	0.0	48.5	6.2
12	30.8	0.0	.2	0.0	0.0	0.0	0.0	0.0	53.5	15.5
13	6[illegible]	0.0	.4	0.0	0.0	0.0	0.0	0.0	35.0	4.6
14	42.1	.2	[illegible]	0.0	0.0	0.0	0.0	0.0	46.9	10.9

308

TEST 13 YEAR 1 NUMBER OF CHILDREN = 247 NUMBER OPERATIONS

VAR	1	2	3	4	5	6	7	8	9	0
1	77.7	.4	0.0	0.0	0.0	0.0	0.0	0.0	21.9	0.0
2	67.6	17.0	6.5	0.0	0.0	0.0	0.0	0.0	6.9	0.0
3	59.9	19.4	3.2	0.0	0.0	0.0	0.0	0.0	17.4	0.0
4	81.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.2	0.0
5	61.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.1	.4
6	76.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	23.5	0.0
7	31.6	12.6	34.0	0.0	0.0	0.0	0.0	0.0	21.9	0.0
8	33.6	36.0	5.7	0.0	0.0	0.0	0.0	0.0	24.7	0.0
9	47.4	32.8	1.2	0.0	0.0	0.0	0.0	0.0	18.6	0.0
10	85.4	.4	0.0	0.0	0.0	0.0	0.0	0.0	13.8	.4
11	52.6	.4	0.0	0.0	0.0	0.0	0.0	0.0	45.7	1.2
12	40.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	57.9	1.6
13	69.2	.4	0.0	0.0	0.0	0.0	0.0	0.0	29.6	.8
14	55.5	.8	0.0	0.0	0.0	0.0	0.0	0.0	42.1	1.6

TEST 13 YEAR 2 NUMBER OF CHILDREN = 132

VAR	1	2	3	4	5	6	7	8	9	0
1	74.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.8	0.0
2	65.9	22.7	3.8	0.0	0.0	0.0	0.0	0.0	7.6	0.0
3	67.4	20.5	2.3	0.0	0.0	0.0	0.0	0.0	9.8	0.0
4	79.5	.8	0.0	0.0	0.0	0.0	0.0	0.0	19.7	0.0
5	72.0	0.0	.8	0.0	0.0	0.0	0.0	0.0	27.3	0.0
6	80.3	0.0	0.0	0.0	-0.0	0.0	0.0	0.0	19.7	0.0
7	35.6	19.7	28.0	0.0	0.0	0.0	0.0	0.0	16.7	0.0
8	46.2	36.4	1.5	0.0	0.0	0.0	0.0	0.0	15.9	0.0
9	50.8	30.3	.8	0.0	0.0	0.0	0.0	0.0	18.2	0.0
10	88.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.4	0.0
11	53.0	0.0	2.3	0.0	0.0	0.0	0.0	0.0	44.7	0.0
12	48.5	0.0	.8	0.0	0.0	0.0	0.0	0.0	50.0	.8
13	62.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	37.1	.8
14	57.6	0.0	.8	0.0	0.0	0.0	0.0	0.0	41.7	0.0

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 FAC  ITEM                                                  PHI CORRELATION MATRIX, N OF CASES =  978
 96.9 323
 82.6 312  689
 81.8 623  210 315
 81.3  10  158 229  99
 78.2   5  255 274 181 197
 77.2 612  217 387 867 139 226
 75.3   2   90  99 140 101 179 179
 74.3 823  172 255 232 192 296 272 135
 73.6 123  264 255 214 180 281 256 153 301
 72.7 423  221 245 234 170 204 241 106 386 314
 72.5 812  179 261 237 197 301 281 145 953 296 378
 68.9   9  194 234 232 204 368 251 193 338 318 262 338
 68.2 412  188 237 224 173 224 267 132 367 321 897 385 276
 66.7   3  447 648 262 222 241 323 112 244 214 228 259 195 215
 62.9  11  183 273 186 385 225 238 172 265 200 180 275 247 170 255
 60.7   6  189 283 586 140 179 675 179 237 232 174 254 179 197 439 231
 52.8   7  127 133 153 176 210 178 121 283 185 238 288 276 268 165 209 199
 47.8  13  167 271 222 322 221 260 135 318 244 254 332 243 261 267 509 253 228
 47.3  12  287 294 234 307 267 276 168 294 221 286 313 265 278 266 385 292 277 479
 44.8   8  127 163 169 121 191 185 135 529 214 150 554 218 164 235 202 277 246 234 216
 44.6 112  212 248 183 192 194 222 171 300 545 332 303 211 365 241 246 261 275 279 336 309
 35.1   4  158 167 168 110 164 179 143 235 268 450 246 211 501 265 192 292 257 228 264 462 375
 33.9  14  189 266 226 271 247 266 175 267 241 303 282 271 308 264 358 235 310 507 557 235 377 274
 31.8   1  165 178 155 151 160 184 153 246 407 266 249 158 296 239 208 271 217 249 277 370 747 476 332

           323 312 623  10   5 612   2 823 123 423 812   9 412   3  11   6   7  13  12   8 112   4  14   1
```

NUMBER OPERATIONS

309

310

CSMS

POSITIVE and NEGATIVE NUMBERS

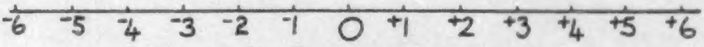
NAME .... TODAY'S DATE ....

BOY or GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ... .
day month year

READ THIS SECTION BEFORE STARTING

Here is a line marked off in equal parts:



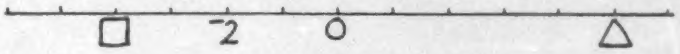
Starting from the point 0 going to the right, the numbers go $^{+}1$, $^{+}2$, $^{+}3$;

going to the left the numbers go $^{-}1$, $^{-}2$, $^{-}3$.

$^{+}1$, $^{+}2$, $^{+}3$ .... These are called POSITIVE numbers.

$^{-}1$, $^{-}2$, $^{-}3$ .... These are called NEGATIVE numbers.

1. If this line is numbered in the same way,



what number would go in the box ☐ ? ......(1).......

what number would go in the box △ ? ....(2).......

---

2. Look at this example.

The number line is marked off in equal steps.

John walks 4 steps FORWARDS and then 1 step BACKWARDS.

This would be the same as the one move : ..3 Forwards....

Now try this one :

John walks 2 steps FORWARDS and then 3 steps BACKWARDS.

This would be the same as the one move: ......(3)..................

Give another pair of moves which would have the same effect as this

one move : ....(4)...

311

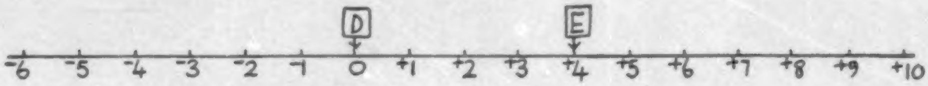
John walks 3 steps FORWARDS and then 2 steps BACKWARDS and then 4 steps FORWARDS.

This would be the same as the one move:..............(5)...

---

3. Here is a day's timetable:

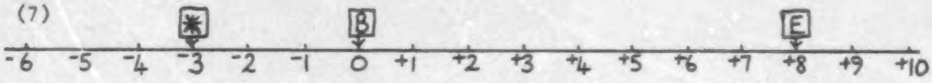
A	7.00 a.m.	get up
B	8.00 a.m.	start breakfast
C	10.00 a.m.	start work
D	12.00 noon	have lunch
E	4.00 p.m.	have tea



(i) On this number line if we put D at 0 then E is at $^{+}4$. Put in A, B and C.

(6)

(7)



(8)

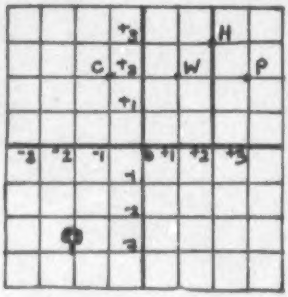
(ii) We now have a new number line; if we put B at 0 then E is at $^{+}8$.

Put in A, C, D.

The milkman comes at ✳. What time is this? ....

(9)

4(i) This picture is a map of a village.



There is a windmill (W) at $(^{+}1, {}^{+}2)$.

$^{+}1$ is one across from 0.

$^{+}2$ is two up from 0.

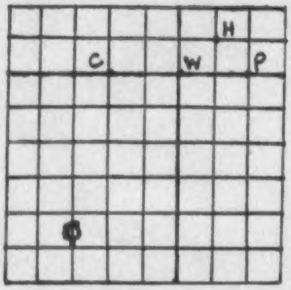
(a) At $(^{-}1, {}^{+}2)$ there is a church (C).

(10) (b) Why do we write $^{-}1$ and not $^{+}1$? ....

(11) (c) Where is the tree ? ?........................

(12) (d) What letter is at $(^{+}2, {}^{+}3)$?

(13) (e) A man M is at $(^{-}3, {}^{+}1)$. Put M on the diagram.



4(ii) In this new diagram the windmill (W) is at $(0,0)$. The new address of the Tree is $(^-3,^-5)$.

What are the new addresses for

(14) (a) the Church (C) ....?

(15) (b) the man (M) ....?

---

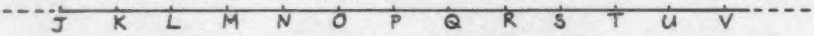
5. $^+$ means to the RIGHT. $^-$ means to the LEFT.

$^+4$ means 4 steps forwards (to the RIGHT).

$^-4$ means 4 steps backwards (to the LEFT).

(i) Moving $^+2$ and then $^-3$ is the same as the single move: ......

(16)

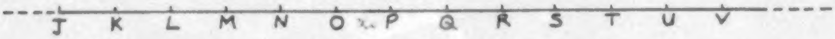


(ii) John starts at L.

John makes a move $^+1$ along the line and then a move of $^+3$; at what

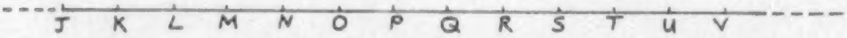
(17) letter does he end up? ......

How could he do this in one move? ......

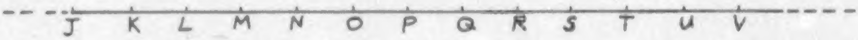


(18) (iii) John starts at M and ends at O. In doing this John made two moves.

One move was $^-3$ along the line. What was the other move? ......



(19) (iv) John makes two moves $^+2$ then $^+4$. He ends at R. Where did he begin? ......



(20) (v) John starts at N and ends at M. He made two moves. Write down what these two moves might have been ....

6.

Write down one move which will give you the same result as the two moves in each of the following:

(21) (i) $^{-}3$, then $^{+}1$ gives ......

(22) (ii) $^{+}2$, then $^{-}3$ gives ......

REMINDER: $^{+}5$ means 5 to the RIGHT,
$+$ means ADD.

We can think of combining moves as "adding" them: so $^{+}3$ then $^{+}1$ can be written as $^{+}3 + ^{+}1$.

(a) Moving $^{+}2$ then $^{+}3$ gives $^{+}5$; this can be written $^{+}2 + ^{+}3 = ^{+}5$

(b) Moving $^{+}3$ then $^{-}1$ gives $^{+}2$; this can be written $^{+}3 + ^{-}1 = ^{+}2$

(23) (c) $^{+}5$ then $^{-}4$ gives ....; this can be written ... + ... = ...

(24) (d) $^{+}6$ then $^{-}3$ gives ....; this can be written ... + ... = ...

---

7. Do the following:

(25) (i) $^{+}2 + ^{+}6 =$ ......

(26) (ii) $^{+}8 + ^{-}4 =$ ......

(27) (iii) $^{-}8 + ^{+}3 =$ ......

(28) (iv) $^{-}9 + ^{-}4 =$ ......

(29) (v) $^{+}5 + ^{-}5 =$ ......

8(i) $^{+}7-{}^{+}5=\square$ means "what must I add to $^{+}5$ to get to $^{+}7$?"

We can rewrite this as an addition sum: $^{+}5+\square={}^{+}7$.

(30) What is the number that goes in the box? ....

(ii) (a) $^{+}8-{}^{+}3=\square$ can be written as an addition sum: $^{+}3+\square={}^{+}8$

(31) What is the number that goes in the box? ....

(b) $^{+}6-{}^{+}7=\square$ can be written as an addition sum: $\ldots+\square=\ldots$

(32) What is the number that goes in the box? ....

(c) $^{+}6-{}^{+}6=\square$ can be written as an addition sum: $\ldots+\square=\ldots$

(33) What is the number that goes in the box? ....

REMINDER: $^{-}3$ means negative 3,

— means SUBTRACT.

Now $^{+}3-{}^{+}3=0$ and also $^{+}3+{}^{-}3=0$.

So subtracting $^{+}3$ gives the same effect as adding $^{-}3$.

So for example $^{+}6-{}^{+}3$ is the same as $^{+}6+{}^{-}3$.

Now work out:

(34) (d) $^{+}6-{}^{+}8=$

(35) (e) $^{-}6-{}^{+}3=$

(iii) Now $^{-}3-{}^{-}3=0$ and also $^{-}3+{}^{+}3=0$.

So subtracting $^{-}3$ is the same as adding $^{+}3$.

So for example $^{+}3+{}^{+}3={}^{+}6$ is the same as $^{+}3-{}^{-}3={}^{+}6$.

(a) $^{+}6-{}^{-}5=\square$ can be written as an addition sum: $^{+}6+{}^{+}5=\square$

(36) What is the number that goes in the box? ....

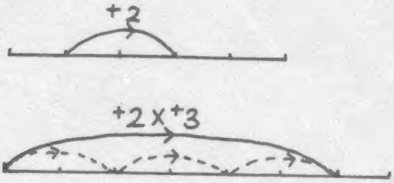
(b) $^{+}8-{}^{-}6=\square$ can be written as an addition sum: $\ldots+\ldots=\square$

(37) What is the number that goes in the box? ....

(c) $^{-}2-{}^{-}5=\square$ can be written as an addition sum: $\ldots+\ldots=\square$

(38) What is the number that goes in the box? ....

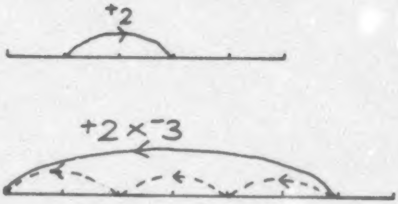
9. (i) "$^{+}2 \times {}^{+}3$" means "$^{+}2$ multiplied by $^{+}3$".
We can think of this as a step of $^{+}2$ taken 3 times in the SAME direction as the $^{+}2$.



So $^{+}2 \times {}^{+}3 = {}^{+}6$.

(39) So $^{+}3 \times {}^{+}5 =$ ......

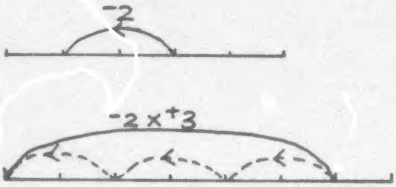
(ii) "$^{+}2 \times {}^{-}3$" means "$^{+}2$ multiplied by $^{-}3$".
We can think of this as a step of $^{+}2$ taken 3 times in the OPPOSITE direction to the $^{+}2$.



So $^{+}2 \times {}^{-}3 = {}^{-}6$.

(40) So $^{+}3 \times {}^{-}4 =$ ......

(iii) "$^{-}2 \times {}^{+}3$" means "$^{-}2$ multiplied by $^{+}3$".
We can think of this as a step of $^{-}2$ taken 3 times in the SAME direction as the $^{-}2$.



So $^{-}2 \times {}^{+}3 = {}^{-}6$.

(41) So $^{-}3 \times {}^{+}3 =$ ......

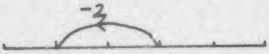
(iv) "$^-2 \times {}^-3$" means

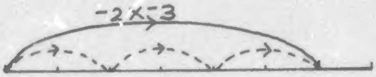
"$^-2$ multiplied by $^-3$".

We can think of this as a

step of $^-2$ taken 3 times

in the OPPOSITE direction to the $^-2$.





So $^-2 \times {}^-3 = {}^+6$

(42) So $^-2 \times {}^-5 =$ ......

(v) Do these:

(43) $^+4 \times {}^-4 =$ ......

(44) $^-3 \times {}^+4 =$ ......

(45) $^-2 \times {}^-3 =$ ......

---

10. Tick each of the statements which you think are true, in section (i), (ii) and (iii).

(i) When you add a negative number to a negative number:

(a) you always get a negative number ......

(46) (b) you always get a positive number ......

(c) you sometimes get a positive number ......

(ii) When you add a positive number to a negative number:

(a) you always get a positive number ......

(47) (b) you always get a negative number ......

(c) you sometimes get a positive number ......

(d) you sometimes get a negative number ......

(iii) When you subtract a negative number from a positive number:

(a) you always get a positive number ......

(48) (b) you always get a negative number ......

(c) you sometimes get a negative number ......

(d) you sometimes get a negative number ......

11. Put these numbers in order of size; start with $^{-}7$.

$^{-}3, ^{-}7, ^{+}3, ^{-}5, ^{+}4$. .... (49)

---

12. A thermometer shows a temperature of 5° Centigrade. The temperature drops 8° Centigrade.

What is the new reading on the thermometer? ...(50)

Write this down as a sum .... = ...(51)

---

13. Do these:

$^{+}5 \times {}^{-}2 =$ .... (52)

$^{+}2 + {}^{+}4 =$ .... (53)

$^{+}6 \div {}^{-}2 =$ .... (54)

$^{+}8 - {}^{+}4 =$ .... (55)

$^{-}4 \times {}^{-}2 =$ .... (56)

$^{+}5 - {}^{-}5 =$ .... (57)

Marking Codes. Positive and Negative Numbers

Question	1	2	3	4	5	6	7	8
1a (1)	-4				5 Or +5	+4,4	-5	
1b (2)	+5	5			-6	-5	6 or +6	
2a (3)	-1 Forward 1 Back					1 Forward or 1 or -1 Back		5, +5, -5
2b (4)	Any pair when add give -1	incl 3B, 2F		2F, 3B		Pair giving +1		Any pair not single
2c (5)	+5 5 forward	5				5 back -5	1 forw +1	9, +9, -9
3(i) A/B (6)	-5,-4	-5 only	-4 only				(-4,-3)	-7,-8 7,8
3(i)C, 3(i)F (7)	C A -2,-1	C -2 only	A -1 only		(-2,-5) C A		(-1,-2) (C, A)	+10, +7 C, A
3(ii)C&D(8)	C D +2,+4	C +2 only	D +4 only		C D (-2,+4)		(+1,+3)	(+10,+12) (+10,-2) C D
3(ii) milk(9)	5am	5 o'clock			9am	11am	4am	-3,3
4(1)b (10)	other side opp.divn W is at +1	negative				"either will do"		they are different
c (11)	(-2,-2½) (-2, -3)	(-3, -2)	(-3, 2)			(2,3)(2,2½) any sign except correct	(-1½,-3) (-3, -4)	(2, 1)
d (12)	H	P				none		

Qu.	1	2	3	4	5	6	7	8
4(i)e (13)	(or dot)M at (-3, +1)	(+1, -3)				(3, 1) any wrong signs	(-4, +2)	
4(ii)a (14)	(-2, 0)	(0, -2)	(-2)		(-1, +2)	(2, 0)	(-3, 1) (-3, 0)	(4, 7) (3, 6)
b (15)	(-4, -1)	(-1, -4)	(+1, -3)		(-3, +1)	(4, 1) any sign	(-3, -0)	(1, 5)
5(i) (16)	1 back 1 left					+1		+5, 5, -5
5(ii) a&b (17)	P, +4	P only	+4 only			-4 and line extended to H	0, +4	
5(iii) (18)	+5, 5	5 forward 5 right			-1	-5		+2
5(iv) (19)	L				0		M	J
5(v)a & b (20)	Any pair when added give -1					Any pair giving +1	Any pair giving -2	-1
6(i) (21)	-2	2 back 2 left				+2, 2		4, +4, -4
6(ii) (22)	-1					+1, 1		5, +5, -5
6c (23)	1, +1 +5 + -4=+1	+1 only	+5 + -4=+1 only			-1 +5 +-4= -1		+5 + -4 =9 any sign 9, +9, -9
6d (24)	+3, 3 +6 + -3= +3	+3 only	+6 + -3=+3			-3 +6 + -3= -3		9, +9, -9 +6, + -3=9 any sign

Qu	1	2	3	4	5	6	7	8
7(i) (25)	+8, 8				4, +4, -4	-8		12, +12
7(ii) (26	+4, 4				32 any sign	-4		12, +12, -12
7(iii) (27)	-5				24 any sign	+5, 5		11 any sign
7(iv) (28)	-13				36 any sign	+13		5 any sign
7(v) (29)	0				25 any sign			10 any sign
8(i) (30)	+2, 2				35 any sign	-2		12 any sign
8(ii)a (31)	5, +5				24 any sign	-5		11 any sign
8(ii)b (32)	only -1	+7+□=+6 -1	equn. only		42 any sign	+i		13 any sign
8(ii)c (33)	only 0	+6+□=+6 0	equn. only		36 any sign			12 any sign
8(ii)d (34)	-2				48 any sign	+2	correct rewrite	14 any sign
8(ii)e (35)	-9				18 any sign	+9	correct rewrite	3 any sign
8(iii)a (36)	+11, 11				30 any sign	-11		1 any sign

Qu.	1	2	3	4	5	6	7	8
8(iii)b (37)	14 +14	+8 + +6=☐ +14	equn only		48 any sign	-14		2 any sign
8(iii)c (38)	3 +3	-2+ +5=☐ +3	equn only		10 any sign	-3		7 any sign
9(i) (39)	15 +15				2 any sign	-15		8 any sign
9(ii) (40)	-12				1 any sign	12, +12		7 any sign
9(iii) (41)	-9				0	9, +9		6 any sign
9(iv) (42)	+10				3 any sign	-10		7 any sign
9(v)a (43)	-16				0	+16		8 any sign
b (44)	-12				1 any sign	12, +12		7 any sign
c (45)	+6, 6				1 any sign	-6		5 any sign
10(i) (46)	(a)				(c)	(a) & (c)	(a) & (b)	(b)
10(ii) (47)	c and d	c only	d only		c,d,a	c,d,b	a	b
10(iii) (48)	(a)				a,b	a,c and/or d	c and/or d	b
11 (49)	-7 -5 -3 +3 +4	+4, +3, -3, -5, -7			-3, -7, +3, -5, +4	-3, +3,+4, -5, -7		3, 3, 4 5, 7 any signs

Qu	1	2	3	4	5	6	7	8
12a (50)	-3 or 3 below 0					+3, 3		13 any form
b (51)	+5- +8=3	+5+ -8=3			8-5=3 or all pluses	5+8=3 or all pluses	5 - -8= -3	8+5 =13
13(a) (52)	-10				7 any sign	+10	correct rewrite	3 any sign
(b) (53)	+6, 6				2 any sign	-6		8 any sign
(c) (54)	-3				4,8 any sign	+3		12 any sign
(d) (55)	+4, 4				32 any sign	-4		12 any sign
(e) (56)	+8, 8				2 any sign	-8		6 any sign
(f) (57)	+10,10				0	-10		25 any sign

^
Column number

322

TEST 11 YEAR 2 NUMBER OF CHILDREN = 334 POSITIVE/NEGATIVE NUMBERS

VAR	1	2	3	4	5	6	7	8	9	0
1	91.6	0.0	0.0	0.0	.3	1.2	.3	0.0	5.4	1.2
2	94.0	1.5	0.0	0.0	0.0	.3	.3	0.0	2.7	1.2
3	79.6	0.0	0.0	0.0	0.0	6.6	0.0	.3	8.4	5.1
4	65.3	.6	0.0	0.0	0.0	6.3	.3	11.1	7.5	9.0
5	72.2	1.8	0.0	0.0	0.0	.9	2.7	0.0	14.7	7.8
6	51.5	0.0	6.3	0.0	0.0	0.0	0.0	15.6	16.8	9.9
7	45.8	11.4	7.5	0.0	.3	0.0	.3	4.8	20.7	9.3
8	52.4	2.7	.9	0.0	0.0	0.0	0.0	.6	23.4	20.1
9	36.8	13.2	0.0	0.0	5.4	.9	0.0	13.8	19.2	10.8
10	47.0	6.3	0.0	0.0	0.0	0.0	0.0	1.2	19.2	26.3
11	71.0	13.5	.3	0.0	0.0	2.7	0.0	.3	6.9	5.4
12	77.8	9.3	0.0	0.0	0.0	.3	0.0	0.0	4.8	7.8
13	77.5	9.6	0.0	0.0	0.0	.3	0.0	0.0	6.6	6.0
14	59.6	5.4	3.3	0.0	1.2	1.5	6.3	0.0	16.8	6.0
15	27.8	.9	1.5	0.0	12.6	3.0	.9	.3	26.3	26.6
16	68.0	0.0	0.0	0.0	4.2	3.6	0.0	1.2	6.9	16.2
17	76.0	4.8	5.1	0.0	.6	0.0	.3	0.0	8.7	4.5
18	63.2	.3	0.0	0.0	3.6	.3	0.0	3.9	21.6	7.2
19	79.0	0.0	0.0	0.0	.9	0.0	7.8	.9	6.9	4.5
20	69.2	0.0	0.0	0.0	.3	8.1	3.3	1.5	11.4	6.3
21	77.2	0.0	0.0	0.0	0.0	3.0	0.0	5.4	6.3	8.1
22	81.1	0.0	0.0	0.0	0.0	2.7	0.0	2.4	5.4	8.4
23	76.6	4.8	3.9	0.0	0.0	4.8	0.0	3.0	3.0	3.9
24	78.4	3.3	3.9	0.0	0.0	4.5	0.0	3.3	2.7	3.9
25	94.9	0.0	0.0	0.0	.9	1.2	0.0	0.0	.9	2.1
26	86.5	0.0	0.0	0.0	0.0	5.7	0.0	4.5	.6	2.7
27	76.6	0.0	0.0	0.0	0.0	3.9	0.0	9.3	6.9	3.3
28	63.5	0.0	0.0	0.0	0.0	6.3	0.0	21.9	4.8	3.6
29	90.4	0.0	0.0	0.0	0.0	0.0	0.0	6.0	.9	2.7
30	91.0	0.0	0.0	0.0	0.0	.6	0.0	1.5	.6	6.3
31	90.4	.3	0.0	0.0	0.0	.6	0.0	2.1	2.4	4.2
32	14.4	29.6	.6	0.0	0.0	35.3	0.0	6.6	6.0	7.5
33	21.6	58.7	.3	0.0	0.0	0.0	0.0	7.2	5.7	6.6
34	55.7	0.0	0.0	0.0	0.0	17.4	5.7	6.0	6.0	9.3
35	25.4	0.0	0.0	0.0	0.0	4.5	4.5	50.9	4.8	9.9
36	76.0	0.0	0.0	0.0	0.0	.9	0.0	15.6	.9	6.6
37	12.6	53.0	1.2	0.0	0.0	.3	0.0	21.3	3.0	8.7
38	10.8	28.7	.3	0.0	0.0	8.4	0.0	38.0	3.6	10.2
39	86.8	0.0	0.0	0.0	0.0	0.0	0.0	1.2	2.1	9.9
40	80.8	0.0	0.0	0.0	.9	3.0	0.0	0.0	6.3	9.0
41	78.7	0.0	0.0	0.0	2.4	6.3	0.0	3.3	1.8	7.5
42	76.6	0.0	0.0	0.0	.6	7.8	0.0	2.4	4.5	8.1
43	71.6	0.0	0.0	0.0	1.8	9.0	0.0	5.4	3.9	8.4
44	70.4	0.0	0.0	0.0	1.5	15.3	0.0	.9	3.3	8.7
45	72.8	0.0	0.0	0.0	.9	14.4	0.0	1.5	1.8	8.7
46	42.5	0.0	0.0	0.0	11.4	2.4	.3	36.5	.9	6.0
47	31.4	6.6	6.9	0.0	0.0	0.0	14.4	30.2	3.0	7.5
48	23.4	0.0	0.0	0.0	.3	2.1	35.0	29.3	1.8	8.1
49	74.6	.3	0.0	0.0	8.4	0.0	5.1	0.0	3.9	7.8
50	66.8	0.0	0.0	0.0	0.0	8.4	0.0	4.5	9.3	11.1
51	42.5	11.1	0.0	0.0	4.2	.6	8.1	4.5	17.4	11.7
52	68.9	0.0	0.0	0.0	.6	14.4	0.0	3.9	5.1	7.2
53	63.8	0.0	0.0	0.0	1.2	2.1	0.0	26.6	.3	6.0
54	51.2	0.0	0.0	0.0	5.1	24.6	0.0	3.6	4.2	11.4
55	77.2	0.0	0.0	0.0	2.4	6.6	0.0	3.3	3.0	7.5
56	62.9	0.0	0.0	0.0	6.9	15.0	0.0	5.4	2.1	7.8
57	21.3	0.0	0.0	0.0	61.7	1.5	0.0	4.2	3.6	7.8
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

TEST 11 YEAR 3 NUMBER OF CHILDREN = 302 POSITIVE/NEGATIVE NUMBERS

VAR	1	2	3	4	5	6	7	8	9	0
1	93.4	0.0	0.0	0.0	.3	2.3	0.0	0.0	3.3	.7
2	94.7	1.7	0.0	0.0	0.0	.7	.7	0.0	1.7	.7
3	87.1	0.0	0.0	0.0	0.0	3.0	0.0	0.0	6.6	3.3
4	77.5	1.3	0.0	.3	0.0	4.3	0.0	6.3	4.6	5.6
5	78.8	3.6	0.0	0.0	0.0	0.0	2.3	0.0	11.3	4.0
6	69.9	.7	4.3	0.0	0.0	0.0	.7	9.9	6.6	7.9
7	64.2	8.3	3.3	0.0	1.3	0.0	0.0	3.6	12.3	7.0
8	66.9	3.3	1.3	0.0	.3	0.0	.3	.7	14.2	12.9
9	52.0	15.9	0.0	0.0	3.0	.3	0.0	8.3	12.9	7.6
10	58.6	7.3	0.0	0.0	0.0	0.0	0.0	0.0	12.6	21.5
11	76.5	10.6	.3	0.0	0.0	1.7	0.0	0.0	8.3	2.6
12	84.1	7.0	0.0	0.0	0.0	.3	0.0	0.0	4.6	4.0
13	79.5	8.3	.3	0.0	0.0	1.3	0.0	0.0	7.3	3.3
14	69.5	6.0	2.3	0.0	1.3	1.3	2.6	.3	11.6	5.0
15	38.1	.7	1.0	0.0	10.6	3.3	1.0	0.0	25.2	20.2
16	79.1	0.0	0.0	0.0	4.6	2.0	0.0	.3	3.6	10.3
17	82.5	7.0	2.6	0.0	0.0	0.0	0.0	1.0	5.0	2.0
18	73.5	.3	0.0	0.0	2.3	.3	0.0	3.0	14.9	5.6
19	83.4	0.0	0.0	0.0	.3	0.0	8.3	1.0	3.0	4.0
20	74.2	0.0	0.0	0.0	0.0	8.6	2.3	.3	10.9	3.6
21	84.8	0.0	0.0	0.0	0.0	1.7	0.0	3.6	4.3	5.6
22	82.1	0.0	0.0	0.0	0.0	1.7	0.0	2.3	7.3	6.6
23	78.8	3.3	6.0	0.0	0.0	5.3	0.0	3.3	.3	3.0
24	78.1	3.0	7.3	0.0	0.0	5.3	0.0	2.3	1.0	3.0
25	97.0	0.0	0.0	0.0	0.0	.3	0.0	0.0	1.0	1.7
26	87.1	0.0	0.0	.3	0.0	6.0	0.0	3.6	1.0	2.0
27	85.1	0.0	0.0	0.0	0.0	3.0	0.0	6.3	3.0	2.6
28	78.1	0.0	0.0	0.0	0.0	6.6	0.0	10.6	1.7	3.0
29	91.1	0.0	0.0	0.0	0.0	0.0	0.0	5.3	.3	3.3
30	93.0	0.0	0.0	0.0	0.0	.7	0.0	1.0	1.0	4.3
31	94.0	0.0	0.0	0.0	0.0	1.3	0.0	1.0	.7	3.0
32	8.9	48.3	0.0	0.0	0.0	26.8	0.0	3.3	6.6	6.0
33	14.6	67.9	1.0	0.0	0.0	0.0	0.0	5.0	5.0	6.6
34	60.3	0.0	0.0	0.0	0.0	13.6	9.9	4.0	3.6	8.6
35	30.5	0.0	0.0	0.0	0.0	2.6	5.3	46.7	6.3	8.6
36	85.8	0.0	0.0	0.0	0.0	.3	0.0	8.3	1.3	4.3
37	9.3	65.6	2.0	0.0	0.0	.3	0.0	12.9	3.6	6.3
38	7.6	35.1	1.3	0.0	0.0	7.0	0.0	36.8	4.0	8.3
39	89.7	0.0	0.0	0.0	0.0	.7	0.0	1.3	1.7	6.6
40	87.1	0.0	0.0	0.0	.3	2.3	0.0	.3	3.6	6.3
41	84.8	0.0	0.0	0.0	2.3	3.6	0.0	2.6	1.3	5.3
42	85.4	0.0	0.0	0.0	1.3	2.6	0.0	1.7	2.3	6.6
43	79.1	0.0	0.0	0.0	3.0	8.6	0.0	2.0	1.3	6.0
44	79.5	0.0	0.0	0.0	2.3	10.3	0.0	.3	1.7	6.0
45	85.4	0.0	0.0	0.0	1.3	6.0	0.0	1.3	0.0	6.0
46	42.7	0.0	0.0	0.0	6.3	1.0	0.0	44.7	1.0	4.3
47	27.8	7.6	4.3	0.0	.3	0.0	10.9	43.0	1.3	4.6
48	22.2	0.0	0.0	0.0	0.0	.7	34.1	35.8	1.0	6.3
49	83.4	.3	0.0	0.0	5.0	0.0	1.0	.3	5.6	4.3
50	75.8	0.0	0.0	0.0	0.0	4.6	0.0	3.3	7.6	8.6
51	57.9	5.6	0.0	0.0	2.6	0.0	6.0	2.6	15.9	9.3
52	77.8	0.0	0.0	0.0	.7	10.6	0.0	2.0	2.6	6.3
53	59.9	0.0	0.0	0.0	1.3	1.7	0.0	31.1	0.0	6.0
54	70.9	0.0	0.0	0.0	3.3	13.6	0.0	1.0	2.0	9.3
55	77.2	0.0	0.0	0.0	2.6	7.6	0.0	3.3	3.0	6.3
56	75.8	0.0	0.0	0.0	2.3	11.6	0.0	2.6	.7	7.0
57	30.5	0.0	0.0	0.0	53.0	2.6	0.0	6.3	1.0	6.6
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

TEST 11 YEAR 4 NUMBER OF CHILDREN = 182 POSITIVE/NEGATIVE NUMBERS

VAR	1	2	3	4	5	6	7	8	9	0
1	96.2	0.0	0.0	0.0	0.0	.5	0.0	0.0	2.2	1.1
2	91.8	4.4	0.0	0.0	0.0	.5	.5	0.0	1.6	1.1
3	86.8	0.0	0.0	0.0	0.0	5.5	0.0	.5	5.5	1.6
4	75.3	.5	0.0	0.0	0.0	6.6	0.0	7.1	3.8	6.6
5	76.9	1.6	0.0	0.0	0.0	.5	1.6	.5	13.2	5.5
6	72.0	0.0	8.2	0.0	0.0	0.0	.5	4.4	9.9	4.9
7	65.9	12.6	5.5	0.0	.5	0.0	0.0	1.6	8.8	4.9
8	70.3	5.5	1.1	0.0	0.0	0.0	0.0	0.0	9.9	13.2
9	48.4	18.7	0.0	0.0	5.5	.5	0.0	3.3	17.6	6.0
10	57.1	9.9	0.0	0.0	0.0	0.0	0.0	.5	11.0	21.4
11	79.7	8.2	0.0	0.0	0.0	1.6	0.0	.5	6.0	3.8
12	85.2	6.0	0.0	0.0	0.0	.5	0.0	0.0	1.6	6.6
13	79.1	4.4	0.0	0.0	0.0	1.6	0.0	0.0	10.4	4.4
14	68.7	7.7	1.6	0.0	.5	.5	1.6	0.0	13.7	5.5
15	39.0	1.6	.5	0.0	10.4	3.8	0.0	.5	21.4	22.5
16	75.3	0.0	0.0	0.0	3.8	1.6	0.0	.5	4.4	14.3
17	73.1	4.9	8.8	0.0	.5	0.0	1.6	.5	6.6	3.8
18	69.2	0.0	0.0	0.0	3.3	1.1	0.0	2.2	19.2	4.9
19	81.9	0.0	0.0	0.0	0.0	0.0	7.1	.5	5.5	4.9
20	72.5	0.0	0.0	0.0	0.0	8.8	1.6	1.6	9.9	5.5
21	81.9	.5	0.0	0.0	0.0	3.3	0.0	6.0	3.3	4.9
22	83.5	.5	0.0	0.0	0.0	2.2	0.0	4.4	3.8	5.5
23	78.6	3.8	5.5	0.0	0.0	3.8	0.0	1.6	1.6	4.9
24	78.0	5.5	5.5	0.0	0.0	2.2	0.0	1.1	2.7	4.9
25	96.2	.5	0.0	0.0	0.0	.5	0.0	0.0	0.0	2.7
26	86.3	0.0	0.0	0.0	0.0	6.6	0.0	3.3	.5	3.3
27	80.8	0.0	0.0	0.0	0.0	4.4	0.0	5.5	6.0	3.3
28	72.5	0.0	0.0	0.0	0.0	9.3	0.0	9.9	4.9	3.3
29	88.5	0.0	0.0	0.0	0.0	0.0	0.0	4.9	2.7	3.8
30	94.0	0.0	0.0	0.0	0.0	.5	0.0	0.0	0.0	5.5
31	90.7	0.0	0.0	0.0	0.0	0.0	0.0	.5	2.7	6.0
32	17.6	36.8	.5	0.0	0.0	29.1	0.0	2.2	4.9	8.8
33	24.2	57.7	0.0	0.0	0.0	0.0	0.0	2.2	5.5	10.4
34	62.6	.5	0.0	0.0	0.0	12.1	4.9	2.7	2.2	14.8
35	33.0	0.0	0.0	0.0	0.0	2.2	3.3	43.4	4.4	13.7
36	74.7	0.0	0.0	0.0	0.0	.5	0.0	10.4	2.7	11.5
37	15.4	48.4	2.7	0.0	0.0	.5	0.0	16.5	4.4	12.1
38	8.8	31.3	.5	0.0	0.0	6.0	0.0	39.0	1.1	13.2
39	84.6	.5	0.0	0.0	0.0	0.0	0.0	0.0	1.1	13.7
40	79.1	0.0	0.0	0.0	.5	2.2	0.0	.5	3.8	13.7
41	78.0	0.0	0.0	0.0	0.0	4.4	0.0	2.7	1.1	13.7
42	78.0	0.0	0.0	0.0	0.0	5.5	0.0	.5	1.1	14.8
43	74.2	0.0	0.0	0.0	1.6	4.4	0.0	2.2	3.8	13.7
44	74.7	0.0	0.0	0.0	1.1	8.2	0.0	.5	1.6	13.7
45	76.4	0.0	0.0	0.0	0.0	7.7	0.0	1.6	.5	13.7
46	44.0	0.0	0.0	0.0	5.5	1.1	.5	37.4	0.0	11.5
47	29.7	6.6	8.2	0.0	.5	.5	8.2	33.0	1.6	11.5
48	30.2	0.0	0.0	0.0	1.6	0.0	27.5	27.5	0.0	13.2
49	72.0	1.6	0.0	0.0	8.2	0.0	4.9	0.0	1.1	12.1
50	75.3	0.0	0.0	0.0	0.0	3.8	0.0	2.7	3.8	14.3
51	51.1	8.2	0.0	0.0	2.7	.5	5.5	2.7	13.2	15.9
52	71.4	0.0	0.0	0.0	1.6	9.9	0.0	1.6	.5	14.8
53	63.2	0.0	0.0	0.0	0.0	1.6	0.0	20.3	1.1	13.7
54	58.8	0.0	0.0	0.0	4.4	17.6	0.0	.5	1.6	17.0
55	69.8	0.0	0.0	0.0	1.1	9.3	0.0	3.3	2.2	14.3
56	65.9	0.0	0.0	0.0	1.1	14.3	0.0	2.7	1.1	14.8
57	31.3	0.0	0.0	0.0	49.5	2.7	0.0	1.6	.5	14.3
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

PHI CORRELATION MATRIX POSITIVE & NEGATIVE NUMBERS N=818

96.1	25																														
96.0	2	118																													
93.3	1	147	540																												
92.4	30	447	199	218																											
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	370																										
91.9	29	490	225	297	434	431																									
89.5	12	177	283	305	398	249	249																								
87.5	39	343	264	223	451	486	308	292																							
87.4	24	398	184	236	330	374	482	182	306																						
86.9	23	407	166	185	[illegible]	364	471	174	350	781																					
86.7	26	366	212	237	363	372	397	193	331	577	498																				
86.6	13	123	210	223	266	336	[illegible]	445	285	196	167	214																			
86.2	11	120	224	232	269	310	237	498	277	200	170	228	454																		
84.0	3	264	263	348	240	262	372	361	309	330	324	319	287	337																	
83.4	17	231	208	207	293	253	261	307	308	240	235	259	295	258	306																
82.8	40	225	169	265	370	423	329	276	640	324	264	297	266	286	321	274															
82.2	22	391	245	270	336	319	437	339	374	352	330	343	265	322	527	349	378														
81.9	33	249	194	216	381	432	355	273	389	338	260	329	252	345	279	310	497	328													
81.3	19	226	172	209	289	318	295	346	302	328	269	282	380	325	380	442	337	447	303												
81.2	21	326	250	270	334	316	399	385	414	345	323	298	286	341	463	347	392	763	342	442											
80.8	41	189	199	203	329	414	268	318	548	292	252	274	290	272	252	274	648	324	407	331	376										
80.7	27	268	261	289	362	373	506	316	377	383	397	436	297	315	461	388	367	491	349	424	493	296									
80.2	42	168	147	234	309	370	299	269	536	264	234	247	217	281	276	272	609	337	380	280	349	676	300								
79.3	36	270	186	188	287	336	269	228	355	325	303	315	188	333	328	250	366	314	293	281	287	349	331	367							
78.2	45	199	177	272	307	333	245	282	473	291	278	255	217	269	302	289	543	342	375	332	367	623	342	667	389						
78.1	5	91	116	164	150	136	174	223	232	164	162	140	138	173	333	176	196	286	166	216	259	200	227	189	219	229					
77.9	49	196	265	233	281	361	349	354	378	355	299	328	256	358	385	299	419	474	384	363	496	428	403	414	353	454	273				
75.6	55	149	85	108	245	301	224	129	234	169	184	226	217	151	193	150	327	195	212	179	198	278	197	274	271	251	146	238			
74.9	43	188	168	250	282	325	255	307	464	307	252	238	226	275	308	256	624	312	395	330	327	570	310	533	380	590	192	418	261		
74.7	44	241	259	214	322	343	234	313	486	236	232	243	207	255	236	268	568	279	398	290	338	673	313	557	369	592	256	407	264	578	
73.7	16	180	253	272	280	251	298	338	279	292	254	305	212	300	526	330	293	468	318	375	458	329	411	309	326	344	335	457	190	276	297
72.9	4	166	182	209	292	243	234	301	227	232	203	237	242	265	445	259	260	318	263	278	310	247	355	234	238	277	202	297	170	211	245
72.7	52	232	223	263	312	302	266	336	425	255	242	244	204	272	334	287	556	381	354	311	372	496	333	481	406	528	193	493	341	634	534
72.2	50	183	218	234	325	325	326	301	341	280	258	220	241	318	388	299	385	399	336	378	396	346	391	373	320	416	269	494	342	349	407
71.8	20	181	202	189	271	262	323	369	261	285	264	225	309	307	362	354	317	360	248	444	385	273	353	287	303	281	239	306	173	295	296
71.4	14	137	158	186	268	269	274	488	293	257	233	229	384	428	380	247	291	354	341	327	394	364	382	316	258	347	253	470	149	333	328
70.9	28	217	114	150	294	274	350	298	255	270	270	271	268	281	380	292	284	418	286	362	421	282	504	269	278	288	240	345	199	325	295
70.4	37	166	267	221	269	319	237	249	322	230	233	273	192	237	308	199	342	306	321	286	283	384	273	397	621	424	207	357	241	379	392
68.9	51	150	224	252	267	295	330	295	322	254	225	242	213	259	333	282	365	384	322	328	399	377	347	389	329	433	207	456	294	367	423
68.8	61	[illegible]	216	246	196	227	213	311	233	253	208	217	214	265	353	224	230	285	287	299	317	234	265	301	314	323	218	372	206	280	269
68.6	18	175	222	196	293	224	282	386	254	259	252	230	250	309	408	355	283	365	283	465	394	292	362	258	279	300	259	368	227	307	320
68.3	56	187	87	153	212	251	260	272	339	209	188	243	147	245	204	204	385	238	335	226	250	449	266	466	353	577	122	358	279	400	468
67.6	7	162	203	252	166	197	211	316	228	264	204	212	224	275	367	231	223	298	285	297	328	239	283	318	298	318	208	367	213	262	264
66.1	34	228	128	168	263	309	294	276	265	273	251	221	179	252	314	270	302	341	354	299	322	340	330	312	349	336	239	384	218	343	373
65.3	8	130	199	234	264	245	261	361	268	272	250	242	262	340	395	253	285	357	317	341	410	303	332	346	331	362	259	452	236	307	331
63.1	6	[illegible]	193	189	182	229	242	315	261	267	244	206	203	281	342	202	227	311	291	328	357	231	305	293	322	314	238	403	230	282	288
62.8	81	66	228	257	181	218	249	346	238	220	211	212	238	329	367	240	251	335	315	331	373	299	322	347	313	348	266	431	239	296	332
62.6	71	117	213	216	179	188	255	319	235	254	224	224	228	283	365	211	255	325	279	277	357	277	325	357	298	345	262	403	247	293	322
62.2	53	128	147	133	115	139	158	111	163	99	79	147	158	126	141	139	158	104	118	130	102	170	148	118	138	132	136	125	307	137	155
61.0	10	97	180	185	187	186	230	297	252	232	187	195	206	261	265	194	298	236	223	272	262	329	250	344	248	319	207	304	99	301	330
60.4	9	94	262	211	183	200	263	349	246	250	219	212	244	327	402	215	259	366	294	291	390	290	338	350	302	354	278	435	237	304	322
60.1	54	119	150	170	224	235	262	209	312	150	150	151	140	180	230	152	441	246	317	198	240	389	246	384	281	460	112	342	321	451	416
51.6	32	145	137	199	193	242	241	210	234	251	226	232	163	200	290	185	296	276	339	269	265	304	288	316	248	336	132	344	205	337	308
42.9	46	85	140	124	145	166	152	87	132	150	123	136	110	132	136	121	114	133	125	124	120	96	148	71	100	38	76	182	171	108	118
42.9	47	22	89	124	98	129	152	120	117	120	57	114	153	118	163	115	49	158	60	168	145	108	161	102	106	98	142	200	200	68	50
41.6	38	93	109	157	157	204	185	205	198	178	172	162	143	157	172	156	260	244	248	239	247	291	236	313	246	318	188	318	214	293	279
35.1	15	82	124	115	152	161	198	226	168	201	171	145	259	227	230	156	193	242	219	254	236	247	236	230	242	244	190	280	173	265	262
33.5	35	89	105	138	144	162	181	164	158	163	213	163	143	164	182	170	173	195	172	174	209	194	222	196	304	223	200	234	168	260	228
26.9	57	65	96	63	111	129	116	180	128	103	161	156	93	154	174	122	169	181	170	163	200	232	198	239	255	233	154	250	210	248	257
24.4	48	85	58	95	98	147	179	130	120	147	136	105	107	112	116	101	176	116	171	119	99	161	98	132	156	148	53	166	118	184	147
FACILITY	VAR.	25	2	1	30	31	29	12	39	24	23	26	13	11	3	17	40	22	33	19	21	41	27	42	36	45	5	49	55	43	44

PHI CORRELATION MATRIX POSITIVE & NEGATIVE NUMBERS CONTD.

73.7	16																													
72.9	4	372																												
72.7	52	301	305																											
72.8	50	376	335	407																										
71.8	20	390	307	317	340																									
71.4	14	377	361	377	376	348																								
70.9	28	351	347	321	349	331	386																							
70.4	37	361	248	397	299	259	277	262																						
68.9	51	343	261	431	681	277	335	297	352																					
68.8	61	335	269	322	344	298	333	283	292	346																				
68.6	18	427	309	348	410	441	381	360	294	314	312																			
68.3	56	232	270	498	324	232	325	223	318	366	296	218																		
67.6	7	340	305	309	342	302	357	263	295	337	915	313	286																	
66.1	34	388	266	362	411	331	347	383	360	373	304	300	279	288																
65.3	8	363	311	406	397	295	430	301	331	370	651	314	320	685	340															
63.1	6	360	307	328	369	352	378	301	308	335	868	333	296	829	319	681														
62.8	81	374	287	369	388	315	425	292	327	353	640	346	297	667	347	900	690													
62.6	71	353	295	372	356	306	309	255	312	343	614	309	310	652	322	815	701	827												
62.2	53	130	85	174	171	127	109	94	163	180	156	108	103	161	167	205	151	220	210											
61.0	10	319	239	304	310	272	370	271	267	265	333	306	285	351	254	354	333	360	376	111										
60.4	9	437	326	385	385	324	444	306	334	374	577	351	330	619	328	753	659	768	846	178	362									
60.1	54	223	210	561	315	204	236	226	292	322	233	207	465	215	245	235	230	231	258	102	209	269								
51.6	32	288	244	329	338	223	274	257	267	338	282	245	292	280	387	321	308	318	327	108	263	325	295							
42.9	46	136	106	131	264	132	144	201	129	149	162	129	16	167	181	134	156	130	154	155	80	181	19	103						
42.9	47	175	184	114	171	143	182	207	139	170	135	139	27	151	155	201	151	196	205	166	131	212	4	128	461					
41.6	38	244	191	360	269	192	297	228	340	330	278	255	275	292	351	302	300	284	288	145	201	307	265	300	165	160				
35.1	15	293	235	271	275	279	374	279	257	282	290	282	269	300	271	320	323	326	308	144	283	370	185	230	128	180	289			
33.5	35	259	159	265	234	226	260	243	352	246	237	223	221	225	381	283	263	277	275	88	206	294	159	257	91	112	378	265		
26.9	57	255	221	303	273	209	261	224	332	293	283	262	252	278	270	303	315	306	309	125	241	333	229	278	120	137	433	281	381	
24.4	48	145	34	137	158	160	171	138	181	117	149	139	130	132	178	140	164	119	116	73	145	117	137	169	156	58	183	183	186	219
FACILITY VAR.		16	4	52	50	20	14	28	37	51	61	18	56	7	34	8	6	81	71	53	10	9	54	32	46	47	38	15	35	57

CSMS

MATRICES 1 : A

NAME .... TODAY'S DATE ....

BOY OR GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ....

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1. A company has three factories, one at Aberdeen (A), one at Bristol (B), and one at Coventry (C). Each factory uses steel, tin and copper to make machinery, and in January :

   the factory at Aberdeen uses 3 tons of steel, 2 tons of tin and 1 ton of copper,
   the factory at Bristol uses 6 tons of steel, 4 tons of tin and 6 tons of copper.

   These figures together with the figures for the Coventry factory are shown in this matrix ⟶

$$\begin{array}{cc} & \begin{array}{ccc} s & tin & c \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 3 & 2 & 1 \\ 6 & 4 & 6 \\ 4 & 3 & 2 \end{pmatrix} \end{array}$$

   How much tin does the factory at Coventry (C) use in January ? ....1....

   For February the figures for the factories at Aberdeen and Bristol are shown in this matrix ⟶

   The factory at Coventry uses : 4 tons of copper,
   5 tons of steel
   and 5 tons of tin.

   Complete the matrix.

$$\begin{array}{cc} & \begin{array}{ccc} s & tin & c \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ & & \end{pmatrix} \end{array} \; 2$$

   Complete this matrix ⟶ which represents the amounts of steel, tin and copper used by each of the factories for January and February together.

$$\begin{array}{cc} \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 10 & 7 & 5 \\ & & \\ & & \end{pmatrix} \end{array} \; 3$$

At the start of March this matrix represents the amounts of metal in store at each factory. $\longrightarrow$

$$\begin{array}{c|ccc} & s & tin & c \\ A & 7 & 4 & 3 \\ B & 6 & 6 & 5 \\ C & 7 & 5 & 3 \end{array}$$

This matrix $\longrightarrow$ represents the amounts of metal used from store by each of the factories during March.

$$\begin{array}{c|ccc} A & 5 & 1 & 1 \\ B & 2 & 3 & 5 \\ C & 4 & 4 & 3 \end{array}$$

Fill in this matrix $\longrightarrow$ so it represents the amount of metal left in the stores at the end of March.

$$\begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$ 4

This matrix $\longrightarrow$ represents the deliveries of steel, tin and copper to the factories for a total of 4 weeks.

$$\begin{pmatrix} 16 & 16 & 12 \\ 28 & 20 & 24 \\ 24 & 20 & 16 \end{pmatrix}$$

The same amount is delivered each week.
What does this matrix represent ? $\longrightarrow$

$$\begin{pmatrix} 4 & 4 & 3 \\ 7 & 5 & 6 \\ 6 & 5 & 4 \end{pmatrix}$$

....

.... 5

Complete this equation $\longrightarrow$

$$3\begin{pmatrix} 4 & 4 & 3 \\ 7 & 5 & 6 \\ 6 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 12 & 12 & 9 \\ 21 & & \\ & & \end{pmatrix}$$ 6

How many weeks deliveries would this represent ? 7

2. The two matrices below show the amounts of screws and nails (in tons) produced by a factory in London (L), and a factory in Manchester (M) for June and July.

a) Complete the addition.

$$\begin{array}{ccc} & sc & n \\ L & 1 & 3 \\ M & 4 & 12 \end{array} + \begin{array}{ccc} & sc & n \\ L & 6 & 5 \\ M & 5 & 3 \end{array} = \begin{array}{cc} sc & n \\ 7 & 8 \\ & \end{array}$$ 8

Now work out :

b) $\begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} + \begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$ c) $\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$ 9

d) Compare your answers to b) and c). What do you notice ? 10

Why is this ? (The questions are not the same, the matrices have been swopped over.)

.... 11

.... 12

Now work out :

e) $\begin{pmatrix} 7 & 9 \\ 8 & 3 \end{pmatrix} - \begin{pmatrix} 5 & 7 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$ 13

f) $3 \begin{pmatrix} 2 & 1 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$ 14

---

3. Find what the letters stand for :

a) $\begin{pmatrix} 3 & 4 \\ 7 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 7 \\ a & b \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ 9 & 5 \end{pmatrix}$

a = ...15

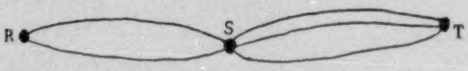
b = ...16

b) $4 \begin{pmatrix} 5 & c \\ d & 3 \end{pmatrix} = \begin{pmatrix} 20 & 16 \\ 4 & 12 \end{pmatrix}$

c = ...17

d = ...18

4. This diagram shows the roads between Rainford, Selham and Talton.



There are 2 roads direct from Rainford to Selham and 3 roads direct from Selham to Talton. There are 6 ways of getting from Rainford to Talton, going through Selham. Explain how we get 6 ways, label or number the roads if it will help you.

.... 19 ....

....

If a new road is built from Rainford to Selham, so that there are 3 roads from Rainford to Selham and 3 roads from Selham to Talton, how many ways are there of going from Rainford to Talton going through Selham ?

20 ....

If there are 6 roads from L to M and 5 roads from M to N, how many ways can you get from L to N, going through M ?

21 ....

---

5. Two boys, Roger and Simon, decided to build camp stools.

Roger's stool needed 5 metres of canvas, 3 metres of wood and 4 metres of string. Simon's stool needed 4 metres of wood, 3 metres of canvas and 2 metres of string. Roger suggested that this matrix represented the information they needed to give the woodwork teacher. He was wrong. Why ?

$$\begin{array}{c@{\;}c} & \begin{array}{ccc} c & w & s \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{pmatrix} 5 & 3 & 4 \\ 4 & 3 & 2 \end{pmatrix} \end{array}$$

.... 22

....

Roger and Simon decided to build smaller stools instead of the larger ones. Roger's new stool needed 1 metre of canvas, 50 cms. of wood and 3 metres of string. Simon's new stool needed 1 metre of canvas, 1 metre of wood and 2 metres of string. Simon suggested that this matrix represented the information they needed to give the woodwork teacher.

He was wrong. Why ?

$$\begin{array}{c@{\;}c} & \begin{array}{ccc} c & w & s \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{pmatrix} 1 & 50 & 3 \\ 1 & 1 & 2 \end{pmatrix} \end{array}$$

.... 23

....

6. $\begin{pmatrix} 5 & 6 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 10 & 11 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 15 & 17 \\ 9 & 12 \end{pmatrix}$ This is a correct matrix addition. Imagine you are writing a maths text-book and wish to use this in a problem. Think of a story in real life using these numbers and fill in what each number would stand for . Make sure the addition makes sense.

Var 24, 25, 26, 27

5 represents ....

6 represents ....

2 represents ....

4 represents ....

10 represents ....

11 represents ....

7 represents ....

8 represents ....

15 represents ....

17 represents ....

9 represents ....

12 represents ....

7. In an athletics match the points for each race were awarded as follows :

For coming first : 3 points
For coming second : 2 points
For coming third : 1 point

or $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ as a column matrix.

Blackley school had : 2 first places, 5 second places and 3 third places.

This can be written as a row matrix like this $\begin{pmatrix} 2 & 5 & 3 \end{pmatrix}$

Blackley school got 6 points for first places.

How many points did Blackley school get for second places ? 28

How many points did Blackley school get for third places ? 29

How many points did Blackley school get altogether ? 30

We can also work this out by multiplying the two matrices together to give us a matrix containing the total points gained, like this : $\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 19 \end{pmatrix}$

Explain how we get the total points gained.

....31..............

....

Cradley school had : 4 first places, 1 second place and 3 third places.

Write this information as a row matrix using these brackets .

Now work out how many points Cradley school got altogether using these matrices. ⟶ $\begin{pmatrix} \text{Var 32} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}$ Var 33

Which school won the match ? 34

By how many points ? 35

At the end of the match someone suggested that the matrix $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ should be used for the scoring.

How many points are given for a first place in this system ? 36

334

The next year the two schools met again, and the results are shown in this matrix

$$\begin{array}{l c} & \begin{array}{ccc} \text{1st} & \text{2nd} & \text{3rd} \end{array} \\ \begin{array}{l} \text{Blackley} \\ \text{Cradley} \end{array} & \begin{pmatrix} 2 & 5 & 2 \\ 4 & 1 & 4 \end{pmatrix} \end{array}$$

How many second places did Cradley school get ? ....37...

One scorer thought the scoring system was 3 points for a first place, 2 for a second and 1 for a third, and worked out the result by multiplying these two matrices together.

$$\begin{pmatrix} 2 & 5 & 2 \\ 4 & 1 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 18 \\ \ \end{pmatrix} 38$$

The 18 means that Blackley school got 18 points altogether.

Finish off the working out. What does the number you have just filled in stand for ? ....39..........................

The other scorer used the other system.

$$\begin{pmatrix} 2 & 5 & 2 \\ 4 & 1 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 20 \\ \ \end{pmatrix} 41$$

What does the 20 mean ? 40

....

Finish off the working out.

Which school wins when $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ is used for scoring ? ......42.........

Which school wins when $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ is used for scoring ? ....43..........

8. This matrix shows the number of buses (b) and coaches (c) used by two factories, one at Poplar (P) and one at Stepney (S), to bring their workers to work.

$$\begin{array}{c} \\ P \\ S \end{array}\begin{array}{c} \begin{array}{cc} b & c \end{array} \\ \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \end{array}$$

It costs £12 to run a bus and £10 to run a coach.

How much does it cost to bring people to the factory in Poplar ? ....44.......

How much does it cost to bring people to the factory in Stepney ? ....45.......

9. Two wine shops, Browns (B) and Carvers (C), sell red wine (r) and white wine (w). The amounts (in bottles) they sell on Monday are shown in this matrix ⟶

$$\begin{array}{c} \\ B \\ C \end{array}\begin{array}{c} \begin{array}{cc} r & w \end{array} \\ \begin{pmatrix} 5 & 4 \\ 3 & 6 \end{pmatrix} \end{array}$$

Red wine costs £4 a bottle.

White wine costs £2 a bottle.

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

We can multiply these two matrices together :

$$\begin{array}{c} \\ B \\ C \end{array}\begin{array}{c} \begin{array}{cc} r & w \end{array} \\ \begin{pmatrix} 5 & 4 \\ 3 & 6 \end{pmatrix} \end{array}\begin{array}{c} \\ \begin{pmatrix} 4 \\ 2 \end{pmatrix} \end{array} = \begin{array}{c} \\ \begin{pmatrix} 28 \\ 24 \end{pmatrix} \end{array}$$

What does the 28 represent ? ....46..................

....

What does the 24 represent ? ....47..................

....

MARKING SCHEME MATRICES A

Variable	Code 1	Other Codes
1	3 tons	
2	$(5 \ \ 5 \ \ 4)$	$(4 \ \ 5 \ \ 5)$: 8
3	$\begin{pmatrix} 11 & 8 & 12 \\ 9 & 8 & 6 \end{pmatrix}$	$\begin{pmatrix} 11 & 8 & 12 \\ 8 & 8 & 7 \end{pmatrix}$: 4 allow one error on code 1
4	$\begin{pmatrix} 2 & 3 & 2 \\ 4 & 3 & 0 \\ 3 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 12 & 5 & 4 \\ 8 & 9 & 10 \\ 11 & 9 & 6 \end{pmatrix}$ or similar : 8 allow one error on code 1
5	one week's deliveries	4 weeks :$\boxed{8}$
6	$\begin{pmatrix} & 15 & 18 \\ 18 & 15 & 12 \end{pmatrix}$	allow one error on code 1
7	3	
8	$(9 \ \ 15)$	
9	$\begin{pmatrix} 9 & 9 \\ 4 & 8 \end{pmatrix}$: twice	only one matrix correct :$\boxed{8}$
10	same	
11	addition of matrices (or numbers) is commutative	when adding change of order is not important :$\boxed{2}$ the matrices have just changed places with no mention of addition :$\boxed{8}$
12	example given for 11 e.g 7+2=2+7	no example given $\boxed{3}$ use code 0 only if 11 is code 0
13	$\begin{pmatrix} 2 & 2 \\ 7 & 3 \end{pmatrix}$	$\begin{pmatrix} 12 & 16 \\ 9 & 3 \end{pmatrix}$:8 allow one error on code 1
14	$\begin{pmatrix} 6 & 3 \\ 12 & 24 \end{pmatrix}$	allow one error on code 1
15	2	
16	3	
17	4	16 :$\boxed{7}$ 12 :$\boxed{8}$
18	1	4 :$\boxed{7}$ 0 : $\boxed{8}$
19	demonstration of 2x3	
20	9	6 :$\boxed{7}$ 7 : $\boxed{8}$
21	30 with diagram	30 without diagram :2 11:$\boxed{7}$ 12 : $\boxed{8}$
22	4 and 3 in wrong places or any mention of error in Simon's row	Not enough data size not given etc :8
23	Units are mixed	

Marices A Marking Scheme contd.

Variable	Code 1	Other Codes
24	1st matrix +	
25	2nd matrix $\begin{pmatrix}A_1 & A_2\\ B_1 & B_2\end{pmatrix}$ =	Row structure only correct :6 Column structure only correct :7
26	3rd matrix structure	$\begin{pmatrix}A & B\\ C & D\end{pmatrix}$ structure :8 $\begin{pmatrix}A & A\\ A & A\end{pmatrix}$ structure:5
27	To add makes sense. All 3 matrices have similar structure and are related by a correct addition	
28	10	2 : $\boxed{7}$ 5 : $\boxed{8}$
29	3	1 : $\boxed{7}$ 3 : $\boxed{8}$ only if 28 coded 8
30	19	Incorrect answer consistent with error in 28 and/or 29 : $\boxed{4}$ 13 : $\boxed{8}$
31	$(2 \times 3) + (5 \times 2) + (3 \times 1) = 19$ or multiply places by points	Perform matrix multiplication or row by column : 2
32	(4 1 3)	(12 2 3) or similar : 8
33	17	$15 \leq x \leq 20$: $\boxed{7}$
34	Blackley	Wrong answer consistent with 33 : 4 Wrong answer consistent with 30 : 5
35	2	Wrong answer consistent with 33 : 4 Wrong answer consistent with 30 : 5
36	4	2 or 1 : $\boxed{8}$
37	1	5 : $\boxed{8}$
38	18	Triple given : 8
39	Cradley's score	Just points : 8
40	Blackley's score	Just points : 8
41	22	
42	Neither (both, draw, tie)	Unproven correcr : 3 Wrong consistent with 38 : 4
43	Cradley	Unproven correct : 3 Wrong consistent with 41 : 4
44	64	68 : $\boxed{8}$
45	42	46 : $\boxed{8}$
46	Amount sold at Brown's, implication of money	Total number of bottles sold at Browns :8 1st row x column : 3
47	Amount sold at Carvers, implication of money	Total number of bottles sold by Carvers:8 2nd row x column : 3

TEST 6 YEAR 3 N=135 MATRICES A

VAR	1	2	3	4	5	6	7	8	9	0
1	80.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.8	4.4
2	79.3	0.0	0.0	0.0	0.0	0.0	0.0	6.7	8.1	5.9
3	71.9	0.0	0.0	5.9	0.0	0.0	0.0	0.0	12.6	9.6
4	79.3	0.0	0.0	0.0	0.0	0.0	0.0	3.7	8.1	8.9
5	63.7	0.0	0.0	0.0	0.0	0.0	0.0	3.0	14.8	18.5
6	83.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.0	14.1
7	70.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.3	16.3
8	89.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.2	5.2
9	85.9	0.0	0.0	0.0	0.0	0.0	0.0	3.0	5.2	5.9
10	79.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.9	8.9
11	2.2	9.6	0.0	0.0	0.0	0.0	0.0	42.2	20.0	25.9
12	3.0	0.0	71.1	0.0	0.0	.7	0.0	0.0	0.0	25.2
13	79.3	0.0	0.0	0.0	0.0	0.0	0.0	8.1	7.4	5.2
14	76.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.3	10.4
15	74.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.3	11.9
16	72.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.1	13.3
17	61.5	0.0	0.0	0.0	0.0	0.0	0.0	1.5	19.3	17.8
18	60.7	0.0	0.0	0.0	0.0	0.0	1.5	0.0	15.6	22.2
19	54.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.5	26.7
20	54.1	0.0	0.0	0.0	0.0	0.0	6.7	4.4	21.5	13.3
21	5.9	27.4	0.0	0.0	0.0	0.0	12.6	5.9	28.9	19.3
22	48.1	0.0	0.0	0.0	0.0	0.0	0.0	1.5	25.9	24.4
23	46.7	0.0	0.0	0.0	0.0	0.0	0.0	2.2	20.0	31.1
24	11.9	0.0	0.0	0.0	3.0	0.0	3.0	36.3	6.7	39.3
25	11.1	0.0	0.0	0.0	4.4	.7	2.2	33.3	7.4	40.7
26	9.6	0.0	0.0	0.0	5.2	.7	1.5	33.3	8.1	41.5
27	4.4	20.7	11.9	0.0	0.0	0.0	0.0	0.0	21.5	41.5
28	77.8	0.0	0.0	0.0	0.0	0.0	5.2	4.4	8.1	4.4
29	78.5	0.0	0.0	0.0	0.0	0.0	6.7	0.0	10.4	4.4
30	74.8	0.0	0.0	5.2	0.0	0.0	0.0	2.2	13.3	4.4
31	54.1	8.1	0.0	0.0	0.0	0.0	0.0	0.0	11.1	26.7
32	74.8	0.0	0.0	0.0	0.0	0.0	0.0	6.7	6.7	11.9
33	63.0	0.0	0.0	0.0	0.0	0.0	9.6	3.0	13.3	11.1
34	72.6	0.0	0.0	9.6	.7	0.0	0.0	0.0	4.4	12.6
35	55.6	0.0	0.0	17.8	.7	0.0	0.0	0.0	11.9	14.1
36	62.2	0.0	0.0	0.0	0.0	0.0	0.0	3.7	16.3	17.8
37	83.7	0.0	0.0	0.0	0.0	0.0	0.0	1.5	4.4	10.4
38	50.4	0.0	0.0	0.0	0.0	0.0	0.0	2.2	20.0	27.4
39	34.8	0.0	0.0	0.0	0.0	0.0	0.0	23.0	10.4	31.9
40	30.4	0.0	0.0	0.0	0.0	0.0	0.0	23.0	10.4	36.3
41	41.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	22.2	36.3
42	34.8	0.0	2.2	14.8	0.0	0.0	0.0	0.0	26.7	21.5
43	42.2	0.0	14.1	12.6	0.0	0.0	0.0	0.0	9.6	21.5
44	49.6	0.0	0.0	0.0	0.0	0.0	0.0	2.2	30.4	17.8
45	48.9	0.0	0.0	0.0	0.0	0.0	0.0	2.2	30.4	18.5
46	39.3	0.0	0.0	0.0	0.0	0.0	0.0	15.6	21.5	23.7
47	39.3	0.0	0.0	0.0	0.0	0.0	0.0	14.8	21.5	24.4

TEST 6 YEAR 4 N=122 MATRICES A

VAR	1	2	3	4	5	6	7	8	9	0
1	83.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15.6	.8
2	90.2	0.0	0.0	0.0	0.0	0.0	0.0	4.9	4.1	.8
3	77.9	0.0	0.0	2.5	.8	0.0	0.0	0.0	10.7	8.2
4	85.2	0.0	0.0	0.0	0.0	0.0	0.0	1.6	10.7	2.5
5	66.4	0.0	0.0	0.0	0.0	0.0	0.0	3.3	20.5	9.8
6	88.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6.6	4.9
7	86.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.2	5.7
8	86.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.5	1.6
9	90.2	0.0	0.0	0.0	0.0	0.0	0.0	.8	9.0	0.0
10	86.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.7	7.4
11	.8	27.0	0.0	0.0	0.0	0.0	0.0	38.5	14.8	18.9
12	3.3	0.0	77.0	0.0	0.0	0.0	0.0	0.0	0.0	19.7
13	86.9	0.0	0.0	0.0	0.0	0.0	0.0	5.7	7.4	0.0
14	82.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.1	4.9
15	83.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.4	9.0
16	82.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.4	9.8
17	80.3	0.0	0.0	0.0	0.0	0.0	.8	0.0	9.8	9.0
18	77.0	0.0	0.0	0.0	0.0	0.0	5.7	0.0	6.6	10.7
19	56.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	22.1	21.3
20	59.8	.8	0.0	0.0	0.0	0.0	11.5	1.6	9.0	17.2
21	12.3	33.6	0.0	0.0	0.0	0.0	9.8	2.5	16.4	25.4
22	50.0	0.0	0.0	0.0	0.0	0.0	0.0	4.9	25.4	19.7
23	62.3	0.0	0.0	0.0	0.0	0.0	0.0	1.6	13.1	23.0
24	8.2	0.0	0.0	0.0	3.3	2.5	4.9	43.4	14.8	23.0
25	8.2	0.0	0.0	0.0	3.3	2.5	4.9	42.6	14.8	23.8
26	6.6	.8	0.0	0.0	4.1	2.5	5.7	37.7	17.2	25.4
27	4.1	27.0	14.8	0.0	0.0	0.0	0.0	0.0	31.1	23.0
28	75.4	0.0	0.0	0.0	0.0	0.0	2.5	4.9	8.2	9.0
29	77.9	0.0	0.0	0.0	0.0	0.0	2.5	1.6	9.0	9.0
30	71.3	0.0	0.0	4.1	0.0	0.0	0.0	4.9	10.7	9.0
31	61.5	.8	0.0	0.0	0.0	0.0	0.0	0.0	6.6	31.1
32	78.7	0.0	0.0	0.0	0.0	0.0	0.0	4.9	2.5	13.9
33	63.1	0.0	0.0	0.0	0.0	0.0	4.1	5.7	10.7	16.4
34	75.4	0.0	0.0	5.7	0.0	0.0	0.0	0.0	.8	18.0
35	63.9	0.0	0.0	13.1	0.0	0.0	0.0	0.0	4.1	18.9
36	68.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6	10.7	19.7
37	76.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.4	16.4
38	58.2	0.0	0.0	0.0	0.0	0.0	0.0	1.6	14.8	25.4
39	49.2	0.0	0.0	0.0	0.0	0.0	0.0	21.3	3.3	26.2
40	43.4	0.0	0.0	0.0	0.0	0.0	0.0	27.0	5.7	23.8
41	54.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.5	34.4
42	47.5	0.0	1.6	5.7	0.0	0.0	0.0	0.0	20.5	24.6
43	52.5	0.0	8.2	4.1	0.0	0.0	0.0	0.0	10.7	24.6
44	55.7	0.0	0.0	0.0	0.0	0.0	0.0	1.6	20.5	22.1
45	50.8	0.0	0.0	0.0	0.0	0.0	0.0	.8	25.4	23.0
46	45.9	0.0	.8	0.0	0.0	0.0	0.0	14.8	13.1	25.4
47	45.9	0.0	.8	0.0	0.0	0.0	0.0	13.1	13.1	27.0

PHI CORRELATION MATRIX , MATRICES A N = 257

Facility	Var.	8	9	6	2	10	13	1	4	37	14	15	29	7	16	28	32	3	34	30	17	18	5	36	33	31	35	20	19	23	38
88.3	8																														
87.9	9	683																													
85.0	6	437	420																												
84.4	2	311	401	464																											
82.9	10	510	688	313	417																										
82.9	13	349	465	372	374	341																									
82.1	1	[illegible]	294	242	247	245	138																								
82.1	4	525	512	415	387	380	326	179																							
[illegible]	37	[illegible]	324	212	109	317	240	200	251																						
79.3	14	466	512	659	384	348	374	232	481	174																					
79.0	15	526	512	387	358	424	323	207	456	294	460																				
78.2	24	424	412	239	215	310	285	220	417	517	283	352																			
77.8	7	272	348	581	365	304	279	214	337	133	460	322	262																		
77.4	16	499	514	361	307	421	273	257	403	244	406	909	301	339																	
76.7	28	458	473	350	[illegible]	384	335	240	462	486	392	370	845	347	340																
76.7	32	286	[illegible]	245	219	261	261	126	270	509	324	279	377	281	252	391															
74.7	3	346	444	373	713	329	282	218	545	249	468	425	300	400	392	356	356														
73.9	34	388	297	236	136	200	177	185	323	462	237	215	459	194	209	447	615	286													
73.2	30	436	449	326	272	307	307	198	450	513	398	355	743	373	324	827	433	415	520												
[illegible]	17	454	493	487	309	407	339	209	453	254	586	712	360	413	690	408	327	387	256	376											
[illegible]	18	[illegible]	391	461	332	336	269	207	360	166	554	657	270	383	675	298	278	356	226	326	753										
[illegible]	5	342	304	465	137	229	272	210	380	248	422	382	304	550	384	385	289	417	232	346	417	327									
65.0	36	241	279	279	202	272	224	231	316	493	302	342	363	256	325	346	520	379	548	420	364	362	247								
63.0	33	349	359	351	227	229	208	210	336	447	356	297	474	250	280	549	606	333	665	536	404	348	316	434							
62.3	31	266	303	343	219	200	328	201	348	417	406	307	483	357	290	538	462	377	506	560	427	370	337	438	501						
59.5	35	342	376	339	192	277	214	256	380	424	372	314	486	284	294	538	594	359	702	520	421	362	292	508	879	535					
57.2	20	297	307	339	257	212	233	191	498	299	383	306	419	295	323	470	284	365	328	469	369	310	403	321	347	332	360				
55.6	19	212	270	258	264	217	259	155	420	262	289	289	344	277	304	414	210	345	272	448	313	270	313	329	289	387	253	683			
54.1	23	248	258	356	293	223	161	221	384	265	348	368	289	410	343	359	266	451	306	393	446	433	403	403	394	410	401	402	356		
54.1	38	272	282	289	207	244	286	140	303	383	329	233	440	316	212	506	451	326	484	463	343	332	256	305	539	522	528	386	356	373	
52.5	44	[illegible]	341	320	344	209	209	247	369	308	389	332	423	336	344	451	414	468	429	461	357	344	412	461	401	417	454	406	343	390	421
49.8	45	313	345	275	298	184	225	241	363	280	399	322	393	288	332	439	365	436	361	445	355	324	339	388	327	358	393	389	372	339	386
49.6	22	186	172	269	227	177	115	153	315	[illegible]	238	276	291	205	250	283	283	319	227	207	328	246	246	230	267	297	269	313	295	513	263
47.5	41	272	328	323	236	328	287	199	342	414	337	318	388	301	289	395	432	355	440	400	376	359	338	436	469	499	513	381	346	406	531
47.1	43	294	301	364	275	263	325	237	338	391	371	295	365	353	266	409	428	387	418	430	372	388	381	414	463	461	507	406	340	446	556
42.4	46	238	293	262	325	264	160	256	318	348	307	288	395	230	274	436	361	426	419	484	349	311	217	332	445	392	515	392	370	348	379
42.4	47	238	293	262	325	264	160	256	318	348	288	268	395	249	256	436	361	426	419	484	349	311	217	316	445	408	515	392	370	348	379
41.6	39	233	312	256	253	258	258	188	291	341	241	299	388	298	267	372	372	364	393	404	339	369	239	404	401	445	487	347	356	413	445
40.9	42	252	283	318	269	251	314	202	326	373	350	292	381	310	259	439	421	392	439	449	417	410	328	394	538	516	572	415	423	448	765
39.3	21	218	249	261	213	196	259	188	354	280	317	297	270	237	300	293	293	303	278	289	329	305	206	390	253	347	290	599	574	373	373
36.6	40	200	281	265	281	195	259	207	354	317	272	292	322	249	274	342	342	404	340	387	350	376	252	371	313	457	379	395	434	407	440
28.0	27	172	204	206	243	168	168	178	291	245	279	215	308	207	233	282	262	323	232	299	309	292	240	330	226	289	267	382	347	400	383
19.5	11	117	182	173	102	223	118	152	178	170	229	205	164	238	194	178	155	195	90	209	253	291	298	195	132	240	144	226	240	334	275
10.1	24	81	124	27	72	83	83	122	89	102	141	78	177	85	88	185	185	106	169	203	132	172	56	192	123	101	145	159	221	179	153
9.7	25	78	121	22	104	79	79	119	119	97	137	72	173	80	51	181	181	130	194	198	126	166	48	213	142	93	163	177	240	170	144
8.6	26	67	113	85	131	102	65	106	142	117	157	89	161	96	98	168	168	146	181	185	167	177	78	195	119	94	138	208	245	198	170
3.1	12	65	66	73	15	81	-156	83	83	89	92	92	94	95	96	98	-7	52	4	108	116	121	37	37	44	47	56	64	24	165	30

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FAC  ITEM                                        PHI CORRELATI[illegible] N OF CASES = [illegible]
52.5  44
49.8  45  806
49.0  22  262 252
47.5  41  482 486 330
47.1  43  490 510 306 882
42.4  46  374 389 260 492 483
42.4  47  358 373 260 508 499 984
41.6  39  423 342 308 572 579 536 552
40.9  42  457 436 293 620 659 455 455 566
39.3  21  350 377 262 383 374 341 341 403 449
36.6  40  414 406 321 523 546 525 541 800 552 480
28.0  27  384 401 324 378 366 358 341 351 415 385 425
19.5  11  250 218 225 261 265 174 174 242 291 268 239 306
10.1  24  163 156  83 172 148 260 234 240 193 232 281 336  95
 9.7  25  206 198  98 187 163 276 249 255 207 246 295 350 104 934
 8.6  26  207 195 117 210 185 300 272 249 226 266 316 397  95 912 932
 3.1  12  125 135  93   9  10  72  72  30  33  39  49  37 364  14  16  25

           44  45  22  41  43  46  47  39  42  21  40  27  11  24  25  26  12
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PHI CORRELATION MATRIX, MATRICES A contd.

341

CSMS

MATRICES1 : M

NAME .... TODAY'S DATE ....

BOY OR GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ....

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1. A company has three factories, one at Aberdeen (A), one at Bristol (B), and one at Coventry (C). Each factory uses steel, tin and copper to make machinery, and in January :

   the factory at Aberdeen uses 3 tons of steel, 2 tons of tin and 1 ton of copper,
   the factory at Bristol uses 6 tons of steel, 4 tons of tin and 6 tons of copper.

   These figures together with the figures for the Coventry factory are shown in this matrix ⟶

$$\begin{array}{cc} & \begin{array}{ccc} s & \text{tin} & c \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 3 & 2 & 1 \\ 6 & 4 & 6 \\ 4 & 3 & 2 \end{pmatrix} \end{array}$$

   How much tin does the factory at Coventry (C) use in January ? 1

   For February the figures for the factories at Aberdeen and Bristol are shown in this matrix ⟶

   The factory at Coventry uses : 4 tons of copper,
   5 tons of steel
   and 5 tons of tin.

   Complete the matrix.

$$\begin{array}{cc} & \begin{array}{ccc} s & \text{tin} & c \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ & & \end{pmatrix} \end{array}$$ 2

   Complete this matrix ⟶ which represents the amounts of steel, tin and copper used by each of the factories for January and February together.

$$\begin{array}{cc} \begin{array}{c} A \\ b \\ C \end{array} & \begin{pmatrix} 10 & 7 & 5 \\ & & \\ & & \end{pmatrix} \end{array}$$ 3

2. The two matrices below show the amounts of screws and nails (in tons) produced by a factory in London (L), and a factory in Manchester (M) for June and July.

a) Complete the addition.

$$\begin{array}{ccc} & sc & n \\ L & 1 & 3 \\ M & 4 & 12 \end{array} + \begin{array}{ccc} & sc & n \\ L & 6 & 5 \\ M & 5 & 3 \end{array} = \begin{array}{cc} sc & n \\ 7 & 8 \\ & \end{array}$$ 4

Now work out :

b) $\begin{pmatrix}2 & 4\\ 1 & 6\end{pmatrix} + \begin{pmatrix}7 & 5\\ 3 & 2\end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$ 5

c) $\begin{pmatrix}7 & 5\\ 3 & 2\end{pmatrix} + \begin{pmatrix}2 & 4\\ 1 & 6\end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$ 5

d) Compare your answers to b) and c). What do you notice ? 6

Why is this ? (The questions are not the same, the matrices have been swopped over.) 7

.... 8

....

---

3. Two boys, Roger and Simon, decided to build camp stools.

Roger's stool needed 5 metres of canvas, 3 metres of wood and 4 metres of string.

Simon's stool needed 4 metres of wood, 3 metres of canvas and 2 metres of string.

Roger suggested that this matrix represented the information they needed to give the woodwork teacher. He was wrong. Why ?

$$\begin{array}{cccc} & c & w & s \\ R & 5 & 3 & 4 \\ S & 4 & 3 & 2 \end{array}$$ 9

....

....

Roger and Simon decided to build smaller stools instead of the larger ones.

Roger's new stool needed 1 metre of canvas, 50 cms. of wood and 3 metres of string. Simon's new stool needed 1 metre of canvas, 1 metre of wood and 2 metres of string. Simon suggested that this matrix represented the information they needed to give the woodwork teacher.

He was wrong. Why ?

$$\begin{array}{cccc} & c & w & s \\ R & 1 & 50 & 3 \\ S & 1 & 1 & 2 \end{array}$$ 10

....

....

4. In an athletics match the points for each race were awarded as follows :

For coming first : 3 points
For coming second : 2 points
For coming third : 1 point

or $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ as a column matrix.

Blackley school had : 2 first places, 5 second places and 3 third places.

This can be written as a row matrix like this $\begin{pmatrix} 2 & 5 & 3 \end{pmatrix}$

Blackley school got 6 points for first places.

How many points did Blackley school get for second places ? ....11.

How many points did Blackley school get for third places ? ....12.

How many points did Blackley school get altogether ? ....13.

We can also work this out by multiplying the two matrices together to give us a matrix containing the total points gained, like this :

$$\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 19 \end{pmatrix}$$

Explain how we get the total points gained.

....14......

....

Cradley school had : 4 first places, 1 second place and 3 third places.

Write this information as a row matrix using these brackets .

Now work out how many points Cradley school got altogether using these matrices. ⟶

$$\begin{pmatrix} \text{Var 15} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix} \text{ Var 16}$$

Which school won the match ? ....17.....

By how many points ? ....18....

At the end of the match someone suggested that the matrix $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ should be used for the scoring.

How many points are given for a first place in this system ? ....19..........

The next year the two schools met again, and the results are shown in this matrix

$$\begin{array}{l} & \text{1st} & \text{2nd} & \text{3rd} \\ \text{Blackley} & 2 & 5 & 2 \\ \text{Cradley} & 4 & 1 & 4 \end{array}$$

How many second places did Cradley school get ? 20

One scorer thought the scoring system was 3 points for a first place, 2 for a second and 1 for a third, and worked out the result by multiplying these two matrices together.

$$\begin{pmatrix} 2 & 5 & 2 \\ 4 & 1 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 18 \\ \end{pmatrix} \text{Var 21}$$

The 18 means that Blackley school got 18 points altogether.

Finish off the working out. What does the number you have just filled in stand for ? ....22.

The other scorer used the other system.

What does the 20 mean ? 23

....

$$\begin{pmatrix} 2 & 5 & 2 \\ 4 & 1 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 20 \\ \end{pmatrix} 24$$

Finish off the working out.

The referee thought that 4 points for a first place, 3 points for a second place and 1 point for a third place would be the best scoring system. $\longrightarrow \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

He asked the scorers to compare all three scoring systems.

They did so by multiplying these two matrices together.

Complete the multiplication.

$$\begin{pmatrix} 2 & 5 & 2 \\ 4 & 1 & 4 \end{pmatrix} \begin{pmatrix} 3 & 4 & 4 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 18 & 20 & \\ & & 23 \end{pmatrix}$$

Var 25, 26, 27

What does the 23 stand for ? ....

Which school wins when $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ is used for scoring ? ....29..........

Which school wins when $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ is used for scoring ? ....30..........

Which school wins when $\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$ is used for scoring ? ....31..........

5. Two wine shops, Browns (B) and Carvers (C) sell red wine (r) and white wine (w). The amounts (in bottles) they sell on Monday are shown in this matrix. ⟶

$$\begin{array}{ccc} & r & w \\ B & 5 & 4 \\ C & 3 & 6 \end{array}$$

Red wine costs £4 a bottle, and white wine costs £2 a bottle.

This matrix multiplication is correct. ⟶

$$\begin{array}{ccc} & r & w \\ B & 5 & 4 \\ C & 3 & 6 \end{array} \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}28\\24\end{pmatrix}$$

What does the 28 represent ? ....32..

....

What does the 24 represent ? ....33..

....

We can rewrite the square matrix like this :

$$\begin{array}{ccc} & B & C \\ r & 5 & 3 \\ w & 4 & 6 \end{array}$$

Then we multiply :

$$\begin{array}{ccc} & B & C \\ r & 5 & 3 \\ w & 4 & 6 \end{array} \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}26\\28\end{pmatrix}$$

How do we get the 26 ? ....34..

....

The 26 does not stand for the cost of red wine sold at both shops, why not ?

....35..

....

---

6. This matrix shows the number of buses (b) and coaches (c) used by two factories, one at Poplar (P) and one at Stepney (S), to bring their workers to work.

$$\begin{array}{ccc} & b & c \\ P & 2 & 4 \\ S & 1 & 3 \end{array}$$

It costs £12 to run a bus and £10 to run a coach.

How much does it cost to bring people to the factory in Poplar ? ......36......

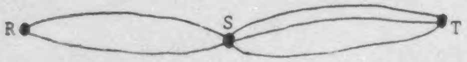
How much does it cost to bring people to the factory in Stepney ? ......37......

Now write this sum out as a matrix multiplication. One matrix has already been written out for you, fill in the rest.

$$\begin{pmatrix}2 & 4\\1 & 3\end{pmatrix}$$

38

7. This diagram shows the roads between Rainford, Selham and Talton.



There are 2 roads direct from Rainford to Selham and 3 roads direct from Selham to Talton. There are 6 ways of getting from Rainford to Talton, going through Selham. Explain how we get 6 ways, label or number the roads if it will help you.

.... 39

....

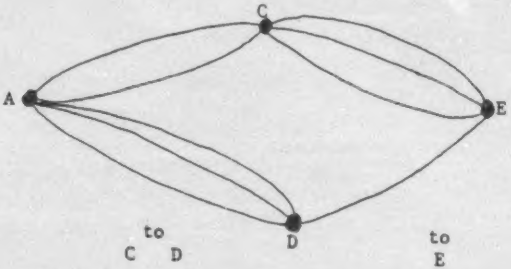
If a new road is built from Rainford to Selham, so that there are 3 roads from Rainford to Selham and 3 roads from Selham to Talton, how many ways are there of going from Rainford to Talton going through Selham ?

40 ....

If there are 6 roads from L to M and 5 roads from M to N, how many ways can you get from L to N, going through M ?

41 ....

This diagram shows the airline routes from A to E. All aircraft must stop at C or D to refuel.



We can show this information in matrix form like this :

$$\text{from}\ \begin{array}{cc} & \text{to} \\ & \begin{matrix} C & D \end{matrix} \\ A & \begin{pmatrix} 2 & 3 \end{pmatrix} \end{array} \qquad \text{from}\ \begin{array}{cc} & \text{to} \\ & E \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} 3 \\ 1 \end{pmatrix} \end{array}$$

To get the number of routes from A to E (stopping at C or at D) we can multiply the matrices together. Complete the multiplication.

$$\begin{array}{cc} & \begin{matrix} C & D \end{matrix} \\ A & \begin{pmatrix} 2 & 3 \end{pmatrix} \end{array} \begin{array}{cc} & E \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} 3 \\ 1 \end{pmatrix} \end{array} = \text{from}\ \begin{array}{cc} & \text{to} \\ & E \\ A & \begin{pmatrix} \quad \end{pmatrix} \end{array} \ 42$$

The airline now operates from new airports at B and F as well, and the routes are shown in these two matrices. → (All aircraft stop at C or D to refuel.)

$$\text{from}\ \begin{array}{cc} & \text{to} \\ & \begin{matrix} C & D \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \end{array} \qquad \text{from}\ \begin{array}{cc} & \text{to} \\ & \begin{matrix} E & F \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \end{array}$$

When we multiply these matrices together we will get another matrix containing routes. Fill in the correct row and column headings for this matrix.

$$\begin{array}{cc} & \begin{matrix} C & D \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \end{array} \begin{array}{cc} & \begin{matrix} E & F \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \end{array} = \text{from}\ \begin{array}{c} \text{to} \\ \begin{pmatrix} 9 & 10 \\ 13 & 10 \end{pmatrix} \end{array} 43$$

8. $\begin{pmatrix} 4 & 7 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 18 \\ 19 \end{pmatrix}$

This is a correct matrix multiplication. Imagine you are writing a maths text-book and wish to use this in a problem. Think of a story in real life using these numbers and fill in what each number would stand for. Make sure the multiplication makes sense.

4 represents .... Var 44 45 46

7 represents ....

3 represents ....

8 represents ....

1 represents ....

2 represents ....

18 represents ....

19 represents ....

---

9. $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 9 & 5 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 7 & 8 \end{pmatrix}$

These are correct matrix multiplications. Look at them carefully and compare them. Write down what they tell you about matrix multiplication.

.... 47

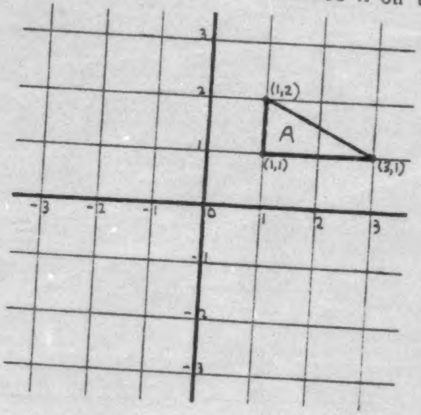
....

10. We can also use matrices to move points and shapes around on a diagram. If we start from the origin (at tne centre (0,0)) the triangle marked A on the diagram has corners at

1 right 2 up or $\begin{pmatrix}1\\2\end{pmatrix}$

1 right 1 up or $\begin{pmatrix}1\\1\end{pmatrix}$

3 right 1 up or $\begin{pmatrix}3\\1\end{pmatrix}$



If we wish to move the triangle A using the matrix $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ we can do so by multiplying like this :

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}1\\-2\end{pmatrix}$ → The corner at $\begin{pmatrix}1\\2\end{pmatrix}$ moves to $\begin{pmatrix}1\\-2\end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix}$ → The corner at $\begin{pmatrix}1\\1\end{pmatrix}$ moves to $\begin{pmatrix}1\\-1\end{pmatrix}$

Finish this multiplication to find the new position of the third corner. $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}=\begin{pmatrix} \\ \end{pmatrix}$ 48

Now draw in the new position of the triangle on the diagram and label it B. 49

Fill in the answers to these three equations to move triangle A using the matrix $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$.

$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix} \\ \end{pmatrix}$ 50

$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix} \\ \end{pmatrix}$ 51

$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}=\begin{pmatrix} \\ \end{pmatrix}$ 52

Draw the new triangle on the diagram and label it C. 53

MATRICES I:M MARKING BOOK

VAR.	CODE 1	OTHER CODES	
(1)	3 TONS		$\boxed{1}$ - $\boxed{9}$
(2)	$(5 \quad 5 \quad 4)$	$(4 \quad 5 \quad 5)$: $\boxed{8}$	
(3)	$\begin{pmatrix} 11 & 8 & 12 \\ 9 & 8 & 6 \end{pmatrix}$	$\begin{pmatrix} 11 & 8 & 12 \\ 8 & 8 & 7 \end{pmatrix}$: $\boxed{4}$	Allow one error on code $\boxed{1}$.
(4).	$(9 \quad 15)$		$\boxed{1}$ - $\boxed{9}$
(5)	$\begin{pmatrix} 9 & 9 \\ 4 & 8 \end{pmatrix}$: TWICE	ONLY 1 MATRIX CORRECT : $\boxed{8}$	-
(6)	SAME		$\boxed{1}$ - $\boxed{9}$
(7)	Addition of matrices (or numbers) is commutative.	When adding change of order is not important : $\boxed{2}$	The matrices have just changed places : $\boxed{8}$ (No mention of addition)
(8)	Example given for ⑦ e.g. 7 + 2 = 2 + 7	No example given : $\boxed{3}$	Use code $\boxed{0}$ only if ⑦ is code $\boxed{0}$. $\boxed{1}$ - $\boxed{3}$ ONLY
9	4 and 3 in wrong places, or any mention of error in the order of Simon's row.	Not enough data, Size not given etc. : $\boxed{8}$	
10	Units are mixed		
(11)	10	2 : $\boxed{7}$	5 : $\boxed{8}$
(12)	3	1 : $\boxed{7}$	3 : $\boxed{8}$ only if ⑪ is code $\boxed{8}$.
(13)	19	Incorrect answer consistent with error in ⑪ and/or ⑫ : $\boxed{4}$	13 : $\boxed{8}$

MATRICES H contd

Var	CODE 1	OTHER CODES	
14	(2x3) + (5x2) + (3x1) = 19 or multiply places by points.	Perform matrix multiplication or row by column : $\boxed{2}$	
15	(4 1 3)	(12 2 3) or similar : $\boxed{8}$	
16	17	15 < answer < 20 : $\boxed{7}$ 14 : $\boxed{8}$	
17	BLACKLEY	Wrong answer given, but consistent with ⑯ : $\boxed{4}$ Wrong answer given, but consistent with ⑬ : $\boxed{5}$	
18	2	Wrong answer given, but consistent with ⑯ : $\boxed{4}$ Wrong answer given, but consistent with ⑬ : $\boxed{5}$	
19	4	2 or 1 : $\boxed{8}$	
20	1	5 : $\boxed{8}$	
21	18	Triple given : $\boxed{8}$	
22	CRADLEY'S SCORE	Just points : $\boxed{8}$	
23	BLACKLEY'S SCORE	Just points : $\boxed{8}$	
24	22		$\boxed{1}$ - $\boxed{9}$
25	18	Wrong answer given consistent with answer to ㉑ : $\boxed{4}$	Square Matrix solution : $\boxed{8}$
26	22	Wrong answer given consistent with answer to ㉔ : $\boxed{4}$	
27	25		
28	CRADLEY'S SCORE UNDER THE REFEREE'S SCORING SYSTEM (4 3 1)	Total Score : $\boxed{5}$ Cradley's Score : $\boxed{6}$ Referee's (new) score : $\boxed{7}$ 3rd places, 3rd place points : $\boxed{8}$ 2nd row x 3rd column or similar : $\boxed{3}$	

MATRICES Contd

Var	CODE 1	OTHER CODES	
29	NEITHER (Both, Draw, Tie)	Unproven correct answer : $\boxed{3}$ Wrong, consistent with ㉕ : $\boxed{4}$ Wrong, consistent with (71) : $\boxed{5}$	If an answer is wrong but consistent with both applicable numerical answers use $\boxed{5}$
30	CRADLEY	Unproven correct answer : $\boxed{3}$ Wrong, consistent with (76) : $\boxed{4}$ Wrong, consistent with ㉔ : $\boxed{5}$	
31	BLACKLEY	Unproven correct answer : $\boxed{3}$ Wrong, consistent with ㉗ : $\boxed{4}$ DRAW : $\boxed{8}$	
32	Amount sold at Browns, implication of money	Total number of bottles sold by Browns : $\boxed{8}$ 1st row x column : $\boxed{3}$	
33	Amount sold at Carvers, implication of money	Total number of bottles sold by Carvers : $\boxed{8}$ 2nd row x column : $\boxed{3}$	
34	(5 x 4) + (3 x 2) or first row by column	Total cost of red wine or no. of bottles of red wine type answer : $\boxed{8}$	Matrices swapped around : $\boxed{3}$
35	£2 is not the cost of red wine		
36	64	68 : $\boxed{8}$	S is given matrix, α, β given answers to ㊱ and ㊲ . *: if r.h.s. transposed code unchanged.
37	42	46 : $\boxed{8}$	
38	$\begin{pmatrix}2 & 4\\1 & 3\end{pmatrix}\begin{pmatrix}12\\10\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix}^{*}$	$S\begin{pmatrix}12\\10\end{pmatrix} = \begin{pmatrix}64\\42\end{pmatrix}^{*}$: $\boxed{2}$	$S\,(12\ \ 10) = (\alpha\ \ \beta)$ *: $\boxed{4}$ $S\,(12\ \ 10) = (64\ \ 42)$ *: $\boxed{3}$
39	DEMONSTRATION OF 2 x 3	$S\begin{pmatrix}12\\10\end{pmatrix}$ only : $\boxed{6}$	(with α and β given.) $\begin{pmatrix}12\\10\end{pmatrix}$ pre.: $\boxed{7}$ $(12\ \ 10)$ pre. : $\boxed{8}$
40	9	6 : $\boxed{7}$ 7 : $\boxed{8}$	
41	30, with diagram drawn	30, without diagram : $\boxed{2}$ 11 : $\boxed{7}$ 12 : $\boxed{8}$	
42	9		$\boxed{1}$ - $\boxed{9}$
43	$\begin{array}{c\|cc} & E & F \\ \hline A & & \\ B & & \end{array}$	$\begin{array}{c\|cc} & A & B \\ \hline E & \boxed{3} & \\ F & & \end{array}$ $\begin{array}{c\|cc} & E & \\ \hline A & \boxed{6} & \\ & & \end{array}$ $\begin{array}{c\|cc} & E & F \\ \hline C & \boxed{7} & \\ D & & \end{array}$ etc. $\begin{array}{c\|cc} & & \\ \hline AC & \boxed{8} & \\ BD & & \end{array}$	

NATRICES IN cont'd

VAR	CODE 1	OTHER CODES
44	$\begin{pmatrix} A_1 & A_2 \\ B_1 & B_2 \end{pmatrix}$ structure	As $\boxed{1}$ but copy of Qu.5 : $\boxed{3}$ Row structure ONLY correct : $\boxed{6}$ Column structure ONLY correct : $\boxed{7}$ $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ structure : $\boxed{8}$
45	1 relates to 4 and 3 AND 2 relates to 7 and 8	Use code $\boxed{3}$ only if price etc. given in code $\boxed{8}$ structure in (64 Weak relationships only : $\boxed{3}$ 1 relates to 4 and 7, 2 relates to 3 and 8 : $\boxed{7}$ Undifferentiated multiplier used, e.g. how much, how much, or 1p, 2p with no relationship to specific elements : $\boxed{8}$
46	To multiply makes sense	To multiply makes only partial sense : $\boxed{3}$ (e.g. mixture of apples and pears for 18 and 19.) Column headings used for 18 and 19 : $\boxed{5}$
47	Matrix multiplication is not commutative	The order (position) is important when multiplying matrices : $\boxed{2}$ Wrong "jargon" used : $\boxed{8}$
48	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	$\boxed{1}$ - $\boxed{9}$
49	Correct drawing of B	$\boxed{1}$ - $\boxed{9}$

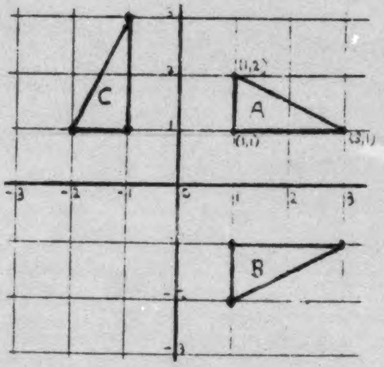


DIAGRAM for Qu. 10

VAR	CODE 1	OTHER CODES
50	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$: $\boxed{7}$ $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$: $\boxed{8}$
51	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$: $\boxed{7}$
52	$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$: $\boxed{7}$ $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$: $\boxed{8}$
53	Correct drawing of C	$\boxed{1}$ - $\boxed{9}$

TEST 5 YEAR 3 N=170 MATRICES M

VAR	1	2	3	4	5	6	7	8	9	0
1	92.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.7	2.4
2	96.5	0.0	0.0	0.0	0.0	0.0	0.0	1.8	1.8	0.0
3	94.1	0.0	0.0	1.2	0.0	0.0	0.0	0.0	4.1	.6
4	97.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.4	.6
5	97.6	0.0	0.0	0.0	0.0	0.0	0.0	1.8	.6	0.0
6	95.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.9	1.2
7	21.8	42.9	0.0	0.0	0.0	0.0	0.0	21.8	7.6	5.9
8	10.0	0.0	84.1	0.0	0.0	0.0	0.0	0.0	0.0	5.9
9	69.4	0.0	0.0	0.0	0.0	0.0	0.0	17.1	9.4	4.1
10	81.2	0.0	0.0	0.0	0.0	0.0	0.0	7.1	7.1	4.7
11	96.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.9	.6
12	96.5	0.0	0.0	0.0	0.0	0.0	.6	0.0	2.4	.6
13	95.3	0.0	0.0	.6	0.0	0.0	0.0	1.2	2.4	.6
14	48.8	41.2	0.0	0.0	0.0	0.0	0.0	0.0	2.9	7.1
15	97.1	0.0	0.0	0.0	0.0	0.0	0.0	2.4	0.0	.6
16	93.5	0.0	0.0	0.0	0.0	0.0	1.8	1.8	2.4	.6
17	97.1	0.0	0.0	1.8	0.0	0.0	0.0	0.0	.6	.6
18	91.8	0.0	0.0	4.1	0.0	0.0	0.0	0.0	3.5	.6
19	91.8	0.0	0.0	0.0	0.0	0.0	0.0	1.2	5.2	1.8
20	95.3	0.0	0.0	0.0	0.0	0.0	0.0	1.8	1.8	1.2
21	91.2	0.0	0.0	0.0	0.0	0.0	0.0	1.2	5.9	1.8
22	87.6	0.0	0.0	0.0	0.0	0.0	0.0	3.5	5.3	3.5
23	82.4	0.0	0.0	0.0	0.0	0.0	0.0	8.2	6.5	2.9
24	81.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.2	7.6
25	84.7	0.0	0.0	1.8	0.0	0.0	0.0	0.0	1.8	11.8
26	80.6	0.0	0.0	4.7	0.0	0.0	0.0	0.0	2.9	11.8
27	80.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.6	11.8
28	52.4	0.0	2.9	0.0	2.4	13.5	.6	11.2	4.7	12.4
29	79.4	0.0	0.0	1.2	1.8	0.0	0.0	0.0	14.1	3.5
30	80.0	0.0	1.2	2.9	1.2	0.0	0.0	0.0	11.2	3.5
31	72.9	0.0	2.9	3.5	.6	0.0	0.0	7.6	7.1	5.3
32	74.1	0.0	.6	0.0	0.0	0.0	0.0	17.1	3.5	4.7
33	73.5	0.0	.6	0.0	0.0	0.0	0.0	17.6	3.5	4.7
34	72.4	0.0	1.2	0.0	0.0	0.0	0.0	7.1	10.0	9.4
35	42.4	0.0	0.0	0.0	0.0	0.0	0.0	9.4	26.5	21.8
36	84.7	0.0	0.0	0.0	0.0	0.0	0.0	.6	10.0	4.7
37	84.1	0.0	0.0	0.0	0.0	0.0	0.0	1.2	9.4	5.3
38	3.5	53.5	0.0	1.2	0.0	11.2	.6	.6	13.5	15.9
39	75.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.7	9.4
40	77.1	0.0	0.0	0.0	0.0	0.0	4.7	5.3	5.3	7.6
41	11.8	57.6	0.0	0.0	0.0	0.0	11.2	1.8	8.2	9.4
42	72.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15.3	12.4
43	50.0	0.0	1.8	0.0	0.0	.6	2.9	0.0	14.1	30.6
44	58.2	0.0	.6	0.0	0.0	4.7	2.9	8.2	1.8	23.5
45	45.9	0.0	4.1	0.0	0.0	0.0	6.5	9.4	9.4	24.7
46	45.3	0.0	6.6	0.0	0.0	0.0	0.0	2.4	21.8	24.1
47	22.9	37.6	0.0	0.0	0.0	0.0	0.0	2.9	20.0	16.5
48	75.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	14.1
49	65.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.8	22.9
50	67.6	0.0	0.0	0.0	0.0	0.0	1.2	5.9	10.6	14.7
51	75.3	0.0	0.0	0.0	0.0	0.0	1.2	0.0	8.2	15.3
52	67.1	0.0	0.0	0.0	0.0	0.0	1.2	6.5	10.6	14.7
53	58.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.8	22.9

TEST 5 YEAR 4 N=365 MATRICES M

VAR	1	2	3	4	5	6	7	8	9	0
1	97.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0
2	98.1	0.0	0.0	0.0	0.0	0.0	0.0	1.6	0.0	.3
3	94.5	0.0	0.0	1.4	0.0	0.0	0.0	0.0	2.2	1.9
4	98.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.4	0.0
5	94.8	0.0	0.0	0.0	0.0	0.0	0.0	.8	4.4	0.0
6	94.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.7	1.4
7	23.3	38.4	0.0	0.0	0.0	0.0	0.0	23.0	8.2	7.1
8	12.3	0.0	80.8	0.0	0.0	0.0	0.0	0.0	0.0	6.8
9	73.2	0.0	0.0	0.0	0.0	0.0	0.0	5.8	14.5	6.6
10	84.1	0.0	0.0	0.0	0.0	0.0	0.0	2.2	6.0	7.7
11	98.1	0.0	0.0	0.0	0.0	0.0	.3	.5	1.1	0.0
12	98.1	0.0	0.0	0.0	0.0	0.0	.8	.3	.8	0.0
13	95.6	.3	0.0	2.2	0.0	0.0	0.0	.8	.8	.3
14	64.1	30.4	0.0	0.0	0.0	0.0	0.0	0.0	3.0	2.5
15	98.9	0.0	0.0	0.0	0.0	0.0	0.0	.8	.3	0.0
16	92.1	0.0	0.0	0.0	0.0	0.0	2.2	1.4	4.4	0.0
17	97.8	0.0	0.0	1.6	.5	0.0	0.0	0.0	0.0	0.0
18	89.6	0.0	0.0	6.8	.8	0.0	0.0	0.0	2.5	.3
19	92.9	0.0	0.0	0.0	0.0	0.0	0.0	1.1	5.2	.8
20	96.7	0.0	0.0	0.0	0.0	0.0	0.0	1.4	1.6	.3
21	92.6	0.0	0.0	0.0	0.0	0.0	0.0	.5	6.6	.3
22	83.3	0.0	0.0	0.0	0.0	0.0	0.0	12.3	2.7	1.6
23	81.4	0.0	0.0	0.0	0.0	0.0	0.0	12.9	3.8	1.9
24	89.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.7	3.3
25	90.4	0.0	0.0	1.9	0.0	0.0	0.0	0.0	5.2	2.5
26	89.9	0.0	0.0	2.5	0.0	0.0	0.0	0.0	4.9	2.7
27	84.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.2	2.5
28	52.3	0.0	5.2	0.0	3.6	12.6	3.0	12.1	3.8	7.4
29	83.0	0.0	.6	3.0	3.6	0.0	0.0	0.0	7.9	1.9
30	88.5	0.0	1.1	1.4	2.5	0.0	0.0	0.0	4.7	1.9
31	83.5	0.0	1.4	5.2	0.0	0.0	0.0	3.3	4.9	2.7
32	80.3	0.0	2.5	0.0	0.0	0.0	0.0	14.8	1.9	.5
33	80.3	0.0	2.5	0.0	0.0	0.0	0.0	14.5	1.9	.8
34	74.5	.3	1.9	0.0	0.0	0.0	0.0	7.4	8.5	7.4
35	47.7	0.0	0.0	0.0	0.0	0.0	0.0	6.0	29.6	16.7
36	86.0	0.0	0.0	0.0	0.0	0.0	0.0	2.2	8.5	3.3
37	87.7	.3	0.0	0.0	0.0	0.0	0.0	2.2	6.3	3.6
38	5.5	57.3	1.6	.5	[illegible]	7.9	1.9	1.4	14.0	9.9
39	81.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.2	7.1
40	83.8	.3	0.0	0.0	0.0	0.0	.5	3.6	7.1	4.7
41	16.2	58.9	0.0	0.0	0.0	0.0	4.4	1.6	11.2	7.7
42	79.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.8	9.0
43	34,2	0.0	3.6	0.0	0.0	.5	2.7	1.1	25.8	32.1
44	53.4	0.0	.3	0.0	0.0	4.1	6.3	11.5	3.8	20.5
45	38.6	0.0	3.0	0.0	0.0	0.0	7.1	13.2	16.4	21.6
46	34.2	0.0	9.0	0.0	0.0	0.0	0.0	1.4	32.9	22.5
47	19.7	42.2	0.0	0.0	0.0	0.0	0.0	3.0	16.2	18.9
48	80.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.3	16.7
49	64.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	9.9	25.5
50	70.4	0.0	0.0	0.0	0.0	0.0	0.0	6.0	5.8	17.8
51	78.4	0.0	0.0	0.0	0.0	0.0	0.0	.3	3.6	17.8
52	71.5	0.0	0.0	0.0	0.0	0.0	0.0	7.1	3.6	17.8
53	58.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.5	24.4

PHI CORRELATION MATRIX. MATRICES M N=535

Facility	Var.	15	4	2	11	12	17	20	1	5	13	6	3	14	16	19	21	18	25	26	37	24	30	36	22	10	27	29	40	23	39
98.3	15																														
98.1	4	83																													
97.6	2	-29	67																												
97.6	11	262	67	53																											
97.6	12	262	67	53	442																										
97.0	17	640	157	53	369	369																									
96.3	20	-25	45	32	96	96	32																								
95.7	1	115	136	325	86	86	86	103																							
95.7	5	-27	38	-33	-33	-33	-33	-41	45																						
95.7	13	187	38	86	604	604	205	55	136	0																					
94.6	6	-31	27	15	15	15	15	-47	71	722	30																				
94.4	3	-31	86	541	119	172	67	268	268	28	148	85																			
93.1	14	78	179	148	196	196	148	62	123	-21	160	0	157																		
92.5	16	424	118	1	139	139	500	56	44	44	149	57	54	118																	
92.5	19	18	118	139	185	232	47	93	79	79	149	151	208	118	0																
92.1	21	69	11	89	89	134	89	89	75	-27	143	22	110	194	207	128															
91.3	18	300	141	71	153	133	480	1	85	23	140	60	84	208	818	2	209														
88.6	25	99	80	172	172	210	57	84	126	10	242	43	193	203	143	166	332	140													
86.9	26	164	110	82	193	190	154	128	109	27	163	53	146	134	184	184	195	153	558												
86.7	37	77	68	224	81	117	81	97	243	-1	134	76	287	219	119	161	193	113	275	174											
86.5	24	203	107	115	222	257	151	95	159	-29	240	2	141	194	179	179	251	147	306	464	184										
85.8	30	113	62	40	213	213	144	145	124	-7	230	68	226	163	148	230	179	137	376	540	298	498									
85.6	36	111	22	142	73	104	108	87	201	18	123	42	200	182	126	187	216	117	255	172	655	165	244								
84.7	22	-15	56	161	202	202	67	162	114	12	267	12	212	190	76	155	165	123	190	142	215	197	228	239							
83.2	10	96	11	58	221	253	91	69	175	77	249	90	194	172	100	176	203	156	263	225	192	217	217	256	127						
83.2	27	96	85	123	156	188	123	69	151	27	224	113	151	172	119	309	203	156	498	477	221	247	289	242	183	225					
81.9	29	89	78	114	240	240	114	137	91	19	282	58	222	215	124	216	421	107	594	306	330	326	489	304	324	229	346				
81.9	40	202	78	177	240	272	146	86	187	67	259	80	264	235	124	272	169	124	273	249	316	226	267	346	257	255	229	320			
81.7	23	88	41	113	239	267	176	110	209	66	256	57	178	156	85	85	95	138	210	145	242	223	250	260	630	174	161	316	266		
79.8	39	151	33	132	223	192	132	97	200	54	214	85	201	211	104	264	130	102	273	177	324	156	288	324	303	184	209	355	693	315	
79.4	31	77	100	69	129	100	99	94	165	28	188	102	97	134	119	294	178	145	312	296	291	219	428	265	181	278	550	360	360	189	251
78.7	48	144	-38	125	184	184	95	41	92	-20	249	16	190	163	181	112	187	152	330	231	240	169	285	241	158	168	180	276	430	178	454
78.3	32	107	94	152	152	152	93	87	89	0	201	34	128	142	57	160	99	87	196	212	262	164	188	236	267	163	224	246	258	255	277
78.1	33	166	93	151	151	151	92	86	88	0	199	33	126	140	55	159	114	85	194	210	272	162	186	246	251	173	221	243	267	263	274
77.4	51	137	-41	146	146	175	117	58	149	39	215	67	178	169	166	118	207	199	354	253	275	166	279	249	154	210	210	279	452	206	451
77.3	42	135	88	115	173	144	173	56	256	59	234	104	175	236	199	182	121	195	293	275	296	292	324	294	248	193	253	319	457	246	433
74.0	34	88	75	72	17	-10	127	40	42	84	0	121	40	224	139	74	128	150	176	187	170	91	137	157	79	75	143	130	196	72	169
73.3	41	150	72	124	233	206	176	125	205	80	267	116	201	218	165	245	184	129	275	216	373	170	298	366	317	225	247	395	669	336	653
72.0	9	80	67	95	117	117	63	30	72	-9	154	71	173	125	59	75	65	90	195	140	136	168	175	158	127	420	142	149	149	166	152
71.1	52	135	30	162	188	188	82	43	223	42	223	41	160	208	186	140	219	199	343	255	285	244	307	267	186	241	273	317	434	239	464
69.5	50	122	-1	159	185	185	86	62	220	39	220	38	174	187	197	120	214	207	350	236	279	226	289	260	214	212	255	342	447	253	476
64.9	49	147	-72	112	188	188	138	123	133	37	230	48	160	154	237	103	207	181	277	155	196	168	227	188	197	160	139	334	344	218	400
62.6	7	79	35	53	72	103	78	71	198	217	140	309	130	185	118	176	90	137	172	112	164	125	194	222	218	199	158	187	268	183	362
61.5	47	45	4	99	74	124	49	127	154	78	135	132	107	132	96	169	140	129	163	137	211	115	162	211	207	219	178	195	315	270	358
60.9	38	73	87	97	97	122	97	124	132	38	151	62	171	219	92	165	179	125	255	211	375	189	255	359	244	192	223	319	349	254	332
58.1	53	65	22	87	161	185	87	72	137	44	193	47	171	141	176	118	160	181	279	210	226	209	262	202	185	236	205	288	367	244	403
55.0	44	86	-13	101	149	149	101	79	141	11	178	65	138	152	142	156	126	134	218	161	265	193	255	270	292	155	175	315	344	231	377
52.3	28	127	34	43	141	165	92	88	148	-17	185	19	157	138	127	226	138	166	187	195	233	248	297	227	352	261	221	327	308	370	331
46.3	35	62	-11	72	96	96	72	82	158	103	158	138	127	148	48	91	143	49	224	191	217	187	214	239	142	184	194	307	268	155	314
43.9	45	79	2	32	82	82	82	83	138	26	120	48	87	152	164	149	186	144	191	142	202	172	197	211	269	181	171	273	342	266	361
39.3	43	15	26	77	101	77	52	57	132	19	132	40	129	128	126	184	163	160	228	118	212	159	162	209	235	228	197	268	328	271	394
37.8	46	71	-34	47	97	97	97	72	108	13	127	67	89	136	162	162	141	138	158	119	191	137	195	220	256	216	175	246	326	229	353
11.6	8	47	49	57	57	57	19	9	76	47	76	35	88	75	58	80	62	59	38	53	55	22	30	48	72	115	37	64	109	35	80

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PHI CORRELATION MATRIX, MATRICES M Contd

79.4	31																							
78.7	48	254																						
78.3	32	248	235																					
78.1	33	245	243	983																				
77.4	51	266	831	246	254																			
77.0	42	260	399	251	259	416																		
74.0	34	99	139	102	88	168	121																	
73.3	41	319	438	307	313	430	422	123																
72.0	9	186	112	216	213	120	163	76	158															
70.1	52	354	726	260	266	788	428	134	444	201														
69.5	50	276	766	252	258	777	420	117	471	165	906													
64.9	49	216	678	220	235	622	295	144	387	133	596	593												
62.6	7	209	220	193	189	237	229	185	301	179	280	252	208											
61.5	47	186	348	236	231	407	297	179	303	181	380	394	342	341										
60.9	38	284	294	303	317	318	345	145	382	199	347	335	277	228	206									
58.1	53	261	585	261	275	618	319	119	412	178	728	714	676	229	309	353								
55.0	44	245	336	234	229	327	308	174	378	179	401	404	340	263	341	283	358							
52.3	28	375	280	260	246	262	252	100	362	220	292	303	253	283	275	355	305	316						
46.0	35	237	278	230	234	274	281	238	319	125	307	292	278	255	244	269	296	239	287					
40.9	45	216	303	271	274	313	301	181	373	147	394	369	294	328	322	331	336	593	360	307				
39.3	43	266	315	200	203	315	311	188	381	194	349	365	278	304	313	306	317	373	368	302	397			
37.8	46	195	301	241	244	319	278	127	357	168	390	373	306	322	338	283	340	612	341	279	849	376		
11.6	8	68	45	48	50	14	114	41	99	82	70	23	70	195	70	26	23	57	99	76	54	-15	55	
		31	48	32	33	51	42	34	41	9	52	50	49	7	47	38	53	44	28	35	45	43	46	8

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CSMS

VECTORS 1

WITH
VARIABLE
NUMBERS

THE VARIABLE NUMBER OF EACH ITEM IS CIRCLED

NAME .... TODAY'S DATE ....

BOY or GIRL .... CLASS ....

SCHOOL .... DATE OF BIRTH ....
day month year

---

1. To make a standard radio we need 3 valves, 2 transistors, 4 batteries and 1 switch. This may be written as the vector $\begin{pmatrix} 3 \\ 2 \\ 4 \\ 1 \end{pmatrix}$.

What we need for a de-luxe radio may be written as $\begin{pmatrix} 0 \\ 7 \\ 4 \\ 2 \end{pmatrix}$.

What parts do we need for a de-luxe radio? .... ①

....

What vector would represent the total parts needed to make 1 standard radio and 1 de-luxe radio? $\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$ ②

How many de-luxe radios can be made from $\begin{pmatrix} 0 \\ 21 \\ 12 \\ 6 \end{pmatrix}$? .... ③

$3\begin{pmatrix} 3 \\ 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 12 \\ 3 \end{pmatrix}$. What could we build from this? .... ④

Work out $2\begin{pmatrix} 0 \\ 7 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$ ⑤ . What could we build from this? .... ⑥

Which vector would represent the parts needed altogether for 3 standard radios and 2 de-luxe radios? $\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$ ⑦

2. Work out the following, putting in your own brackets:

(i) $\begin{pmatrix}4\\1\\3\end{pmatrix} + \begin{pmatrix}1\\0\\2\end{pmatrix} =$ ⑧

(ii) $\begin{pmatrix}2\\3\\7\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix} =$ ⑨

(iii) $\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}-1\\-4\end{pmatrix} =$ ⑩

(iv) $\begin{pmatrix}-2\\-4\end{pmatrix} + \begin{pmatrix}-1\\5\end{pmatrix} =$ ⑪

(v) $2\begin{pmatrix}7\\5\\4\end{pmatrix} =$ ⑫

(vi) Find what the letters stand for:

$4\begin{pmatrix}4\\b\end{pmatrix} = \begin{pmatrix}a\\8\end{pmatrix}$

a = ... ⑬

b = ... ⑭

---

3. Write the following as a single vector: $6\begin{pmatrix}1\\1\end{pmatrix} + 2\begin{pmatrix}0\\2\end{pmatrix} =$ ⑮

The vector $\begin{pmatrix}5\\3\end{pmatrix} = 5\begin{pmatrix}1\\0\end{pmatrix} + 3\begin{pmatrix}0\\1\end{pmatrix}$.

So $\begin{pmatrix}4\\7\end{pmatrix} = \ldots\ldots\begin{pmatrix}1\\0\end{pmatrix} + \ldots\ldots\begin{pmatrix}0\\1\end{pmatrix}$ ⑯

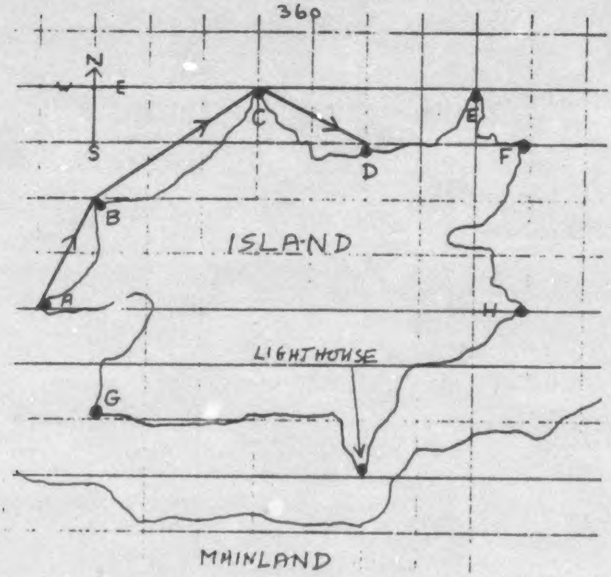
Write $\begin{pmatrix}2\\9\end{pmatrix}$ in the same way, using $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$: $\begin{pmatrix}2\\9\end{pmatrix} =$ ⑰

The vector $\begin{pmatrix}2\\4\end{pmatrix} = 2\begin{pmatrix}1\\1\end{pmatrix} + 1\begin{pmatrix}0\\2\end{pmatrix}$.

Now fill in the missing numbers:

$\begin{pmatrix}3\\7\end{pmatrix} = \ldots\ldots\begin{pmatrix}1\\1\end{pmatrix} + \ldots\ldots\begin{pmatrix}0\\2\end{pmatrix}$ ⑱

4.



The move from A to B can be described as 1 East and 2 North, or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ as a vector.

The move from B to C can be described as 3 East and 2 North, or $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ as a vector.

From C to D the move is described by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Why do we write -1 ?

.... ⑲

Describe the following moves using vectors: D to E is ⑳ $\begin{pmatrix} \\ \end{pmatrix}$. E to F is ㉑ $\begin{pmatrix} \\ \end{pmatrix}$. F to H is $\begin{pmatrix} \\ \end{pmatrix}$. ㉒

Add these three vectors together: ㉓ $\begin{pmatrix} \\ \end{pmatrix}$. What does this tell you?

.... ㉔

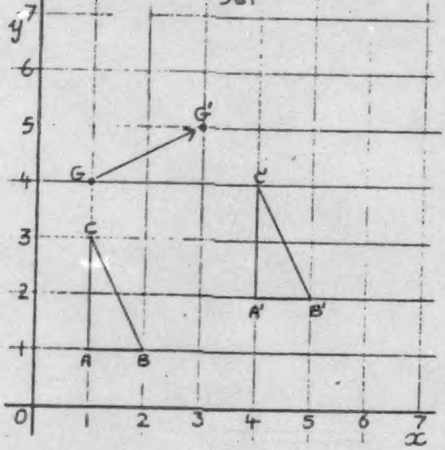
What is the vector for the overland route from G to H? $\begin{pmatrix} \\ \end{pmatrix}$ ㉕

Write down in vectors a sea route from G to H, via the lighthouse: ㉖

How can you check the vectors you have given for the sea route? ㉗

....

....



5. The point G has been translated (or moved) to G' by the vector $\begin{pmatrix}2\\1\end{pmatrix}$. Check that this is true.

The triangle ABC has been translated to A'B'C' by using the vector $\begin{pmatrix}3\\1\end{pmatrix}$. Check that A, B and C have all moved by the vector $\begin{pmatrix}3\\1\end{pmatrix}$.

If we translate the triangle A'B'C' using the vector $\begin{pmatrix}1\\2\end{pmatrix}$ we will get A"B"C". Draw A"B"C" on the diagram. ㉘

What are the coordinates of B" ? .... ㉙

Which vector would translate ABC straight to A"B"C" ? $\begin{pmatrix} \\ \end{pmatrix}$ ㉚

How did you work this out? .... ㉛

....

What would be the effect of a translation of $\begin{pmatrix}9\\8\end{pmatrix}$ followed by a translation of $\begin{pmatrix}-9\\-8\end{pmatrix}$?

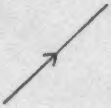
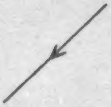
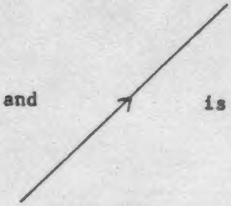
.... ㉜

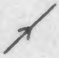
Write this as a vector: $\begin{pmatrix} \\ \end{pmatrix}$ ㉝

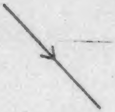
If we translate the line segment DE using the vector $\begin{pmatrix}11\\12\end{pmatrix}$, what would be the new coordinates of D (3,3) and E (4,7) ?

D would be at .... ㉞

E would be at .... ㉟

6. If  is a, then  is -a and  is 2a.

What is  ? .... ㊱

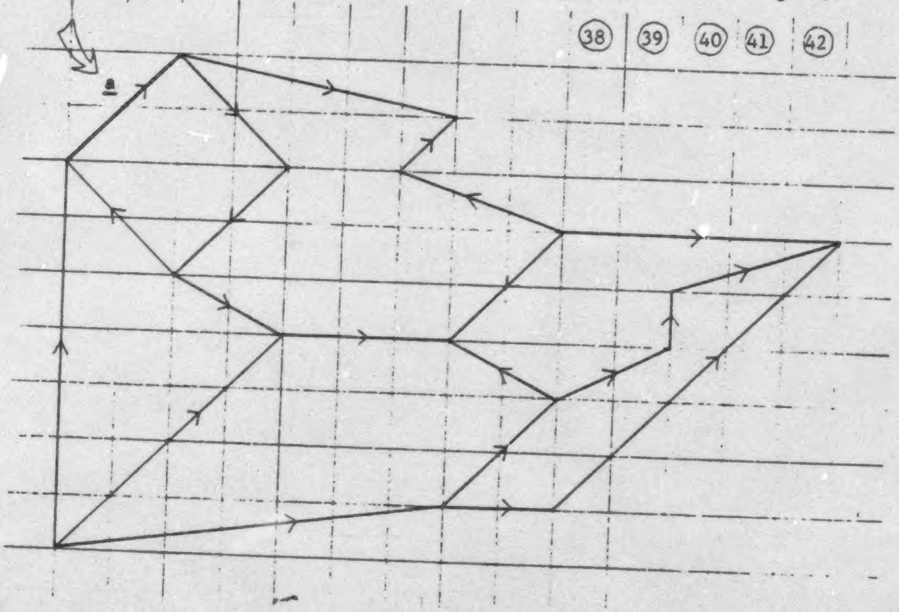
This  is not a. This vector is not related to a. Why not?

.... ㊲

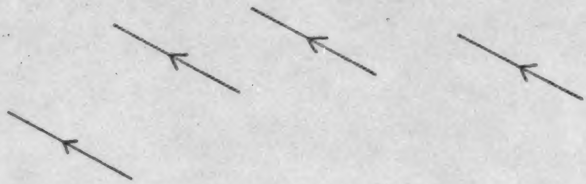
In the diagram find all the vectors you can that are the same as a (equivalent to a) or are related to a, 2a, -a, etc.

Label each vector you find with its "name" (a, -a, etc.).

One vector equivalent to a has been marked for you (top left of diagram).



7. These are the vector c:



In this box draw vector 2c

㊸

In this box draw vector -c

㊹

What can you say about the direction of 2c and the original vector c ?

.... ㊺

What can you say about the size of 2c and the original vector c ?

.... ㊻

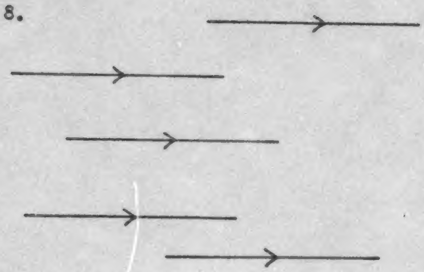
What can you say about the direction of -c and the original vector c ?

.... ㊼

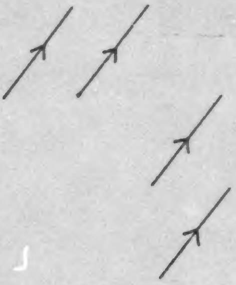
What can you say about the size of -c and the original vector c ?

.... ㊽

8.

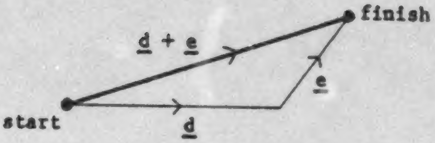


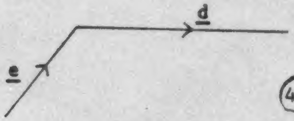
These are the vector $\underline{d}$.



These are the vector $\underline{e}$.

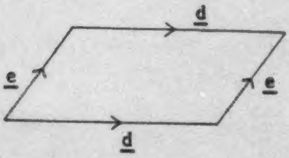
To add them we choose a vector $\underline{d}$ and a vector $\underline{e}$ so that they fit nose to tail. Then, to get the single vector $\underline{d} + \underline{e}$ we draw in the 3rd side of the triangle. Notice that the start and finish tell us the direction of the arrow on $\underline{d} + \underline{e}$.


finish
$\underline{d} + \underline{e}$
$\underline{e}$
start
$\underline{d}$


$\underline{d}$
$\underline{e}$

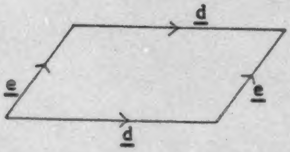
(49)

Draw in the single vector $\underline{e} + \underline{d}$. (Don't forget the arrow.)


$\underline{d}$
$\underline{e}$
$\underline{e}$
$\underline{d}$

(50)

On this diagram draw in the single vector $\underline{e} + \underline{d}$.


$\underline{d}$
$\underline{e}$
$\underline{e}$
$\underline{d}$

(51)

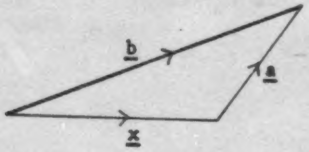
On this diagram draw in the single vector $\underline{d} + \underline{e}$.

Compare the last two diagrams. What does this tell you? ....

.... (52)

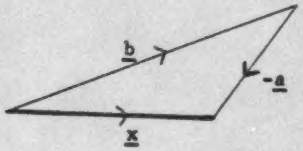
....

9. In this diagram $\underline{x} + \underline{a} = \underline{b}$.

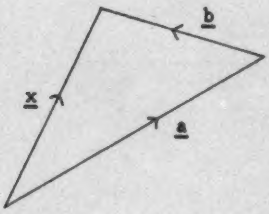


If we wish to write down what $\underline{x}$ is equal to, we can reverse the direction of $\underline{a}$, and then the diagram gives us

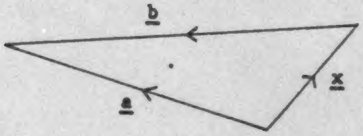
$$\underline{x} = \underline{b} - \underline{a}.$$



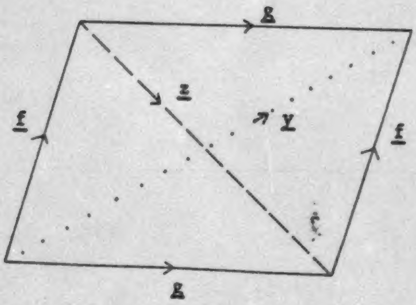
For each of the following write down $\underline{x}$ in terms of $\underline{a}$ and $\underline{b}$.



$\underline{x}$ = ... (53)



$\underline{x}$ = ... (54)

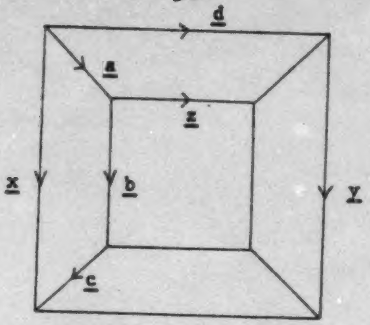


Write the vectors $\underline{y}$ and $\underline{z}$ in terms of $\underline{f}$ and $\underline{g}$. ($\underline{y}$ is the dotted vector and $\underline{z}$ is the dashed vector.)

$\underline{y}$ = ... (55)

$\underline{z}$ = ... (56)

10.



$\underline{d}$
$\underline{a}$
$\underline{z}$
$\underline{x}$
$\underline{b}$
$\underline{y}$
$\underline{c}$

Find $\underline{x}$ in terms of $\underline{a}$, $\underline{b}$, and $\underline{c}$. $\underline{x}$ = ... (57)

Mark any vector equivalent to $\underline{a}$ with an $\underline{a}$ and an arrow. (58)

Mark any vector equivalent to $\underline{b}$ with a $\underline{b}$ and an arrow. (59)

Mark any vector equivalent to $\underline{c}$ with a $\underline{c}$ and an arrow. (60)

Find $\underline{y}$ in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$.
You must use all the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$. $\underline{y}$ = ... (61)

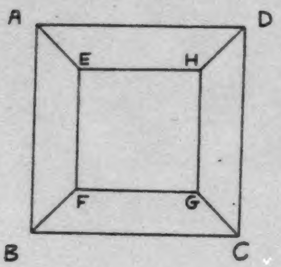
Find $\underline{y}$ in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$,
but not using all of $\underline{a}$, $\underline{b}$ and $\underline{c}$. $\underline{y}$ = ... (62)

Which vector, or vectors, are equivalent to $\underline{x}$? .... (63)

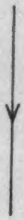
Why? .... (64)

Find $\underline{z}$ in terms of $\underline{a}$, $\underline{c}$ and $\underline{d}$. $\underline{z}$ = ... (65)

11.



Sam the spider is sitting at A and sees a fly at B. He reaches the fly by walking along AE, EF and FB. In vectors we can describe his journey by $\overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FB}$. His journey can be represented by so in vectors $\overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FB} = \overrightarrow{AB}$.



Which single vector would represent the journey from B to A ? .... (66)

Find a journey of 3 vectors that is equivalent to $\overrightarrow{AG}$: ..... + ... + ... (67)

Find one vector equivalent to the route $\overrightarrow{HD} + \overrightarrow{DC} + \overrightarrow{CG}$: .... (68)

Which single vector is the same as $\overrightarrow{AB}$? .... (69)

Find one vector equivalent to $\overrightarrow{AE} + \overrightarrow{HG}$: .... (70)

On a different diagram
which vector do you think is equivalent to the route $\overrightarrow{SJ} + \overrightarrow{JT} + \overrightarrow{TP}$? .... (71)

(72)

Complete the following: $\overrightarrow{SJ}$ + ... = $\overrightarrow{SZ}$. $\overrightarrow{LS}$ + ... + $\overrightarrow{TP}$ = $\overrightarrow{LP}$. (73)

12. A fly is sitting on Alan's nose, which has a position vector of $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Alan tries to swat it and the fly makes the following movements: $\begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$, then $\begin{pmatrix} -3 \\ 0 \\ -1 \end{pmatrix}$, then $\begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}$.

Where does the fly finish its journey? .... (74)

Alan again tries to swat the fly, and the fly lands on the corner of a table, position vector $\begin{pmatrix} 5 \\ 8 \\ 3 \end{pmatrix}$. If the fly now wishes to land on a cake, position vector $\begin{pmatrix} 7 \\ 9 \\ 3 \end{pmatrix}$, what move must it make? $\begin{pmatrix} \\ \\ \\ \end{pmatrix}$ (75)

When it has landed on the cake the fly wishes to go back to the corner of the table. What move must it make? $\begin{pmatrix} \\ \\ \\ \end{pmatrix}$ (76)

Now look through the test again and check your answers and try any questions that you have left out.

Vectors Coding Scheme

Var.	QUESTION	CODE 1	OTHER CODES
1	$\begin{pmatrix}0\\7\\4\\2\end{pmatrix}$ MEANS	No Valves, 7 Transistors 4 Batteries & 2 switches	
2	1 ST.&DE-L	$\begin{pmatrix}3\\9\\8\\3\end{pmatrix}$	1 number wrong : $\boxed{C\ 3}$
3	FROM $\begin{pmatrix}0\\21\\12\\6\end{pmatrix}$	3	
4	FROM $\begin{pmatrix}9\\6\\12\\3\end{pmatrix}$	3 STANDARD RADIOS	
5	$2\begin{pmatrix}0\\7\\4\\2\end{pmatrix} =$	$\begin{pmatrix}0\\14\\8\\4\end{pmatrix}$,	1 number wrong : $\boxed{C\ 3}$
6	$\begin{pmatrix}0\\14\\8\\4\end{pmatrix}$	2 DE-LUXE RADIOS	
7	FOR 3 ST. & 2 DE-L	$\begin{pmatrix}9\\20\\20\\7\end{pmatrix}$	1 number wrong : $\boxed{C3}$

Vectors contd

Var	QUESTION	CODE 1	OTHER CODES
8	$\begin{pmatrix}4\\1\\3\end{pmatrix}+\begin{pmatrix}1\\0\\2\end{pmatrix}=$	$\begin{pmatrix}5\\1\\5\end{pmatrix}$	
9	$\begin{pmatrix}2\\3\\7\end{pmatrix}-\begin{pmatrix}2\\2\\6\end{pmatrix}$	$\begin{pmatrix}0\\1\\1\end{pmatrix}$	
10	$\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}-1\\-4\end{pmatrix}=$	$\begin{pmatrix}1\\-1\end{pmatrix}$	
11	$\begin{pmatrix}-2\\-4\end{pmatrix}+\begin{pmatrix}-1\\5\end{pmatrix}=$	$\begin{pmatrix}-3\\1\end{pmatrix}$	
12	$2\begin{pmatrix}7\\5\\4\end{pmatrix}$	$\begin{pmatrix}14\\10\\8\end{pmatrix}$	
13	$4\begin{pmatrix}4\\b\end{pmatrix}=\begin{pmatrix}a\\8\end{pmatrix}$	a = 16	1 : $\boxed{C\ 4}$ 4 : $\boxed{C\ 8}$
14		b = 2	32 : $\boxed{C\ 4}$ 8 : $\boxed{C\ 8}$
15	$6\begin{pmatrix}1\\1\end{pmatrix}\div 2\begin{pmatrix}0\\2\end{pmatrix}=$	$\begin{pmatrix}6\\10\end{pmatrix}$	$\begin{pmatrix}8\\10\end{pmatrix}$: $\boxed{C\ 3}$ $\begin{pmatrix}1\\5\end{pmatrix}$ or $\begin{pmatrix}6\\8\end{pmatrix}$: $\boxed{C\ 4}$ $8\begin{pmatrix}1\\3\end{pmatrix}$ or $\begin{pmatrix}x\\24\end{pmatrix}$ or $\begin{pmatrix}1\\3\end{pmatrix}$: $\boxed{C\ 8}$ $\begin{pmatrix}10\\6\end{pmatrix}$: $\boxed{C\ 6}$
16	$\begin{pmatrix}4\\7\end{pmatrix}=\cdot\cdot\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}$	4 , 7	
17	$\begin{pmatrix}2\\9\end{pmatrix}=$	$2\begin{pmatrix}1\\0\end{pmatrix}+9\begin{pmatrix}0\\1\end{pmatrix}$	
18	[illegible]	3 , 2	One number correct : $\boxed{C\ 4}$ (e.g. 3 , 7 or 4 , 2)

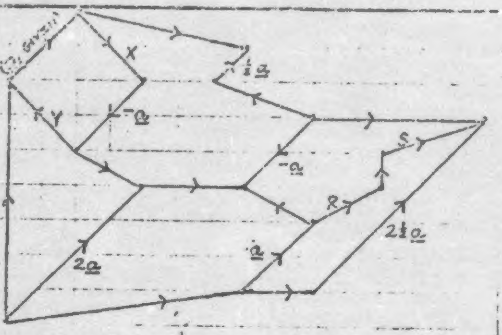
Var.	QUESTION	CODE 1	OTHER CODES
19	Why -1	1 SOUTH , 1 DOWN SOUTH or DOWN	
20	D to E	$\begin{pmatrix}2\\1\end{pmatrix}$	$\begin{pmatrix}-2\\-1\end{pmatrix}$: $\boxed{C\ 5}$ $\begin{pmatrix}1\\2\end{pmatrix}$: $\boxed{C\ 6}$
21	E to F	$\begin{pmatrix}1\\-1\end{pmatrix}$	$\begin{pmatrix}-1\\1\end{pmatrix}^{*}$: $\boxed{C\ 5}$ $\begin{pmatrix}-1\\1\end{pmatrix}^{*}$: $\boxed{C\ 6}$ $\begin{pmatrix}1\\1\end{pmatrix}$: $\boxed{C\ 8}$ * Code ⑤ and Code ⑥ answers the same lock at answers to ⑳ and ㉒ to see which error made
22	F to H	$\begin{pmatrix}0\\-3\end{pmatrix}$	$\begin{pmatrix}0\\3\end{pmatrix}$: $\boxed{C\ 5}$ $\begin{pmatrix}-3\\0\end{pmatrix}$: $\boxed{C\ 6}$ $\begin{pmatrix}3\\0\end{pmatrix}$: $\boxed{C\ 8}$
23	Add	$\begin{pmatrix}3\\-3\end{pmatrix}$	$\begin{pmatrix}-3\\3\end{pmatrix}^{*}$: $\boxed{C\ 5}$ $\begin{pmatrix}-3\\3\end{pmatrix}^{*}$: $\boxed{C\ 6}$
24	What	D to H (any mention of) Can only be correct if ㊸ is $\boxed{C1}$.	Just 3 East 3 South : $\boxed{C\ 4}$ H to D : $\boxed{C\ 5}$
25	Overland Route	$\begin{pmatrix}8\\2\end{pmatrix}$	$\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}3\\0\end{pmatrix}$ etc : $\boxed{C\ 3}$ $\begin{pmatrix}-8\\-2\end{pmatrix}$: $\boxed{C\ 5}$ $\begin{pmatrix}2\\8\end{pmatrix}$: $\boxed{C\ 6}$ ANY TRIPLE : $\boxed{C\ 8}$
26	Sea Route	$\begin{pmatrix}5\\-1\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\left[\text{or } \begin{pmatrix}0\\-1\end{pmatrix}+\begin{pmatrix}5\\0\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\right]$	Route $\equiv \begin{pmatrix}8\\2\end{pmatrix}$ but partly over land : $\boxed{C\ 3}$ Route $\equiv \begin{pmatrix}-8\\-2\end{pmatrix}$: $\boxed{C\ 5}$ Route $\equiv \begin{pmatrix}2\\8\end{pmatrix}$: $\boxed{C\ 6}$ ANY TRIPLE : $\boxed{C\ 8}$
27	Check	Add Vectors = overland route	Just "Add Vectors" : $\boxed{C\ 4}$

Vectors contd

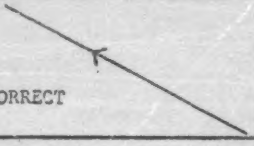
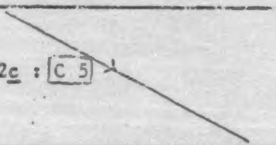
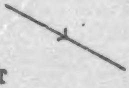
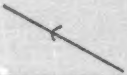
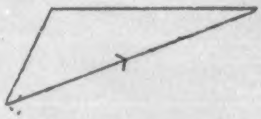
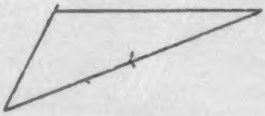
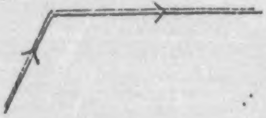
Var.	QUESTION	CODE 1	OTHER CODES
28	DRAW	Drawn correctly: (B" at (6,4))	
29	Co-ORDS	(6, 4)	(4,6) : $\boxed{C\ 6}$ (7,3) : $\boxed{C\ 8}$
30	MOVE	$\begin{pmatrix}4\\3\end{pmatrix}$	$\begin{pmatrix}3\\4\end{pmatrix}$: $\boxed{C\ 6}$
31	HOW	Count Squares	Add Vectors : $\boxed{C2}$
32	EFFECT	NO MOVE OR BACK TO START	"at $\begin{pmatrix}0\\0\end{pmatrix}$" or "at 0" : $\boxed{C3}$
33	WRITE AS VECTOR	$\begin{pmatrix}0\\0\end{pmatrix}$	
34	D at	(14, 15)	1 Co-ord correct : $\boxed{C\ 4}$ (15,14)*: $\boxed{C\ 6}$ (15,14)*: $\boxed{C\ 8}$
35	E at	(15, 19)	1 co-ord correct : $\boxed{C\ 4}$ (19,15) : $\boxed{C\ 6}$ (16,18) : $\boxed{C\ 8}$

* CODE $\boxed{6}$ and CODE $\boxed{8}$ answers the same, see answer to ㉟ to see which error made.

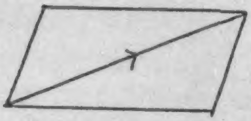
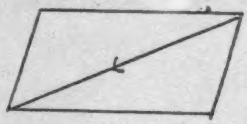
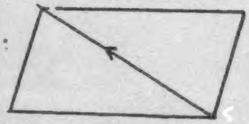
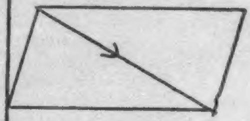
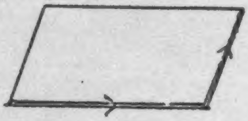
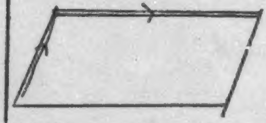
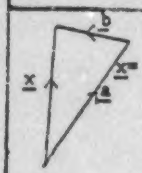
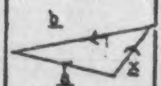
Vectors contd

Var.	QUESTION	CODE 1	OTHER CODES
36		$\frac{1}{2}\underline{a}$ or $\frac{\underline{a}}{2}$	 $\underline{a}$ given X $-\frac{1}{2}\underline{a}$ Y $-\underline{a}$ $-\underline{a}$ S R $2\frac{1}{2}\underline{a}$ $\underline{a}$ $2\underline{a}$
37	Why Not?	Different Direction	
38	MARK 6 VECTORS ON MAZE	5 CORRECTLY MARKED	6 CORRECT : $\boxed{C\ 2}$ 4 CORRECT : $\boxed{C\ 3}$ 3 CORRECT : $\boxed{C\ 4}$ 2 or 1 CORRECT : $\boxed{C\ 8}$ 0 CORRECT : $\boxed{C\ 9}$
39	ERRORS IN MARKING 6 POTENTIAL CORRECT VECTORS	NO ERROR MADE	VECTOR MARKED WITH CORRECT SIGN BUT WRONG LENGTH (e.g. $2\frac{1}{2}\underline{a}$ as $2\underline{a}$) : $\boxed{C\ 4}$ VECTOR MARKED WITH WRONG SIGN BUT CORRECT LENGTH (e.g. $-\underline{a}$ as $\underline{a}$) : $\boxed{C\ 5}$ BOTH MISTAKES MADE : $\boxed{C\ 8}$ (On same vector, $2\frac{1}{2}\underline{a}$ as $-2\underline{a}$ or separately e.g. $-\underline{a}$ as $\underline{a}$ $2\frac{1}{2}\underline{a}$ as $2\underline{a}$)
40	ERROR MADE R or S or both	No mark on R or S	MARK ON R OR S OR BOTH : $\boxed{C\ 8}$
41	ERROR MADE X or Y or both	No mark on X or Y	MARK ON X OR Y OR BOTH : $\boxed{C\ 8}$
42	MARKS ON REST	NO MARK ON ANY OF REST	ONLY 1 OF REST MARKED : $\boxed{C\ 4}$ MORE THAN 1 OF REST MARKED : $\boxed{C\ 8}$

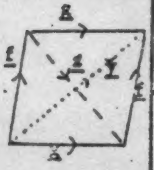
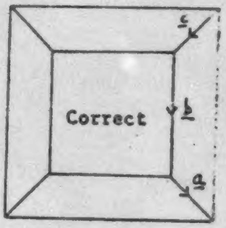
VECTORS contd

Var	QUESTION	CODE 1	OTHER CODES
43	Draw $2\underline{c}$	 CORRECT	DRAW $-2\underline{c}$: $\boxed{\text{C 5}}$ 
44	Draw $-\underline{c}$	 CORRECT	DRAW $\underline{C}$: $\boxed{\text{C 5}}$ 
45	Direction $2\underline{c}$ and $\underline{c}$	SAME	
46	Size $2\underline{c}$ amd $\underline{c}$	DOUBLE	"Longer" , "Bigger" etc. : $\boxed{\text{C9}}$.
47	Direction $-\underline{c}$ and $\underline{c}$	OPPO^ITE or "$\underline{\text{the}}$ other way"	"Different" : $\boxed{\text{C9}}$.
48	Size $-\underline{c}$ and $\underline{c}$	SAME	
49	$\underline{c} + \underline{d}$		 : $\boxed{\text{C 5}}$  : $\boxed{\text{C 8}}$

Vectors contd

Var			
50	$\underline{e} + \underline{d}$		 , : $\boxed{C\ 5}$
51	$\underline{d} + \underline{e}$		 OR : $\boxed{C\ 7}$   OR : $\boxed{C\ 8}$ 
52	What	$\underline{d} + \underline{e} = \underline{e} + \underline{d}$ or commutative answer (NOT 'the same') can only be correct if 15 and 16 are C1	
53		$\underline{a} + \underline{b}$ or $\underline{b} + \underline{a}$	$-\underline{b} - \underline{a}$ or $-\underline{a} - \underline{b}$: $\boxed{C\ 5}$ $-\underline{b} + \underline{a}$ or $-\underline{a} + \underline{b}$ or $\underline{a} - \underline{b}$ or $\underline{b} - \underline{a}$: $\boxed{C\ 7}$
54	 $\underline{x} =$	$\underline{a} - \underline{b}$, or $-\underline{b} + \underline{a}$	$\underline{b} - \underline{a}$ or $-\underline{a} + \underline{b}$: $\boxed{C\ 5}$ $\underline{a} + \underline{b}$ or $\underline{b} + \underline{a}$ or $-\underline{a} - \underline{b}$ or $-\underline{b} - \underline{a}$: $\boxed{C\ 7}$

Vectors contd.

Var				
55 $\underline{y}$ = 	$\underline{f} + \underline{g}$ or $\underline{g} + \underline{f}$	$-\underline{g} - \underline{f}$ or $-\underline{f} - \underline{g}$: $\boxed{C\ 5}$ $\underline{g} - \underline{f}$ or $-\underline{f} + \underline{g}$ or $\underline{f} - \underline{g}$ or $-\underline{g} + \underline{f}$: $\boxed{C\ 7}$	
56 $\underline{z}$ =	$\underline{g} - \underline{f}$ or $-\underline{f} + \underline{g}$	$\underline{f} - \underline{g}$ or $-\underline{g} + \underline{f}$: $\boxed{C\ 5}$ $\underline{f} + \underline{g}$ or $\underline{g} + \underline{f}$ or $-\underline{f} - \underline{g}$ or $-\underline{g} - \underline{f}$: $\boxed{C\ 7}$	
$\underline{x}$ = 57	$\underline{a} + \underline{b} + \underline{c}$	$2\underline{b}$ or $2\underline{a} + 2\underline{c}$: $\boxed{C\ 2}$ $-\underline{a} - \underline{b} + \underline{c}$: $\boxed{C\ 5}$ Correct Vectors 1 sign wrong : $\boxed{C\ 7}$ Correct Vectors 2 signs wrong : $\boxed{C\ 8}$	
Mark $\underline{a}$ 58	Correct Marking	$\underline{a}$ marked, arrow wrong way : $\boxed{C\ 5}$ $\underline{a}$ marked as $\underline{c}$: $\boxed{C\ 8}$	
Mark $\underline{b}$ 59	Correct marking	$\underline{b}$ marked, arrow wrong way : $\boxed{C\ 5}$	
Mark $\underline{c}$ 60	Correct marking	$\underline{c}$ marked, arrow wrong way : $\boxed{C\ 5}$ $\underline{c}$ marked as $\underline{a}$: $\boxed{C\ 8}$	
$\underline{y}$ = 61	$\underline{c} + \underline{b} + \underline{a}$ (or $\underline{a} + \underline{b} + \underline{c}$)	$-\underline{c} - \underline{b} - \underline{a}$: $\boxed{C\ 5}$* Correct Vectors 1 sign wrong : $\boxed{C\ 7}$ Correct Vectors 2 signs wrong : $\boxed{C\ 8}$	
$\underline{y}$ = 62	$2\underline{b}$	$2\underline{a} + 2\underline{c}$ or $2(\underline{a} + \underline{c})$: $\boxed{C\ 2}$** $-2\underline{b}$ or $-2\underline{a} - 2\underline{c}$: $\boxed{C\ 5}$ $\underline{b}$ or $\underline{a} + \underline{c}$ or $\underline{a} + \underline{b} - \underline{a}$: $\boxed{C\ 8}$	
$\underline{x}$ equivalen 63	$\underline{y}$	$2\underline{b}$ or $2\underline{a} + 2\underline{c}$: $\boxed{C\ 2}$ $\underline{y}$ and $\underline{d}$ or $\underline{y}$ and $\underline{b}$: $\boxed{C\ 4}$ $-\underline{y}$: $\boxed{C\ 5}$ $\underline{d}$ or $\underline{b}$: $\boxed{C\ 8}$	

Var			
64	Why?	Definitive (same length and direction)	Commutative argument $\underline{a} + \underline{b} + \underline{c} = \underline{c} + \underline{b} + \underline{a}$ etc : $\boxed{C\ 2}$ just length or direction : $\boxed{C\ 4}$
65	$\underline{z}$ "	$-\underline{a} + \underline{d} + \underline{c}$	$\frac{1}{2}\underline{d}$ or $\underline{a} - \underline{c}$: $\boxed{C\ 2}$ $\underline{a} - \underline{d} - \underline{c}$: $\boxed{C\ 5}$ Correct vectors 1 sign wrong : $\boxed{C\ 7}$ Correct vectors 2 signs wrong, : $\boxed{C\ 8}$
	** If $\underline{a} + \underline{b} + \underline{c}$ or $\underline{c} + \underline{b} + \underline{a}$ shown here put $\boxed{C1}$ in ㊻.		* If $2\underline{b}$ or $2\underline{a} + 2\underline{c}$ shown here put $\boxed{C1}$ or $\boxed{C2}$ in ㊼.
66	B to A	$\overrightarrow{BA}$ or $\overrightarrow{CD}$ or $\overleftarrow{AB}$	$-\overrightarrow{AB}$: $\boxed{C\ 3}$ $\overleftarrow{BA}$: $\boxed{C\ 5}$
67	$\overrightarrow{AG}$	$\overrightarrow{AX} + \overrightarrow{XY} + \overrightarrow{YG}$ $X, Y \in \{A,B,C,D,E,F,G,H\}$	
68	$\overrightarrow{HD} + \overrightarrow{DC} + \overrightarrow{CG}$	$\overrightarrow{HG}$ (or $\overrightarrow{EF}$)	
69	Same as $\overrightarrow{AB}$	$\overrightarrow{DC}$ or $-\overrightarrow{BA}$	$\overrightarrow{CD}$: $\boxed{C\ 5}$ $\overrightarrow{AB}$ and $\overrightarrow{BA}$ variants other than $-\overrightarrow{BA}$: $\boxed{C\ 8}$
70	$\overrightarrow{AE} + \overrightarrow{HG}$	$\overrightarrow{AF}$	$\overrightarrow{HC}$: $\boxed{C\ 2}$ $\overrightarrow{EH}$: $\boxed{C4}$. $\overrightarrow{AG}$: $\boxed{C\ 8}$
71	$\overrightarrow{SJ} + \overrightarrow{JT} + \overrightarrow{TP}$	$\overrightarrow{SP}$	
72	$\overrightarrow{SJ} + \ldots = \overrightarrow{SZ}$	$\overrightarrow{JZ}$	$\overrightarrow{JS}$: $\boxed{C\ 8}$
73	$\overrightarrow{LS} + \ldots + \overrightarrow{TP} = \overrightarrow{LP}$	$\overrightarrow{ST}$	

Vectors contd.

QUESTION	CODE 1	OTHER CODES
Where 74	Back on Nose	at $\begin{pmatrix}0\\0\\0\end{pmatrix}$: $\boxed{C\ 3}$
Move 75	$\begin{pmatrix}2\\1\\0\end{pmatrix}$	One number wrong : $\boxed{C4}$. $\begin{pmatrix}-2\\-1\\0\end{pmatrix}$: $\boxed{C\ 5}$ $\begin{pmatrix}12\\17\\6\end{pmatrix}$: $\boxed{C\ 8}$
Move back 76	$\begin{pmatrix}-2\\-1\\0\end{pmatrix}$	One number wrong : $\boxed{C\ 4}$ $\begin{pmatrix}2\\1\\0\end{pmatrix}$: $\boxed{C\ 5}$ $\begin{pmatrix}0\\1\\2\end{pmatrix}$ or $\begin{pmatrix}0\\-1\\-2\end{pmatrix}$: $\boxed{C\ 6}$ $\begin{pmatrix}3\\7\\3\end{pmatrix}$ or $\begin{pmatrix}5\\8\\3\end{pmatrix}$ or $\begin{pmatrix}7\\9\\3\end{pmatrix}$: $\boxed{C\ 8}$.

TEST 3 YEAR 3 NUMBER OF CHILDREN = 1140 VECTORS 1976

VAR	1	2	3	4	5	6	7	8	9	0
1	89.1	0	0	0	0	0	0	0	9.7	1.1
2	88.0	0	1.6	0	0	0	.1	0	7.1	3.2
3	91.1	0	0	0	0	0	0	0	6.5	2.0
4	75.6	0	.1	0	0	0	0	0	17.6	6.7
5	81.2	0	2.1	0	0	0	0	0	5.4	11.3
6	76.3	0	0	0	0	0	0	0	14.8	8.9
7	63.9	0	10.6	0	0	0	0	0	19.7	5.7
8	93.6	0	0	0	0	0	0	0	5.8	.6
9	85.2	0	0	0	0	0	0	0	13.4	1.4
10	55.2	0	0	0	0	0	0	0	42.5	2.4
11	44.1	0	0	0	0	0	0	0	52.0	3.9
12	88.0	0	0	0	0	0	0	0	6.4	5.6
13	71.3	0	0	3.5	0	0	0	5.7	11.2	8.2
14	76.1	0	0	3.9	0	0	0	5.2	7.0	7.8
15	63.8	0	2.3	2.5	0	0	0	4.0	21.4	6.0
16	66.8	0	0	0	0	0	0	0	13.6	19.6
17	55.4	0	0	0	0	0	0	0	19.0	25.6
18	44.6	0	0	23.3	0	0	0	0	11.3	20.7
19	78.6	0	0	.1	0	0	0	0	9.9	11.4
20	75.8	0	0	0	0	5.6	0	0	14.8	3.8
21	63.0	0	0	0	1.7	4.3	0	12.3	14.6	4.2
22	53.9	0	0	0	5.5	6.4	0	4.5	25.0	4.7
23	40.7	0	0	0	.1	.4	0	0	52.5	6.4
24	22.4	0	0	4.9	.3	0	0	0	33.3	39.1
25	45.1	0	1.3	0	1.6	5.9	0	6.8	28.6	10.6
26	22.0	0	4.9	0	1.1	3.2	0	13.1	22.7	33.1
27	17.9	0	0	5.4	0	0	0	.1	27.3	49.4
28	49.9	0	0	0	0	0	0	0	26.8	23.3
29	45.2	0	0	0	0	1.2	0	3.5	37.5	12.6
30	44.0	.1	0	0	.1	.3	0	0	38.6	16.9
31	43.2	14.5	0	0	0	0	.1	0	16.5	25.7
32	49.4	.1	3.0	0	0	0	0	0	18.3	29.2
33	47.4	0	0	0	0	0	0	0	17.8	34.8
34	31.9	0	0	7.1	0	.1	0	.2	22.2	38.5
35	32.5	0	0	5.6	0	.1	0	.2	22.5	39.1
36	63.0	0	0	0	0	0	0	0	30.1	6.9
37	83.1	.1	0	0	0	0	0	0	7.3	9.6
38	23.4	15.4	22.5	11.3	0	0	0	10.7	2.8	13.6
39	28.7	.1	0	42.6	3.2	0	0	11.2	.3	13.9
40	51.1	0	0	0	0	0	0	34.9	.1	13.9
41	68.2	0	0	.1	0	0	0	17.8	.1	13.9
42	55.2	0	0	10.9	0	0	0	19.9	.2	13.9
43	82.5	0	0	0	.4	0	0	0	11.1	6.1
44	67.3	0	0	0	2.9	0	0	0	22.7	7.1
45	64.2	0	0	0	0	0	0	0	22.8	13.0
46	70.4	0	0	0	0	0	0	0	18.5	11.1
47	64.6	0	0	0	0	0	0	0	22.0	13.3
48	68.4	0	0	0	0	0	0	0	16.5	15.1
49	65.0	0	0	0	16.2	0	.1	5.0	3.6	10.1
50	51.1	0	0	0	12.7	0	5.4	2.5	8.5	19.7
51	39.3	0	.1	0	18.1	0	10.6	2.4	8.9	20.6
52	23.0	0	0	0	0	0	0	0	45.6	31.4
53	31.1	0	0	0	1.1	0	42.2	.1	8.4	17.2
54	16.9	0	0	0	31.8	0	23.1	0	9.2	18.9
55	35.8	0	0	0	.4	0	10.7	0	15.6	36.5
56	14.4	0	0	0	13.2	0	19.5	0	16.0	37.0
57	34.1	.1	0	0	1.2	0	7.7	6.0	18.0	32.9
58	31.5	0	0	0	3.1	0	0	11.9	9.4	44.1
59	42.7	0	0	0	2.9	0	0	.2	6.7	45.5

Vectors Year 3 1976 contd

60	31.1				3.1			12.1	8.2	45.6
61	28.3	0	0	0	1.1	0	7.4	5.6	13.7	43.9
62	3.6	1.3	0	.1	0	0	0	4.0	35.8	55.2
63	28.2	.4	0	11.6	0	0	.1	2.0	11.6	46.1
64	13.8	1.4	0	19.2	0	0	0	0	11.2	54.4
65	8.7	1.1	0	0	1.0	0	8.6	6.1	20.6	53.9
66	58.5	0	.1	0	13.2	0	0	0	9.7	18.5
67	69.4	0	0	0	0	0	0	0	11.2	19.4
68	64.6	0	0	0	.1	0	0	0	13.4	21.9
69	24.8	0	0	.1	1.5	0	0	33.2	12.5	28.0
70	2.5	.1	0	5.2	0	0	0	28.9	28.3	35.0
71	50.4	0	0	0	0	0	0	.1	10.8	38.7
72	29.3	0	0	0	0	0	0	9.4	14.4	46.9
73	36.3	0	0	0	0	0	0	0	15.1	48.6
74	49.6	0	6.5	0	0	0	.1	0	17.0	26.8
75	51.1	0	0	4.2	.8	0	0	5.5	10.4	27.9
76	32.1	0	0	2.9	5.1	1.5	0	15.3	14.9	28.2

TEST 3 YEAR 4 NUMBER OF CHILDREN = 950 VECTORS 76

VAR	1	2	3	4	5	6	7	8	9	0
1	88.7	0	0	0	0	0	0	0	9.8	1.5
2	87.9	0	2.0	.1	0	0	0	0	6.0	4.0
3	93.3	0	.1	0	0	0	0	0	3.3	3.4
4	79.9	0	0	0	0	0	0	0	12.6	7.5
5	83.2	0	1.6	0	0	0	0	0	3.5	11.8
6	80.7	0	0	0	0	0	0	0	10.2	9.1
7	71.3	0	9.3	0	0	0	0	0	12.9	6.5
8	92.1	0	0	0	0	0	0	0	5.8	2.1
9	85.2	0	0	0	0	0	0	0	12.2	2.6
10	70.8	.1	0	0	0	0	0	0	25.1	4.0
11	61.4	0	0	0	0	0	0	0	34.2	4.4
12	90.7	0	0	0	0	0	0	0	3.4	5.9
13	79.1	0	0	1.5	0	0	0	2.5	6.4	8.5
14	80.4	0	0	2.6	0	0	0	3.6	4.8	8.5
15	72.9	0	2.3	2.8	0	.1	0	2.5	10.5	8.7
16	71.3	0	0	0	0	0	0	0	8.4	20.3
17	63.3	0	0	0	0	0	0	0	12.3	24.4
18	55.2	0	0	15.6	0	0	0	0	8.8	20.4
19	78.7	0	0	0	0	0	0	.1	8.8	12.3
20	77.5	0	0	0	0	3.7	0	0	13.1	5.8
21	68.3	0	0	0	1.8	3.9	0	7.9	11.6	6.5
22	59.1	0	0	0	4.1	5.5	0	4.4	20.0	6.9
23	49.2	0	0	0	0	0	0	0	41.2	9.7
24	24.4	0	0	8.0	.2	0	0	0	29.9	37.5
25	50.9	0	1.3	0	1.7	3.4	0	6.6	23.4	12.7
26	33.3	0	3.7	.1	.8	3.5	0	10.4	19.8	28.4
27	24.4	0	.1	7.2	0	0	0	.1	23.5	44.7
28	52.9	0	0	0	0	0	.1	.1	22.6	24.2
29	45.4	0	0	0	0	1.8	0	3.8	28.4	20.6
30	49.4	.1	0	.1	0	.5	0	0	29.3	20.6
31	44.1	17.6	0	0	0	0	0	0	10.3	28.0
32	54.4	0	1.2	0	0	0	0	0	13.9	30.5
33	54.1	0	0	0	0	0	0	0	10.9	34.9
34	32.6	0	0	4.6	0	.3	0	.2	20.0	42.0
35	32.2	0	0	4.4	0	.1	0	.1	20.5	42.6
36	70.0	0	0	0	0	0	.1	0	19.7	10.2
37	82.1	.1	.1	0	0	0	0	0	6.5	11.2
38	21.4	25.8	19.5	8.4	0	0	0	6.7	2.1	16.1
39	43.3	0	0	27.6	4.4	0	0	8.4	.2	16.1
40	62.8	0	0	.1	0	0	0	20.8	.1	16.1
41	71.5	0	0	0	0	0	0	12.4	0	16.1
42	65.1	0	0	7.1	0	0	0	11.7	.1	16.1
43	81.9	0	0	0	.2	0	0	0	7.3	10.6
44	68.6	0	0	0	2.2	0	0	0	18.0	11.2
45	68.0	0	0	0	0	0	0	0	13.6	18.4
46	72.9	0	0	0	0	0	0	0	11.3	15.8
47	68.2	0	0	0	0	0	0	0	12.6	19.2
48	70.6	0	0	0	0	0	0	0	10.4	18.9
49	67.3	0	0	0	12.4	0	0	2.8	2.1	15.4
50	55.2	0	0	0	12.4	0	4.8	2.1	6.2	19.3
51	41.8	0	0	0	19.9	0	8.5	1.7	6.8	21.3
52	27.5	0	0	0	0	0	0	0	39.2	33.4
53	38.5	0	0	0	.7	0	36.8	0	2.9	20.9
54	25.7	0	.1	0	27.6	0	19.6	0	3.7	23.4
55	45.4	0	0	0	.5	0	10.5	0	7.1	36.5
56	21.3	0	0	0	13.3	0	20.4	0	7.1	38.0
57	45.3	.2	0	0	2.0	0	8.3	3.4	8.3	32.5
58	46.9	0	0	0	2.2	0	0	11.4	4.0	35.5
59	57.8	0	0	0	1.8	0	0	.1	4.2	36.1
60	46.4	0	0	0	2.2	0	0	11.8	3.5	36.1

61	40.0	0	0	0	3.5	0	6.9	3.3	6.8	39.5
62	3.8	1.7	0	0	.1	0	0	4.9	34.8	54.6
63	41.4	.6	0	12.2	0	0	0	1.2	4.6	40.0
64	25.7	2.2	0	17.5	0	0	0	0	7.8	46.8
65	16.1	.4	0	0	2.0	0	9.6	7.6	13.5	50.8
66	57.4	0	.7	0	11.8	0	0	.1	6.7	23.3
67	68.9	0	0	0	0	0	0	0	8.3	22.7
68	65.5	0	0	0	0	0	0	0	9.4	25.2
69	44.0	0	0	0	.9	0	0	19.8	6.9	28.3
70	5.9	.3	0	4.7	0	0	0	26.5	25.4	37.2
71	57.1	0	0	0	0	0	0	.1	6.5	36.3
72	43.5	0	0	0	0	0	0	5.4	10.2	40.9
73	47.7	0	0	0	0	0	0	0	10.1	42.2
74	55.8	0	5.4	0	0	0	0	0	13.2	25.7
75	56.8	0	0	6.0	1.2	.1	0	4.2	6.9	24.7
76	39.7	0	0	3.9	4.0	1.2	0	13.2	12.0	26.1

[illegible] CORRELATION MATRIX, N OF CASES = 2210

PHI CORRELATION MATRIX VECTORS 1976 N=2210

		8	3	12	1	2	9	37	43	5	19	6	14	4	20	13	46	41	48	67	16	44	15	7	47	36	45	49	68	21	10
93.2	8																														
92.3	3	[illegible]																													
89.5	12	364	178																												
89.3	1	190	346	[illegible]																											
88.5	2	245	449	363	408																										
85.2	9	326	[illegible]	[illegible]	[illegible]	[illegible]																									
83.8	37	168	262	325	217	[illegible]	[illegible]																								
82.7	43	184	296	[illegible]	[illegible]	[illegible]	[illegible]	433																							
82.7	5	221	343	472	440	371	[illegible]	391	[illegible]																						
79.2	19	179	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]																					
79.0	6	221	434	491	[illegible]	419	[illegible]	336	357	590	433																				
78.4	14	[illegible]	[illegible]	457	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	422	[illegible]	482																			
78.2	4	226	447	478	321	410	[illegible]	[illegible]	322	490	[illegible]	770	439																		
77.2	20	194	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	474	417	[illegible]	[illegible]																	
75.1	13	217	246	[illegible]	214	304	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	432	631	418	[illegible]																
72.5	46	166	[illegible]	349	[illegible]	[illegible]	[illegible]	[illegible]	614	[illegible]	437	430	401	423	429	349															
72.5	41	147	226	252	[illegible]	[illegible]	[illegible]	412	412	[illegible]	302	[illegible]	311	302	348	283	431														
71.4	48	177	270	351	[illegible]	[illegible]	[illegible]	[illegible]	526	[illegible]	432	406	[illegible]	341	419	369	608	627													
70.4	67	158	273	277	210	263	[illegible]	307	[illegible]	323	341	[illegible]	321	350	355	297	468	394	435												
69.9	16	197	303	[illegible]	[illegible]	323	171	[illegible]	[illegible]	424	403	464	418	425	398	425	433	369	455	380											
68.8	44	142	223	286	209	271	131	336	528	333	[illegible]	[illegible]	346	330	387	323	521	399	481	388	394										
68.6	15	198	305	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	374	[illegible]	378	337	371	325	249	311	266	392	283									
67.7	7	[illegible]	348	393	[illegible]	[illegible]	172	[illegible]	[illegible]	402	342	464	[illegible]	438	344	383	361	278	350	335	382	319	309								
66.9	47	168	266	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	489	[illegible]	[illegible]	415	[illegible]	385	307	322	554	350	603	384	404	501	302	318							
66.8	36	186	236	349	[illegible]	[illegible]	[illegible]	370	411	349	373	423	426	347	396	376	485	388	499	381	425	449	321	371	431						
66.7	45	166	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	514	[illegible]	[illegible]	419	[illegible]	403	[illegible]	346	561	433	556	451	417	485	305	341	529	449					
66.7	49	139	221	257	[illegible]	[illegible]	[illegible]	313	418	393	[illegible]	[illegible]	301	310	[illegible]	333	466	395	479	394	403	410	257	291	405	366	468				
66.[illegible]	68	150	269	288	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	402	407	[illegible]	[illegible]	[illegible]	344	485	417	459	751	406	392	308	362	393	419	465	429			
65.9	21	164	276	334	[illegible]	333	159	276	360	349	523	413	374	401	539	307	423	357	436	307	406	370	339	354	377	421	411	342	411		
63.6	10	227	247	325	135	[illegible]	172	[illegible]	293	[illegible]	323	[illegible]	322	328	332	353	357	273	355	297	370	301	317	329	303	390	360	293	310	380	
60.9	42	157	220	256	212	242	112	349	350	274	355	332	289	283	353	246	397	694	409	395	349	381	201	312	358	418	399	373	403	376	308
60.7	31	138	278	324	224	273	131	266	357	319	407	423	396	418	436	345	463	403	424	392	443	397	320	378	410	445	419	368	404	461	345
60.1	17	167	281	332	230	315	150	367	375	393	403	436	404	406	418	431	455	387	455	370	688	396	386	380	395	442	440	383	422	446	409
59.1	66	143	229	224	[illegible]	[illegible]	174	[illegible]	320	246	306	[illegible]	[illegible]	284	322	251	360	335	306	548	325	339	205	281	305	337	378	335	539	320	255
57.9	40	114	183	248	[illegible]	[illegible]	[illegible]	296	310	243	295	294	238	261	320	233	355	543	345	331	309	334	257	268	317	398	357	328	337	341	317
56.9	22	141	256	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	332	[illegible]	439	384	354	370	515	363	420	344	415	337	405	382	365	360	392	447	416	335	390	687	393
54.8	71	161	243	244	[illegible]	244	123	285	352	338	369	341	321	335	371	352	454	407	436	631	431	377	319	335	390	412	454	423	662	400	350
54.[illegible]	75	163	245	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	302	282	263	353	307	331	439	341	282	236	273	305	318	354	314	460	299	282
53.6	53	118	190	212	[illegible]	[illegible]	119	230	356	240	303	274	274	279	317	269	398	346	301	337	332	351	229	253	354	318	387	624	364	314	264
53.1	11	[illegible]	235	[illegible]	[illegible]	[illegible]	[illegible]	175	[illegible]	[illegible]	270	[illegible]	204	200	[illegible]	289	280	248	263	253	304	260	242	257	265	339	303	272	259	322	554
53.1	74	122	258	244	156	[illegible]	[illegible]	249	300	235	278	275	253	258	273	255	345	300	346	398	294	293	239	228	321	305	326	316	405	288	260
53.[illegible]	32	143	241	267	[illegible]	[illegible]	[illegible]	[illegible]	317	[illegible]	300	303	[illegible]	347	347	334	437	370	417	385	413	339	310	328	387	440	415	367	405	449	402
52.2	28	128	223	268	188	[illegible]	115	249	300	303	361	346	343	343	406	344	428	356	408	349	393	379	310	371	374	402	377	379	407	428	315
51.6	33	161	243	252	174	[illegible]	139	274	373	269	332	348	317	328	373	311	418	371	388	376	410	328	291	277	365	424	408	373	394	409	386
51.[illegible]	59	115	206	242	[illegible]	266	145	269	351	300	342	347	320	322	304	332	437	402	448	450	419	394	266	308	379	389	437	398	486	373	349
50.9	18	135	246	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	327	358	365	351	384	385	345	381	363	349	359	516	340	391	364	346	399	391	356	394	424	388
48.2	25	139	193	238	[illegible]	205	[illegible]	154	249	231	331	293	286	286	397	287	314	271	315	307	313	273	279	298	293	317	330	286	327	411	297
47.6	3[illegible]	132	213	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	311	366	[illegible]	305	364	403	365	415	373	401	352	421	365	309	382	378	432	401	378	409	459	351
46.3	29	122	213	249	[illegible]	[illegible]	[illegible]	244	229	[illegible]	359	341	337	339	388	324	417	342	410	342	390	369	291	348	369	397	396	369	386	419	324
45.3	23	145	[illegible]	248	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	307	[illegible]	[illegible]	335	479	345	388	329	391	347	384	342	334	342	361	444	403	336	395	636	428
43.5	38	122	188	249	176	[illegible]	125	258	355	274	310	327	314	324	337	244	442	441	435	307	305	414	270	307	392	593	419	368	387	400	347
42.7	73	124	211	222	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	309	310	309	314	314	391	344	389	515	388	334	259	322	344	367	411	376	520	373	345
41.8	55	125	149	262	147	147	[illegible]	298	275	242	255	300	275	276	254	250	340	282	345	339	314	297	218	246	313	325	345	340	362	299	294
40.5	51	117	151	173	[illegible]	176	[illegible]	[illegible]	[illegible]	[illegible]	220	[illegible]	230	233	251	242	345	277	340	299	326	343	188	257	339	320	339	378	316	263	240
40.5	57	110	183	185	151	147	109	214	292	269	250	274	246	250	229	251	324	271	334	326	308	285	235	247	296	300	323	342	362	282	268
40.1	58	122	167	[illegible]	154	210	177	[illegible]	[illegible]	[illegible]	[illegible]	304	244	282	301	312	367	395	373	389	375	341	259	282	349	372	393	366	419	366	333
39.6	6[illegible]	131	198	[illegible]	[illegible]	[illegible]	[illegible]	260	399	252	295	300	292	286	387	314	382	383	383	397	375	349	253	296	348	374	407	365	428	371	345
36.9	72	125	199	221	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	301	295	313	310	376	334	356	457	371	321	279	316	334	364	395	361	491	366	335
36.6	76	146	159	[illegible]	165	[illegible]	[illegible]	[illegible]	276	[illegible]	273	[illegible]	[illegible]	247	271	294	339	322	345	384	338	305	261	269	302	344	339	315	406	324	311
36.2	39	118	157	[illegible]	141	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	241	245	274	241	347	331	344	301	284	312	195	262	293	431	332	300	309	338	260

FACILITY VAR.

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36.1  63  [illegible] 337 326 338 334 328 304 252 250 313 321 362 296 361 325 323
35.2  53  [illegible] 194 161 198 204 167 177 127 134 191 165 181 203 210 132 156
35.5  61  [illegible] 221 215 205 233 294 273 297 291 307 244 219 231 268 270 285 303 332 259 256
35.5  69  [illegible] 174 230 226 234 313 247 195 163 161 194 213 222 214 303 226 219
33.7  35  [illegible] 307 330 293 348 341 341 341 353 325 266 300 308 376 367 329 384 385 344
33.7  34  [illegible] 305 337 294 361 351 340 340 300 327 253 314 309 384 379 324 385 399 340
27.6  26  [illegible] 255 314 261 298 281 294 265 321 271 250 283 279 312 311 270 314 375 299
25.3  52  [illegible] 217 237 257 229 300 271 300 252 293 278 199 228 290 289 303 385 264 282 252
24.3  24  [illegible] 271 235 271 297 255 289 262 279 266 302 254 249 251 259 318 295 263 297 380 310
22.   64  [illegible] 217 209 236 235 283 269 294 273 277 268 216 210 290 307 301 250 296 287 266
21.4  54  [illegible] 155 157 157 173 171 206 177 191 161 152 125 170 169 200 200 200 140 150
21.3  27  [illegible] 237 201 258 280 253 269 252 290 226 242 243 254 296 294 259 277 310 296
18.5  56  [illegible] 164 148 175 174 238 226 232 245 241 202 149 194 235 227 247 242 268 231 215
13.6  65  [illegible] 177 [illegible] 169 228 204 243 234 214 210 183 184 217 246 249 227 248 229 251
 5.5  62  [illegible] 97 89 107 107 117 125 139 122 111 111 108 107 123 145 141 116 135 115 121
 4.8  70  [illegible] 61 67 101 90 128 135 127 127 124 132 106 100 112 135 131 136 129 130 139
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Facility Var.

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          [illegible] 6 14 4 20 13 46 41 48 67 16 44 15 7 47 36 45 49 68 21 10

FAC   ITEM
36.1  63
35.2  53  219
35.5  61  452 [illegible]
35.   69  338 [illegible] [illegible]
33.7  35  377 189 439 [illegible]
33.7  34  374 158 [illegible] [illegible] [illegible]
27.6  26  320 135 [illegible] [illegible] 301 384
25.3  52  331 191 [illegible] [illegible] [illegible] [illegible] [illegible]
24.3  24  332 176 311 [illegible] 412 415 362 332
22.   64  [illegible] [illegible] [illegible] [illegible] [illegible] 340 310 316 336
21.4  54  241 341 254 [illegible] [illegible] 187 200 217 210 266
21.3  27  332 176 272 [illegible] [illegible] [illegible] [illegible] 345 419 365 199
18.5  56  282 198 312 229 315 306 306 317 316 306 315 296
13.6  65  361 243 366 233 327 314 321 363 325 393 293 319 384
 5.5  62  226 137 229 [illegible] 183 190 187 246 246 220 192 242 226 291
 4.8  70  241 [illegible] [illegible] 249 211 [illegible] 245 248 252 274 233 254 296 305 335

          63  53  61  69  35  34  26  52  24  64  54  27  56  65  62  70
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MMI CORRELATION MATRIX, N OF CASES = 2210
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VECTORS 1976 contd

384

PHI CORRELATION MATRIX VECTORS 1976 contd

60.9	42																														
60.7	31	362																													
60.1	17	371	472																												
59.1	66	317	355	331																											
57.9	40	722	328	334	373																										
56.[illegible]	22	363	450	463	[illegible]	[illegible]																									
54.8	71	424	445	465	476	350	411																								
54.6	75	298	317	352	354	353	[illegible]	451																							
53.6	50	331	344	345	361	269	317	360	204																						
53.1	11	273	240	349	229	261	322	318	234	262																					
53.1	74	363	317	306	324	284	[illegible]	440	443	255	246																				
53.[illegible]	32	359	531	447	328	351	463	458	315	352	342	367																			
52.2	26	344	626	421	322	319	431	402	276	330	269	289	451																		
51.6	33	349	476	437	317	423	427	461	349	338	366	397	665	389																	
51.[illegible]	59	382	427	461	359	326	369	497	388	369	301	366	399	398	427																
50.9	18	367	439	540	316	348	437	434	324	363	331	295	469	401	437	414															
48.2	25	[illegible]	332	334	257	295	415	322	237	239	252	267	337	328	335	313	368														
47.6	30	364	646	[illegible]	342	339	471	425	292	365	294	312	506	609	456	401	453	342													
46.3	29	336	568	437	317	313	421	398	287	333	264	311	466	621	384	376	397	303	777												
45.3	23	364	442	456	[illegible]	[illegible]	[illegible]	416	312	312	491	318	476	420	454	382	453	405	473	415											
43.5	38	455	419	416	318	412	396	404	301	354	325	304	430	390	403	412	374	314	443	408	430										
42.7	73	360	415	414	425	319	[illegible]	676	407	348	327	409	424	374	427	462	431	304	418	379	405	381									
41.8	55	315	336	334	317	303	333	460	315	336	230	331	342	313	358	426	362	270	331	323	334	328	413								
40.5	51	292	362	354	243	294	265	323	247	405	233	235	314	289	327	369	313	217	307	289	292	319	323	312							
40.5	57	286	332	335	272	286	307	395	306	314	253	309	334	289	348	438	331	248	319	264	320	301	366	507	292						
40.1	58	389	418	455	321	361	361	451	334	356	314	341	407	388	417	747	411	311	407	391	403	421	440	386	338	390					
39.6	60	381	413	415	334	357	366	452	351	347	310	347	408	387	419	750	411	318	407	399	406	427	444	399	335	390	950				
36.9	72	342	425	[illegible]	421	341	[illegible]	633	379	236	317	394	433	399	454	460	434	306	429	413	413	392	733	398	319	372	458	467			
36.6	76	339	371	382	329	313	334	453	679	306	270	420	425	335	412	400	359	279	309	348	365	369	438	325	300	338	361	379	444		
36.2	39	422	351	134	272	253	315	[illegible]	259	263	240	267	354	328	322	340	336	288	357	338	351	514	311	293	250	275	371	377	350	299	
36.1	63	349	349	351	293	327	329	416	342	304	292	311	333	310	348	528	349	266	349	328	352	353	438	375	285	439	506	513	434	368	297
35.2	53	183	151	168	175	187	[illegible]	[illegible]	148	209	131	159	126	145	174	224	170	135	144	149	143	164	205	349	172	354	201	209	215	149	164
35.[illegible]	61	273	297	327	253	267	348	365	285	297	272	287	332	260	333	443	337	240	305	286	303	283	375	496	288	692	401	408	381	328	264
35.[illegible]	65	236	256	256	235	[illegible]	[illegible]	[illegible]	200	207	170	248	238	186	247	364	230	190	215	201	220	270	258	251	211	267	346	354	299	256	225
33.7	35	336	470	414	328	337	421	423	294	310	313	321	495	425	462	363	439	338	400	468	445	408	427	373	335	331	402	413	424	380	299
33.7	34	355	475	423	345	349	422	429	293	306	363	316	504	441	475	372	435	329	493	476	453	417	426	364	326	320	403	415	441	392	318
27.6	26	293	360	379	239	303	414	356	259	252	280	270	370	331	359	349	393	467	393	339	450	327	371	313	292	290	374	379	353	339	328
25.3	52	357	304	342	213	273	313	327	255	538	255	239	344	307	352	358	347	243	328	322	312	334	336	337	699	300	364	372	354	319	297
24.3	24	292	371	364	246	268	445	361	255	277	241	255	404	347	386	318	391	295	406	364	565	355	379	330	275	307	345	352	392	333	317
22.[illegible]	64	[illegible]	320	314	242	307	303	362	263	200	248	252	317	259	305	426	314	264	311	289	336	334	375	339	259	343	444	451	394	321	337
21.4	54	188	156	232	145	211	169	239	159	168	177	177	174	157	190	192	200	159	182	186	195	168	242	265	172	264	222	230	264	179	191
21.3	27	276	345	344	[illegible]	273	[illegible]	[illegible]	251	235	271	275	366	308	379	332	362	372	364	346	382	322	360	312	287	293	369	368	344	341	296
18.5	56	238	259	[illegible]	267	243	259	331	237	277	223	270	294	255	263	296	272	230	295	269	299	258	338	377	256	297	340	340	342	295	261
13.6	65	234	246	245	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	221	[illegible]	281	250	292	349	294	229	279	284	298	268	342	332	260	368	412	412	352	300	299
5.5	62	141	154	192	124	100	[illegible]	157	136	105	139	151	166	167	170	205	173	131	174	172	166	167	192	204	187	212	218	221	205	190	193
4.8	79	153	141	135	[illegible]	[illegible]	[illegible]	[illegible]	145	162	151	151	[illegible]	[illegible]	175	144	173	160	159	165	182	200	200	187	185	189	226	229	227	203	201
Facility	Var.	42	31	17	66	40	22	71	75	50	11	74	32	26	33	59	18	25	30	29	23	38	73	55	51	57	58	60	72	76	39

APPENDIX TWO

Discrimination of Ratio Items

RATIO

CHILD NUMBER (RANKED BY TOTAL SCORE) (1)

Item	Facility	[illegible]
1a(ii)	94	11111111111111111111111110111111111111111111111111101111111111111
1a(i)	[illegible]	11111111111111111111111111111111111111111111111111101111111111111
3a(i)	87	11111111111111111111111111111101111111111111111111111111111111111
8 i	84	11111111111111111111111111111111111111111111111111111111111111111
1b(i)	77	11111111111111111111111110111111111111111111111111001111111111111
3a(ii)	75	11111111111111111111111111111101111010111111111111101111011111110
1b(ii)	70	11111111111111111111111110111111011111011111111111100111111111111
4a(v)	62	11111111111111111111110111111111110111011011111010111111111011010
2B	51	1100111101111011011111111111010110110100001010110010111
2A	48	110011110111110110111110111111011111110100011011110110011
2C	47	1110111110111101101111101111111010111111100011010110010011
3a(iii)	46	111111110100111111111101111110111111110001111111110111100
3a(iv)	44	111111101111111011111011111101101111111101111011001111101
3b(i)	43	11111110010111101011110111101011010011010011101001011110101
8 ii	42	11111001111001011011110011110101111000011111001110011001000
4a(iii)	31	11111111110100110000111100011100001011011110011000100000101
4a(iv)	30	10111111111010010010111110101100000101101111100100001001000
6 i	29	1011111100111101001111101110100001100100000000001100010
8 iii	29	111110101000010001011011111000111101001000011011101101101000
5	25	1111111111100001010100011100000110011010010000000100000000000
3b iii	19	111110001110101111101000000010100000010100100111100000000111
3b ii	18	11111100010101000111000000011010000001000000111101100100101
3b iv	18	111110001110001110101000000010100000010110100110100000000111
8 iv	17	11110011001010011111000000000100010010001010001100000001001
1b iii	14	1111111011010100100101000101010000100010101000000000000100
7 ii	9	11001101000110011110000000000000001001000010000100000000100000
6 ii	8	111110100000111000000001001000000000000000000001000000001000000
7 i	7	100100100010001101000000000000100010000000000000000000000000000

All numbers refer to items on final paper.

Question 4b on final paper not included

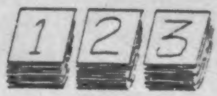
2 A.B.C marked separately

CHILD NUMBER (RANKED BY TOTAL SCORE) (2)

Item		
1a(ii)	11111111111111111111111111111111111111111111111111111111111111111111111111110111	111111111111101111111111111111111111111111101111100100001
1a(i)	111111111111111111111111111111111111111111111111111111111101111111101111	111111111111101111111111111111111111110010111001000000
3a(i)	11111111111011101111111111111111111111111111011101111111111111111111111	0101100111111111011011101111001111111000111001000
8(i)	1111011111011111110111101111110111110111101111111111111111111111111111	11111011111110110111011110110100000000001100110100
1b(i)	111111111111111111101010000111101111011111111011111111111101111111111110	00111111101010110101001011111101100111000000000
3a(ii)	1111011111110111011111011111111111110111111101110101010111110111111111	01011001101011101001110110100011111110000011001000
1b(ii)	1111111011111101111110111101111111111101101111111011111011000111100111	1101111000101000110101100100101000100000000000000
4a(v)	11111011111111011011010011010111111011110101001100011011011001110000	00100100010101000000001000000100100000010010011000
2B	110010110110000000000100110001001000110010001000000001001110000000000	1011001000000110000000000000000001001010100000010010
2A	110010110110111111101001100001001100110010000000100010111100000000100	1010001000000101000000000000000000000101000000000000
2C	1100101101101111111011011000000100011001001010101000100111000000001	10110010000000010000000000000000000000101000000000000
3a(iii)	011110001011010000100011011011000001000011100000000101000000001000000111	10000000000010000100010000000000000000000000000000000
3a(iv)	0100000010101101000100000110010000010000101000001100100000000000011010	0100000000000110000110010000000000000000000000000000
3b(i)	01110110111100010000011010111000000100010110000000100011010010100001	001011111000000001100000000000000000000000000000
8ii	1011000100011110110110010001100111001010110011010010100000010010010	1000101000100000010000000100000000000000000000000
4a(vi)	100001001001010101001110101001011101000100000000000110000000000100000	0000000000010000000000000000000000000000000000000000
4a(HG)	100000001000000000100111000100101100100010010000000001000100011000110001	000000101010000000000000000000000000000000000000000
6i	0000011110100010001000001110001000100100000011000110000010000010000000	000001000000000000000100000101000000100000000000000
8iii	10100000000101101000000100101000001000110000000001000000000000000100000	000000000000000001001000101000000000000000000000000
5	000101100100000100000100001000000001101100000010000000000000000000000	00001001000000000000000100000001000000101000000000
3biii	000000010000000000000101010011000000000000000100000000000000000000000000	00000000000000000000000000000000000000000000000000
3bii	011010000100000000000000000000000000000000000000000000000000100000000000	000000000000000000000001000000000000000000000000000
3biv	00100000100000000000000010101000100000000010000000000000000000000000000000	00000000000000000000000000000000000000000000000000
8iv	00110000000100100100000100000000000000000000010000000000000000000001000100	000000000100000000000000000000000000000000000000
1biii	0000010001000000000000000000000000000000000100000100000000000000000000000	000001000000000100000000000000000000000000000000
7ii	00000000000000000000000000000000000000000000000000000000000000000000000000	0000000000000000000000000000000000000000000000000
6ii	00000000000000000001000000000000000000000000000000000000000000000000000000	0000000000000000000000000000000000000000000000000
7i	00000000000000000000000000000000000000000000000000000000000000000000000000	0000000000000000000000000000000000100000000000000

APPENDIX THREE

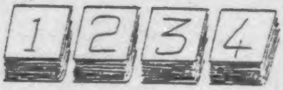
The Piaget Test



1a. We have THREE piles of cards.
One pile has a 1 printed on each card,
one pile has a 2 printed on each card, and
one pile has a 3 printed on each card.

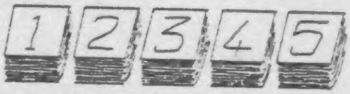
A card is taken from ANY of the piles, and
another card is taken from ANY of the piles;
these two cards are then arranged next to each other, like this:

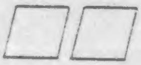
Find out HOW MANY different arrangements you could make: ......



1b. Now we have FOUR piles of cards.
Again, two cards are arranged next to each other, like this:

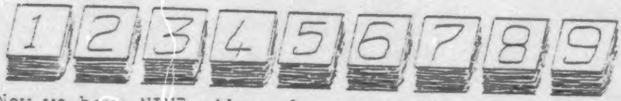
Find out HOW MANY different arrangements you could make: ......

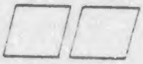




1c. Now we have FIVE piles of cards.
Again, two cards are arranged next to each other, like this:

HOW MANY different arrangements could you make? ......





1d. Now we have NINE piles of cards.
Again, two cards are arranged next to each other, like this:

HOW MANY different arrangements could you make? ......

Explain you answer: ....

....

2. Make an accurate copy of these diagrams. Draw your answers on the BACK PAGE of the test paper.

   Use a cm ruler only, and WRITE DOWN the length of everything you measure.

Diagram A→

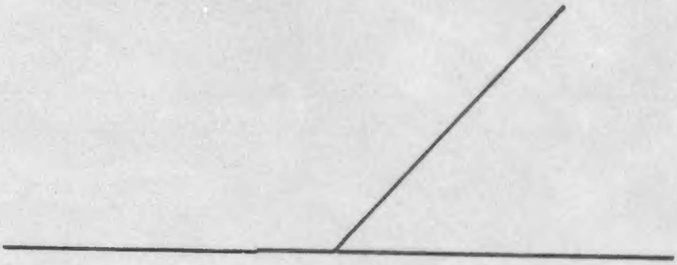


Diagram B→

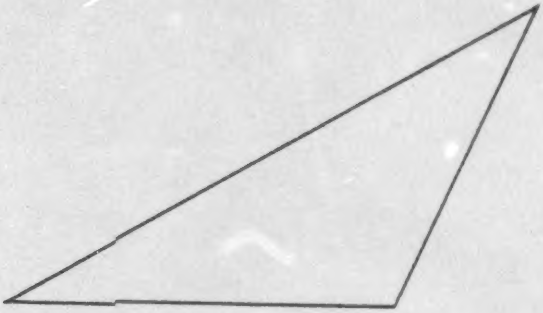
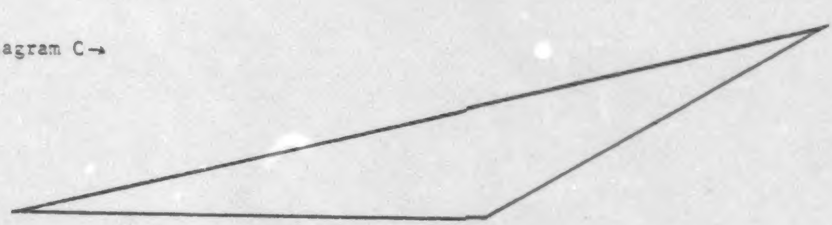
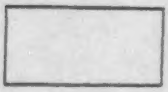


Diagram C→



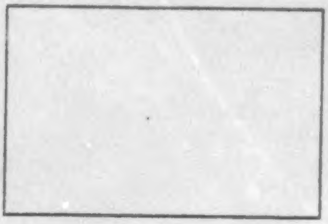
3a. Draw a rectangle exactly the same shape as this rectangle but make your rectangle larger.

Use a cm ruler, and WRITE DOWN the length of everything you measure.



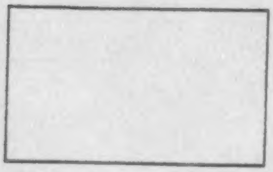
3b. Draw a rectangle exactly the same shape as this rectangle but make your rectangle larger. We have given you the longer side of your new version.

Use a cm ruler, and WRITE DOWN the length of everything you measure.



3c. Draw a rectangle exactly the same shape as this rectangle but make your rectangle larger. We have given you the longer side of your new version.

Use a cm ruler, and WRITE DOWN the length of everything you measure.



4a. 30 sweets are shared between two boys so that one boy has 6 more than the other.

How many sweets does each boy get? ....

Please remember
to write down
your working:

4b. 10 sweets are shared between two boys so that one boy has 4 more than the other.

How many sweets does each boy get? ....

4c. 10 sweets are shared between John and Peter so that John has 9 and Peter has 1.

How many more sweets does John have than Peter? .....

4d. 16 sweets are shared between two boys so that one boy has 6 more than the other.

How many sweets does each boy get? ....

4e. 29 sweets are shared between two boys so that one boy has 7 more than the other.

How many sweets does each boy get? ....

4f. 158 sweets are shared between two boys so that one boy has 42 more than the other.

How many sweets does each boy get? ....

4g. 753 sweets are shared between two boys so that one boy has 29 more than the other.

How many sweets does each boy get? ....

5a. Three coats, one red (R), one yellow (Y) and one blue (B) are hung next to each other on three hooks. (Each hook has one coat on it.)

In how many ways can the coats be arranged? ....

5b. Three girls, A, B, C are seated on three chairs in a row.

In how many ways can the girls be seated? ....

5c. Four girls, A, B, C, D are seated on four chairs in a row.

In how many ways can the girls be seated? ....

5d. Another girl, E, comes along, so there are now five girls seated on five chairs in a row.

In how many ways can the girls be seated? ....

Explain your answer: ....

....

Diagram A:

remember to
write down
the length
of everything
you measure.

Diagram B:

remember to
write down
the length
of everything
you measure.

Diagram C:

remember to
write down
the length
of everything
you measure.

[illegible]

APPENDIX FOUR

The Kolmogorov-Smirnov Test Applied to School Samples (1977)

MEAN IQ AND KOLMOGOROV PROBABILITIES OF SCHOOLS

SCHOOL No.	n	$\widetilde{IQ}$	p	15	25	24	27	28	09	18	06	20	16	11
15	88	108·81	<·01	╲										
25	118	105·51	<·01		╲									
24	141	102·45	<·05		103·8	╲								
27	199	101·56	<·05				╲							
28	197	101·19	>·20			101·71 =·05	101·38 <·10	╲						
09	76	101·12	>·20		103·8	102·0 =·01	101·43 >·20	101·2 >·20	╲					
18	190	101·03	>·20		102·7	101·6 =·05	101·30 <·15	101·1 >·20	101·7 ≫·20	╲				
06	204	100·34	>·20		102·2 <·01	101·2 <·20	100·94 >·20	100·8 >·20	100·5	100·7 >·20	╲			
20	128	99·69	>·20	103·4	102·5	101·1 >·20	100·83 >·20	100·6 >·20	100·2	100·5	100·1	╲		
16	292	98·84	>·20	101·14 >·20	100·8 >·20	100·0	99·94 >·20	99·8 >·20	99·3	99·7	99·5	99·1	╲	
11	135	98·22	<·15	102·4 <·15	101·6 >·20	100·4	100·21 >·20	100·0 >·20	99·3 >·20	99·9	99·5	98·9 >·20	98·6 =·20	
26	221	98·08	<·05	101·1 <·05	100·7	99·8	99·73 >·20	99·6 =·20	98·9 =·15	99·4	99·2	98·7 <·05	98·5 <·05	
12	161	97·76	<·10	101·7 =·20	101·0 >·20	99·9	99·86 >·20	99·7 >·20	98·8 >·20	99·5	99·2	98·6 >·20	98·5 <·05	
08	121	97·07	<·05	102·0 <·20	101·2	100·0 >·20	99·86 >·20	99·6 >·20	98·6 =·20	99·5	99·1	98·4 >·20	98·3	
13	100	97·00	<·05	102·5 =·01	101·6 <·05	100·2	100·0 >·20	99·8 >·20	98·9 <·15	99·6	99·2	98·5 >·20	98·4	
14	198	96·52	<·05	100·3 <·20	99·9	99·0	99·0 >·20	98·8 >·20	97·8	98·7 >·20	98·5 >·20	98·1 <·05	97·9	
04	146	95·14	<·01	100·3 >·20	99·8 >·20	98·7 >·20	98·8 >·20	98·6 >·20	97·2	98·5 >·20	98·2 >·20	97·3	97·6	
07	104	94·04	<·01	100·8 =·20	100·1 >·20	98·9 ≫·20	99·0 >·20	98·7 >·20	97·0	98·6 >·20	98·2 <·20			
05	184	93·23	<·01	98·3 <·15	98·0 <·15	97·23 <·01	98·7 <·01	97·3 <·01	95·5	97·2 <·01	97·0			

School No.	n	$\widetilde{IQ}$	p	15	25	24	09	18	06	20	16	11
15	88	108·21	<·01	╲								
25	118	105·51	<·01		╲							
24	141	102·45	<·05		103·8	╲						
09	76	101·12	>·20		103·8	102·0 =·01	╲					
18	190	101·03	>·20		102·7	101·6 =·05	101·7 ≫·20	╲				
06	204	100·34	>·20		102·2 <·01	101·2 <·20	100·5	100·7 >·20	╲			
20	128	99·69	>·20	103·4	102·5	101·1 >·20	100·2	100·5	100·1	╲		
16	292	98·84	>·20	101·14 >·20	100·8 >·20	100·0	99·3	99·7	99·5	99·1	╲	
11	135	98·22	<·15	102·4 <·15	101·6 >·20	100·4	99·3 >·20	99·9	99·5	98·9 >·20	98·6 =·20	
26	221	98·08	<·05	101·1 <·05	100·7	99·8	98·9	99·4	99·2	98·7 <·05	98·5 <·05	
12	161	97·76	<·10	101·7 =·20	101·0 >·20	99·9	98·8 >·20	99·5	99·2	98·6 >·20	98·5 <·05	
08	121	97·07	<·05	102·0 <·20	101·2	100·0 >·20	98·6 =·20	99·5	99·1	98·4 >·20	98·3	
13	100	97·00	<·05	102·5 =·01	101·6 <·05	100·2	98·8 <·15	99·6	99·2	98·5 >·20	98·4	
14	198	96·52	<·05	100·3 <·20	98·9	99·0	97·8	98·7 >·20	98·5 >·20	98·1 <·05	97·9	
04	146	95·14	<·01	100·3 >·20	99·8 >·20	98·7 >·20	97·2	98·5 >·20	98·2 >·20	97·3	97·6	
07	104	94·04	<·01	100·8 =·20	100·1 >·20	98·9 ≫·20	97·0	98·6 >·20	98·2 <·20			
05	184	93·23	<·01	98·3 <·15	98·0 <·15	97·23 <·01	95·5	97·2 <·01	97·0			

APPENDIX FIVE

VALUES OF THE PHI COEFFICIENT IN EACH HIERARCHY

Test	Level	Mean ϕ	Median ϕ
Ratio '76	1	.31	.17
	2	.39	.35
	3	.41	.37
	4	.41	.38
Number Operations	1	.16	.16
	2	.31	.31
	3	.38	.37
Decimals & Place Value	1	.32	.29
	2	.36	.35
	3	.49	.48
	4	.40	.39
	5	.46	.45
	6	.43	.41
Fractions 1 &2 (problems)	1	.42	.44
	2	.46	.46
	3	.45	.43
	4	.40	.40
Fractions 3 & 4 (problems)	1	.49	.46
	2	.44	.43
	3	.44	.47
	4	.34	.33
Algebra	1	.28	.28
	2	.42	.38
	3	.44	.43
	4	.40	.40
Rotations & Reflections	1	.36	.33
	2	.30	.32
	3	.42	.37
	4	.33	.33
	5	.37	.36
Positive & Negative Numbers	1	.47	.44
	2	.43	.40
	3	.40	.38
	4	.38	.38
Matrices Core	1	.41	.37
	2	.48	.47
	3	.44	.41
	4	.40	.34
Vectors	1	.37	.37
	2	.45	.42
	3	.44	.42
	4	.45	.44
	5	.44	.42
	6	.36	.33
	7	.31	.30
Graphs	1	.17	.16
	2	.31	.29
	3	.41	.36

VALUES OF THE PHI COEFFICIENT CONTD.

Test	Level	Mean ϕ	Median ϕ
Measurement	1	.38	.36
'77	2	.10	.34
	3	.44	.44
	4	.41	.33

APPENDIX SIX

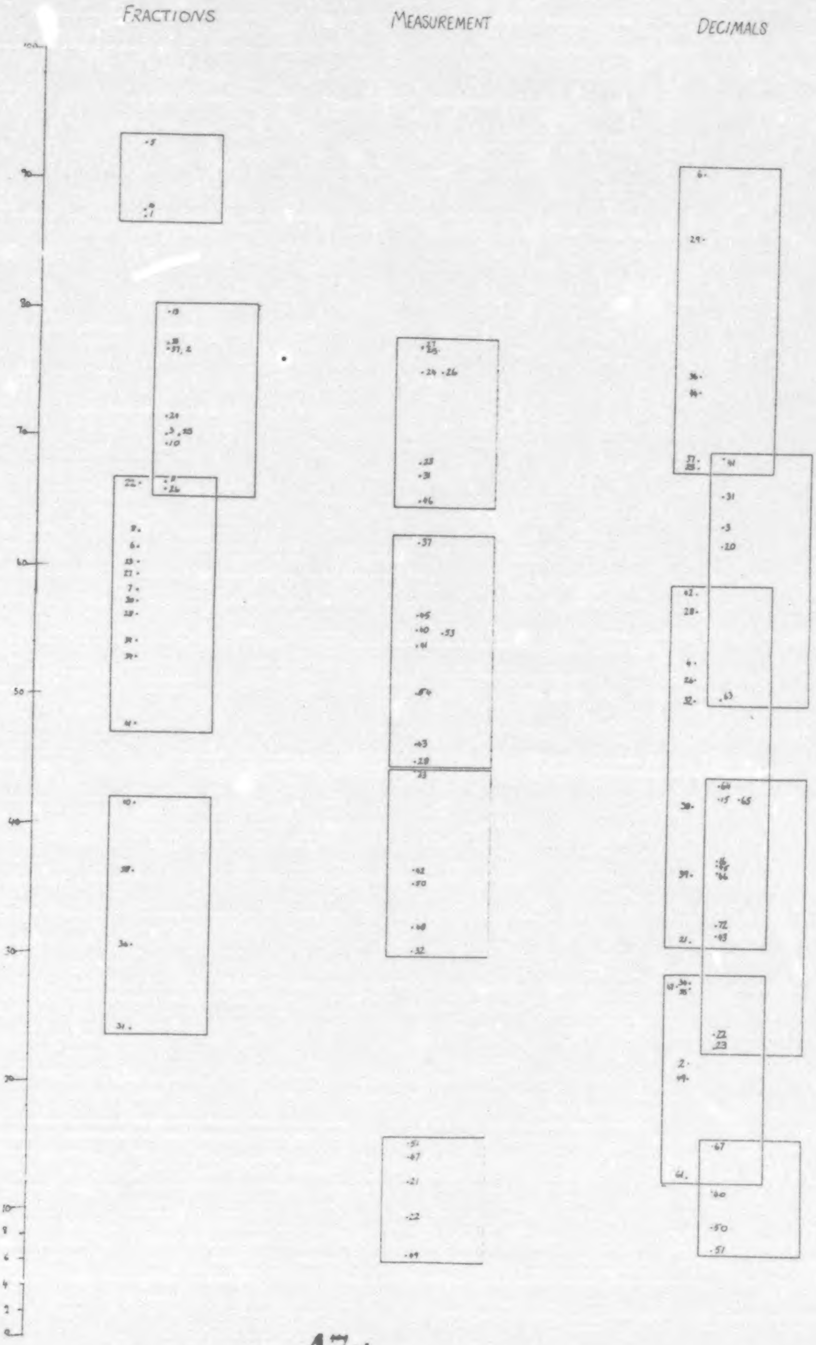
KRUSKAL'S GAMMA TEST / TEST

	Algebra	Ratio	Graphs	Fractions 1, 2	Fractions 3, 4	Measurement	Decimals	Pos. Neg. Nos.
Algebra								
Ratio	.763							
Graphs	.788	.790						
Fractions 1.2	.740	.839						
Fractions 3,4	.778	.853	.739					
Measurement	.709	.790	.795	.629	.664			
Decimals	.765	.800	.733	.810	.802	.605		
Pos. Neg. Nos	.747	.596					.797	
Reflections/ Rotations	.681	.591	.706			.681		.686

APPENDIX SEVEN

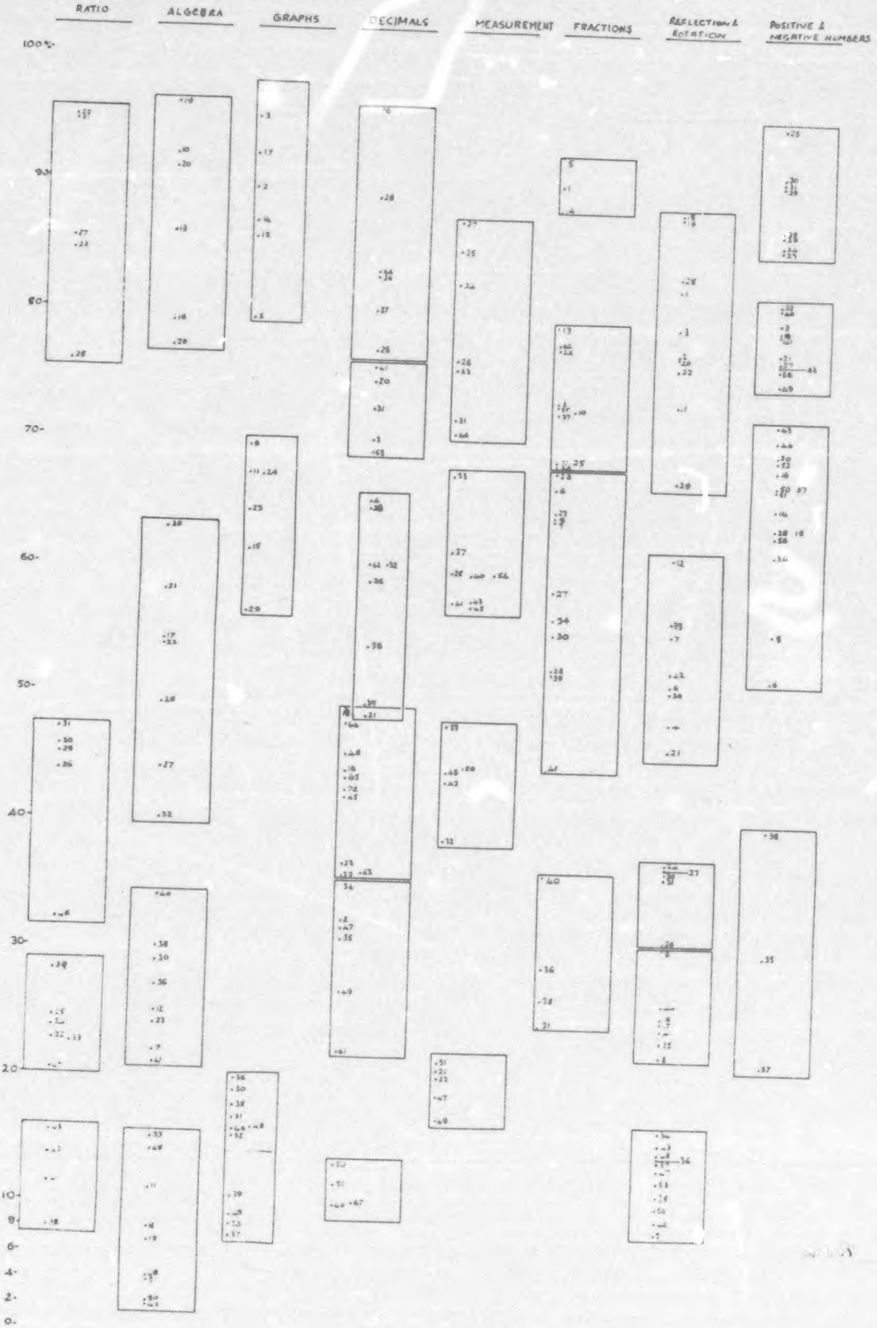
Note: Subtract 20 from each number in the Ratio hierarchy in order to match the variable numbers written on the test paper.

1st YEAR (AGE 12+)



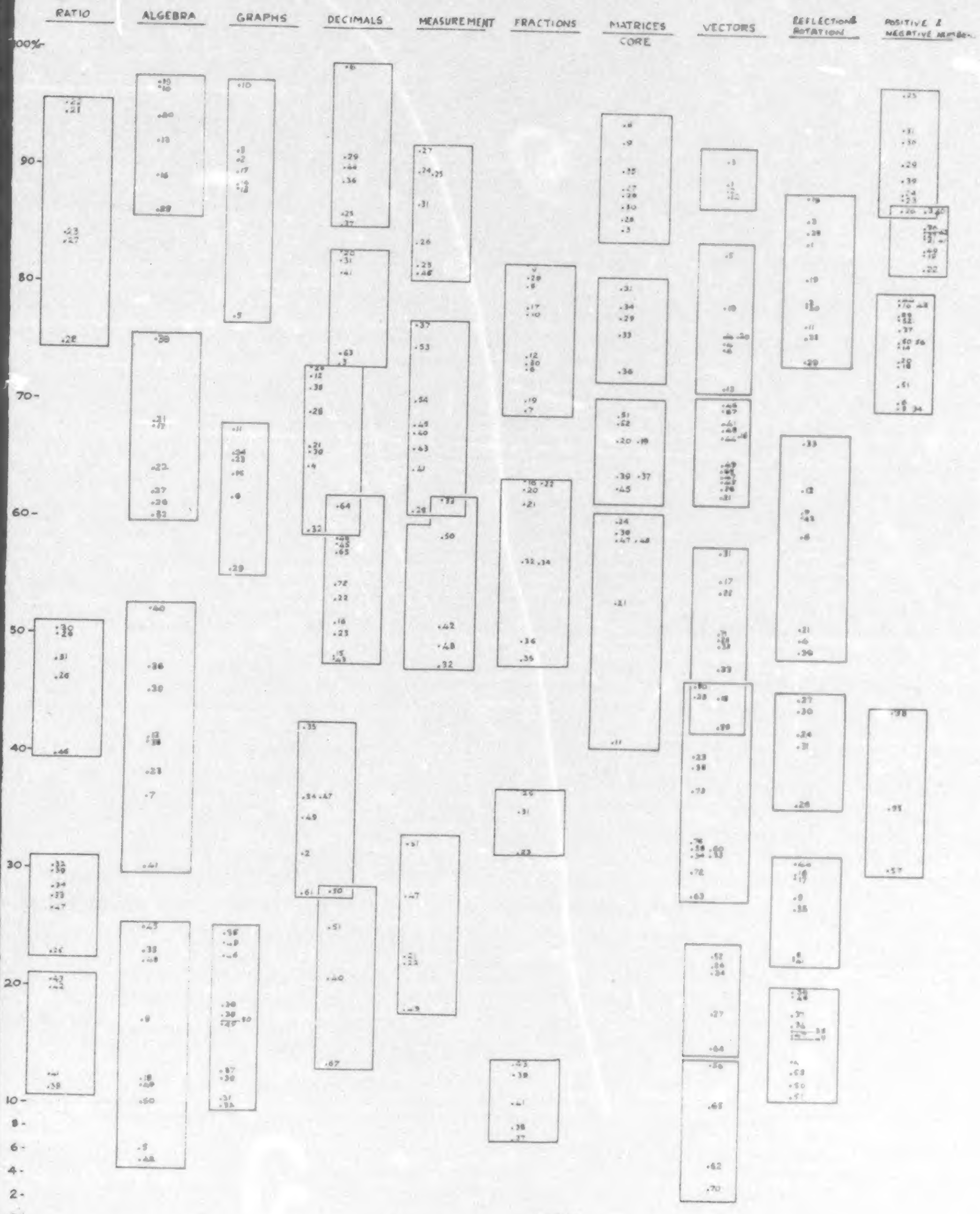
2ND YEAR (AGE 13+)

Note: All numbers are variable numbers



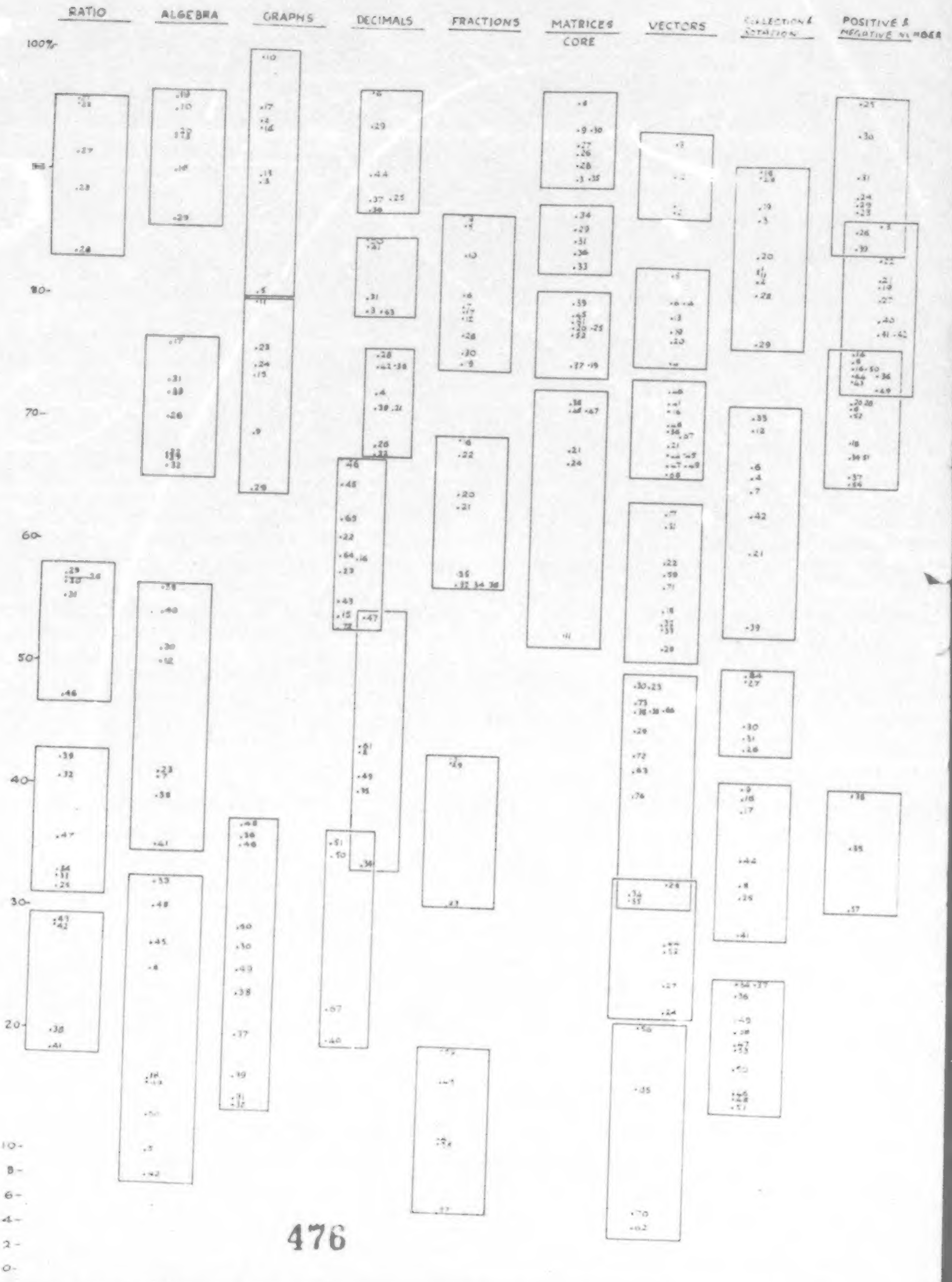
3RD YEAR (AGE 14+)

Note: numbers are variable numbers



4TH YEAR (AGE 15+)

Note: All numbers are variable numbers



APPENDIX SEVEN
(AT END OF BOOK)

Contents of the Topic Hierarchies

Appendix Eight

CSMS(M) Publications List

1974 A.J. Malpas and M. Brown. Cognitive demand and difficulty of GCE O-Level mathematics pre-test items. Br.J.educ.Psychol. 44, 2, 155-162.

An analysis of mathematical items which appear on test papers in terms of Piagetian Cognitive levels of demand.

1976 M. Brown and D.E. Küchemann. "Is it an 'add' Miss?" Mathematics in School, 5, 5, and following issue (part 2).

A description of the interviews and results of testing children in top primary and early secondary classes on the recognition of operations on whole numbers.

1977 D.M. Kerslake. "The Understanding of Graphs", Mathematics in School, 6, 2.

The report of the results of using the Graphs test paper on about 600 Bristol children. This paper summarises Daphne Kerslake's M.Phil. thesis. The sample was enlarged for the CSMS testing but the same test paper was used.

1978 K. Hart. "The Understanding of Ratio in the Secondary School" Mathematics in School, 7, 1, 1978.

A report for teachers of the difficulties children experience when dealing with Ratio and Proportion problems, and the errors they make. The results are from the testing of the CSMS Ratio paper.

1978 K. Hart. "Mistakes in Mathematics", Mathematics Teaching. Winter 1978.

A description for teachers of some of the errors children make, with some reasons for their high occurrence.

1978 D.E. Küchemann. "Children's Understanding of Numerical Variables". Mathematics in School, 7, 4.

A report for teachers of the results of testing 2nd - 4th year secondary pupils on the CSMS Algebra papers.

1978 K. Hart. "Should do Better". The Tablet, Nov.11th 1978.

An article for parents and the non-educationists on aspects of mathematics education which were being debated in 1978. There is an emphasis on the expectations of teachers and parents regarding what children 'should be able to do'.

1978 K. Hart. "The Understanding of Fractions in the Secondary School". Paper given at the second annual conference of the International Group for the Psychology of Mathematics Education, Osnabrück, Sept.1978.

A report given at the Conference of the International Group for the Psychology of Mathematics Education. The contents are basically a description of 1st and 2nd year secondary pupils understanding of fractions (the meaning of fractions and the operations of addition and subtraction). A hierarchy is stated and a comparison made between performance on computations and on problems.

1978 "Cognitive Demand in the Learning of Mathematics". Chapter in Cognitive Development in the School Years by Ann Floyd, Faculty of Educational Studies, The Open University. December 1978.

A chapter in an Open University Book specifically on the demand of mathematics, based on CSMS results.

1978 D.E. Küchemann. "Secondary School Children's Understanding of Reflection and Rotation". Paper given at the second annual conference of the International Group for the Psychology of Mathematics Education, Osnabrück, Sept. 1978.

A paper given at the Conference of the International Group for the Psychology of Mathematics Education detailing the results of the CSMS test paper on Rotations and Reflections. The views children take of the requirements of questions in these aspects of Transformation Geometry.

1979 K. Hart. "Children's Understanding of Mathematics", The Journal for Science and Mathematics Education South East Asia, published by RECSAM, Penang, Jan.1979.

A general description of some of the errors children make when faced with problems in mathematics; the difficulties we have found and the over estimation of the child's capability by the teacher.

1979 K. Hart. "Mathematics Research of CSMS", Mathematics Education for Teaching, 1979.

A general description of the aims and methods of the CSMS Maths research.

1979 K. Hart. "Towards a Hierarchy of Understanding in Mathematics". I.G.P.M.E. Conference, Warwick, July 1979.

A paper presented at the July 1979 meeting of I.G.P.M.E. which demonstrates the hierarchies in Ratio and Fractions. A comparison is made between these two aspects of the same topic. Stage 1 types of item (the easiest) are described for all topics researched by CSMS.

1979 K. Hart. "A Hierarchy of Understanding in Mathematics". Conference on Cognitive Development, Leeds, Sept.1979.

A paper presented at the September 1979 Conference. The hierarchies of three topics are described together with a brief description of the CSMS maths research. Stage 4 (the hardest) items are described across all topics.

1979 K. Hart. "From Whole Numbers to Fractions and Decimals", Journal of I.R.E.M., Orleans, France.

An article published in English in a French Mathematics education book. The content is based on CSMS results dealing purely with number. It emphasises the great gap between being able to operate with small whole numbers and being able to deal with large numbers, fractions and decimals.

1979 K. Hart and M.McCartney. "Item Analysis Procedures for the Classification of Test Items. Working paper.

A description of the form of analysis from the CSMS testing in order to arrive at hierarchies. The homogeneity coefficients used are defined and the rationale for the method is outlined.

1980 Mathematics Test Papers with teachers' guides. CSMS. To be published by NFER.

1980 Children's understanding of mathematics (11-16). CSMS Maths team. Editor K. Hart; published by Murray , London.

A book for teachers which describes the results of the CSMS mathematics research. A chapter is devoted to each topic investigated; the levels of the hierarchy, methods used by children and the errors they make are detailed. The first chapter describes the research methodology and a general chapter gives an overview of the topic hierarchies and their interrelation. The final section deals with implications for teaching.

1980 Dietmar Küchemann. "Children's Difficulties with Single Reflections and Rotations." Mathematics in School, Vol.9, No.2.

1980 Dietmar Küchemann. "Children's Understanding of Integers". Mathematics in School, Vol.9, No.2.